## Complexity driven collapse of economic equilibria

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# Micro(economics) — Macro(economics)

- \* General problem: aggregating microeconomic behaviour and interactions between economic agents into macroeconomic laws
- \* Specific problem: understanding the macroeconomic behaviour of modern industrialized economies
- \* Input-output analysis: understanding the linkages and mutual impact between different productive entities

output from sector / firm  $A \longrightarrow$  input to sector / firm B

# Input-output analysis (I)

### Pre-industrial era: Tableaux économiques (1758)



Figure: François Quesnay (1694-1774)

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# Input-output analysis (II)

\* Economies have evolved to remarkable levels of complexity: the production of technologically sophisticated objects involves multiple production processes feeding each other, often delocalized / outsourced across multiple firms





★ Modern input-output models are written in the language of General Equilibrium (GE) Theory

profit maximizing firms	$\overleftrightarrow$	utility maximizing consumers		
market prices				

# A GE input-output model (I)

#### Ingredients:

- $\star \ C \ {\rm goods}$ 
  - Raw goods:  $x_0^c = 1$
  - Consumer goods:  $k^c = 1$



- $\star$  N technologies / firms
  - $-q_i^c < 0$ : good c is an input to technology i
  - $-q_i^c > 0$ : good c is an output of technology i

# A GE input-output model (II)

Goal: determine an equilibrium price  $p^c$  for each good such that

- ★ Firms maximize profits by optimizing their scales of production s<sub>i</sub> ≥ 0 ("intensity" at which technologies are run)
- $\star$  Consumers maximize their utility function (consumption of consumer goods) given a budget constraint

Problems of standard GE approach

- \* No heterogeneity (representative consumer)
- $\star$  Reliance on precise knowledge of the economy's input-output matrix  $q_i^c$
- $\star$  Analytically intractable even for very small numbers of goods and firms

## GE meets Complexity

\* Solution: "promoting" interactions in the economy to random variables

#### Systems with random interactions

F. Dyson: "What is here required is a new kind of statistical mechanics, in which we renounce exact knowledge not of the state of the system but of the system itself."

- \* Methods: averaging over ensemble of economies in the limit  $N, C \rightarrow \infty$  with n = N/C finite  $\longrightarrow$  Averaging over all possible input-output matrices and all possible representative consumers
- \* Benefits of the approach
  - 1) Analytical access to the economy's typical properties
  - Solution expressed as interaction between a representative consumer and representative firm — "Microscopic" justification for representative consumer assumption

## The economy's phase diagram



- Result: Transition from no-industrialization trap to industrial production happens without need to invoke a "big push"
- \* Result: introduction of new technologies has different impacts in different regimes

## Economic expansion through intermediate goods

- \* Economies can expand via outsourcing / externalization, i.e. the creation of new markets for intermediate goods
- ★ Increase in the number of firms → increase in the number of interactions through market prices → increase in complexity!



## Complexity driven collapse of equilibria



 $\star~\phi =$  fraction of active technologies / firms

- $\star~\mathcal{I}=$  fraction of intermediate goods
- \* Result: expansion of the economy via externalization / outsourcing leads to a sudden shutdown after a critical threshold is exceeded

## Remarks

- Full solution of GE in a simple input-output setting
- \* NOT our predictions  $\rightarrow$  GE's predictions!
- Analytical treatment of combinatorial phase transition in high-dimensional geometry (see Donoho and Tanner (2009))
- \* Similar phase transition describes emergence of arbitrage opportunities from heterogeneity in pricing models
  - M. Bardoscia, G. L., M. Marsili, J. Stat. Mech. (2012)
- Preprint online at http://arxiv.org/abs/1511.09203

#### Figure 4 Greece: Real GDP (2005=100)



Sources: IMF; EC; authors' calculations