# Complexity driven collapse of economic equilibria

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CoSyDy Workshop – Queen Mary University London

July 6, 2016

# Micro(economics) $\longrightarrow$ Macro(economics)

- ⋆ General problem: aggregating microeconomic behaviour and interactions between economic agents into macroeconomic laws

- ⋆ Specific problem: understanding the macroeconomic behaviour of modern industrialized economies

- ⋆ Input-output analysis: understanding the linkages and mutual impact between different productive entities

output from sector / firm $A$ $\longrightarrow$ input to sector / firm $B$

# Input-output analysis (I)

## Pre-industrial era: Tableaux économiques (1758)

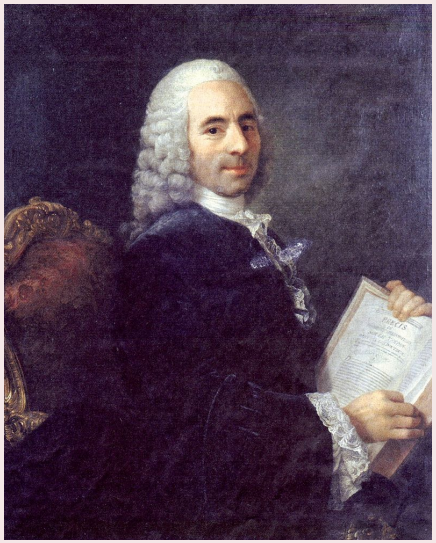


Figure: François Quesnay (1694-1774)

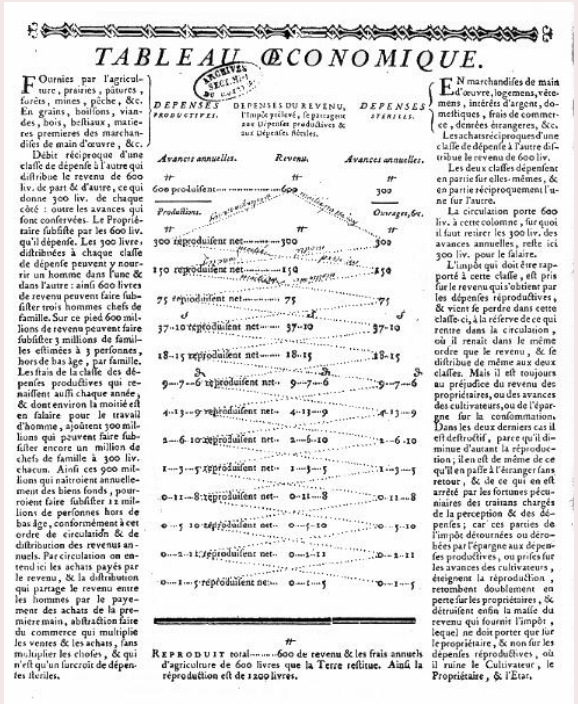


TABLEAU ŒCONOMIQUE.

# Input-output analysis (II)

⋆ Economies have evolved to remarkable levels of complexity: the production of technologically sophisticated objects involves multiple production processes feeding each other, often delocalized / outsourced across multiple firms

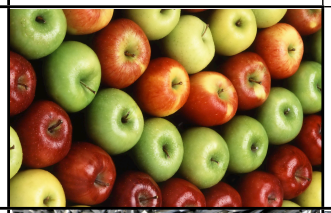
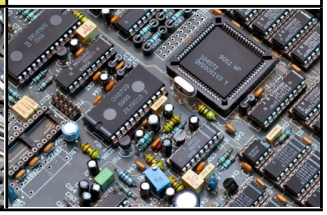

⋆ Modern input-output models are written in the language of General Equilibrium (GE) Theory

$$\text{profit maximizing firms} \underbrace{\longleftrightarrow}_{\text{market prices}} \text{utility maximizing consumers}$$

# A GE input–output model (I)

Ingredients:

- ⋆ $C$ goods
  - Raw goods: $x_0^c = 1$
  - Consumer goods: $k^c = 1$

| | $x_0^c = 1$ | $x_0^c = 0$ |
|---|---|---|
| $k^c = 1$ |  |  |
| $k^c = 0$ |  |  |

- ⋆ $N$ technologies / firms
  - $q_i^c < 0$: good $c$ is an input to technology $i$
  - $q_i^c > 0$: good $c$ is an output of technology $i$

# A GE input–output model (II)

Goal: determine an equilibrium price $p^c$ for each good such that

- ⋆ Firms maximize profits by optimizing their scales of production $s_i \geq 0$ ("intensity" at which technologies are run)
- ⋆ Consumers maximize their utility function (consumption of consumer goods) given a budget constraint

Problems of standard GE approach

- ⋆ No heterogeneity (representative consumer)
- ⋆ Reliance on precise knowledge of the economy's input–output matrix $q_i^c$
- ⋆ Analytically intractable even for very small numbers of goods and firms

# GE meets Complexity

⋆ Solution: "promoting" interactions in the economy to random variables

**Systems with random interactions**

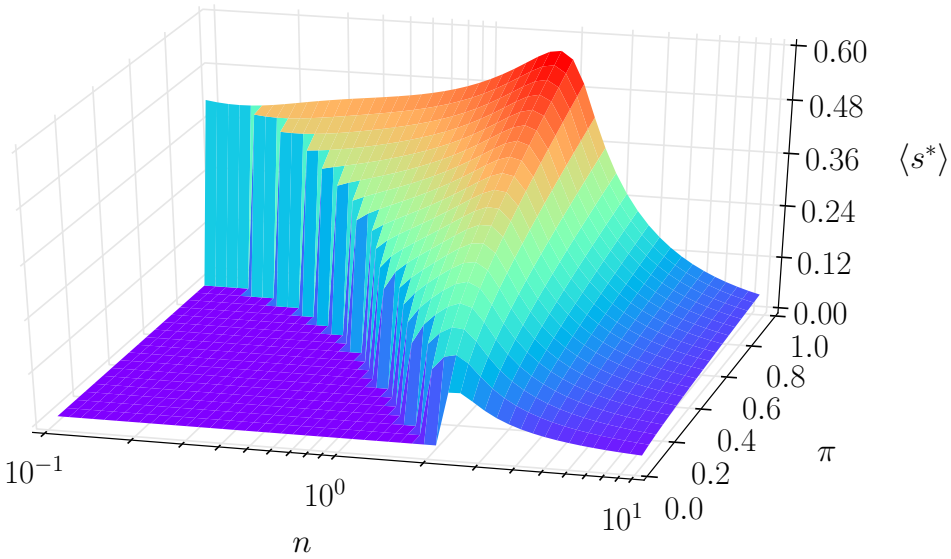
F. Dyson: *"What is here required is a new kind of statistical mechanics, in which we renounce exact knowledge not of the state of the system but of the system itself."*

⋆ Methods: averaging over ensemble of economies in the limit $N, C \to \infty$ with $n = N/C$ finite $\longrightarrow$ Averaging over all possible input–output matrices and all possible representative consumers

⋆ Benefits of the approach

1) Analytical access to the economy's typical properties
2) Solution expressed as interaction between a representative consumer and representative firm $\longrightarrow$ "Microscopic" justification for representative consumer assumption

# The economy's phase diagram

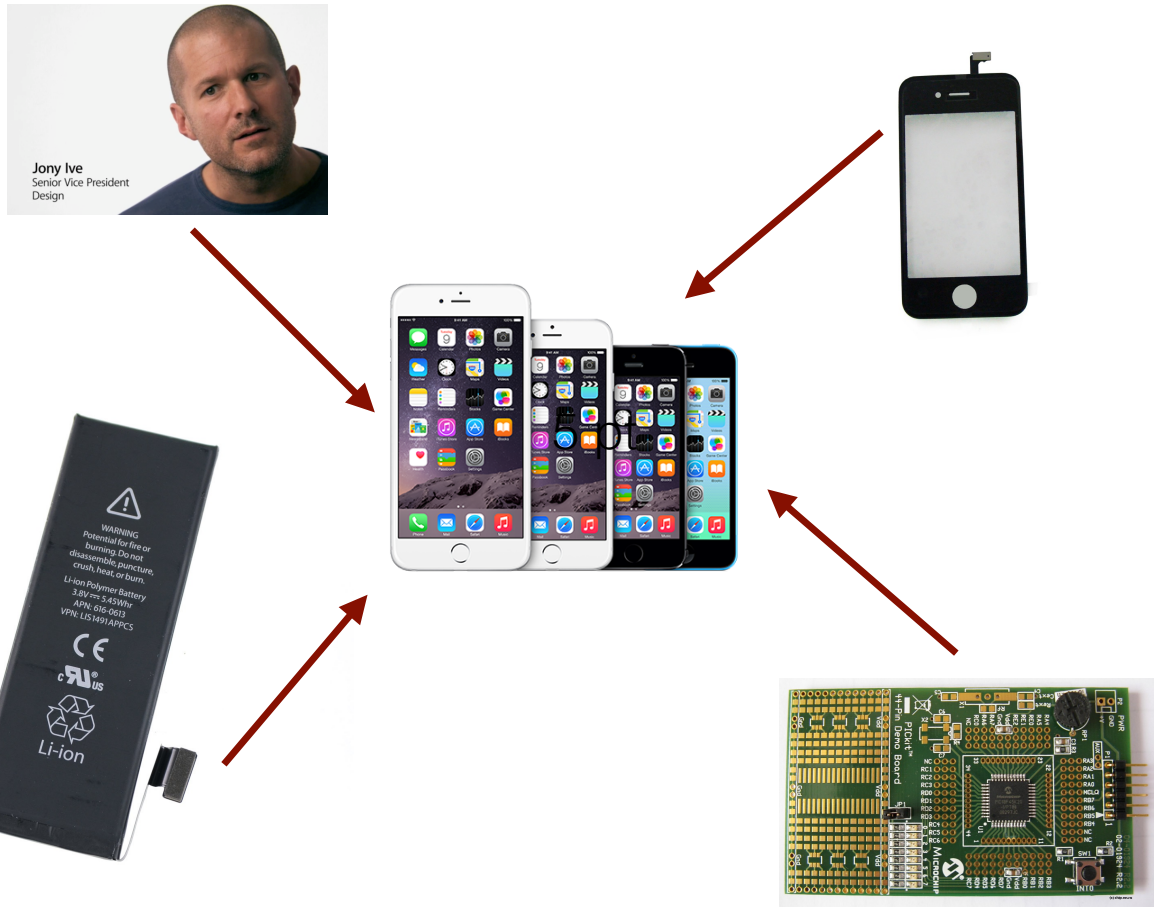



- $n = \frac{N}{C} = \frac{\#\text{ of technologies}}{\#\text{ of goods}}$
- $\pi = \frac{\#\text{ raw goods}}{\#\text{ of goods}}$
- $\langle s^* \rangle =$ average scale of production

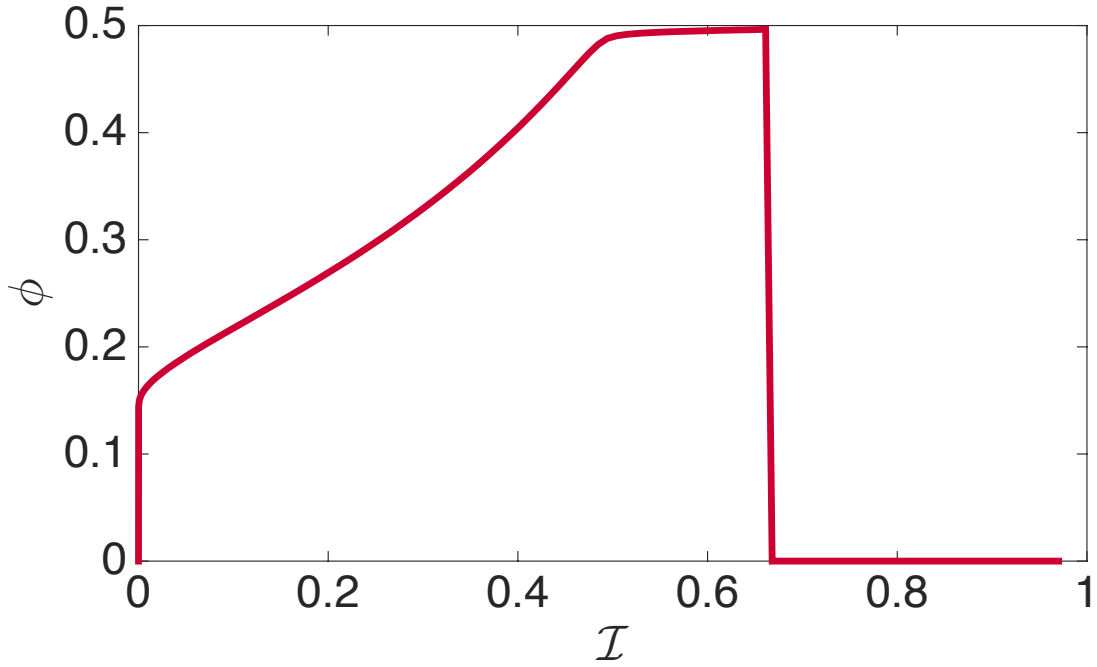
- Result: Transition from no-industrialization trap to industrial production happens without need to invoke a "big push"
- Result: introduction of new technologies has different impacts in different regimes

# Economic expansion through intermediate goods

- ⋆ Economies can expand via outsourcing / externalization, i.e. the creation of new markets for intermediate goods
- ⋆ Increase in the number of firms $\longrightarrow$ increase in the number of interactions through market prices $\longrightarrow$ increase in complexity!



# Complexity driven collapse of equilibria



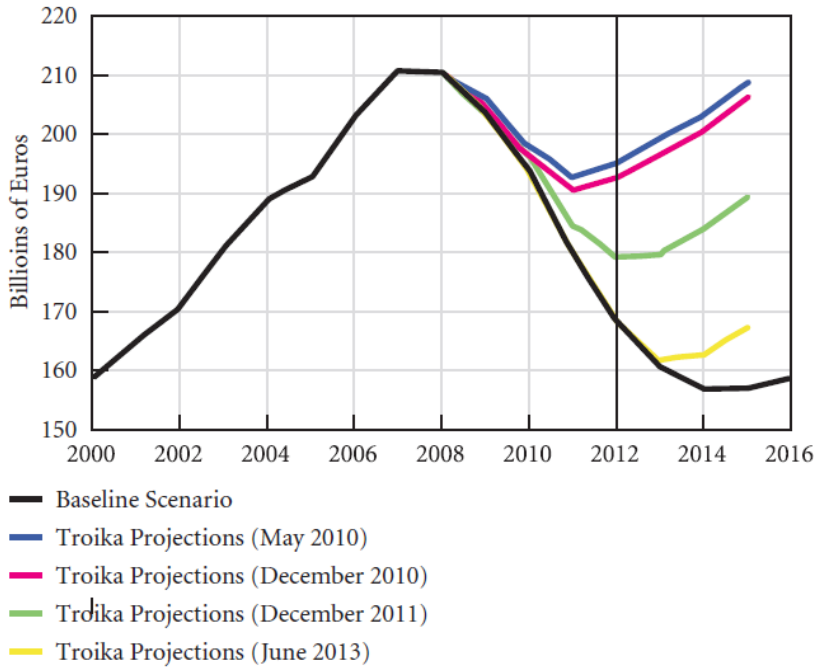

- $\phi =$ fraction of active technologies / firms
- $\mathcal{I} =$ fraction of intermediate goods
- Result: expansion of the economy via externalization / outsourcing leads to a sudden shutdown after a critical threshold is exceeded

# Remarks

- ⋆ Full solution of GE in a simple input-output setting
- ⋆ NOT our predictions → GE's predictions!
- ⋆ Analytical treatment of combinatorial phase transition in high-dimensional geometry (see Donoho and Tanner (2009))
- ⋆ Similar phase transition describes emergence of arbitrage opportunities from heterogeneity in pricing models

  M. Bardoscia, G. L., M. Marsili, *J. Stat. Mech.* (2012)
- ⋆ Preprint online at http://arxiv.org/abs/1511.09203

**Figure 4** Greece: Real GDP (2005=100)




*Sources:* IMF; EC; authors' calculations