Multi-relational social dynamics:

interactions, opinion formation and the dissemination of cultures

Federico Battiston

School of Mathematical Sciences, Queen Mary University of London, UK

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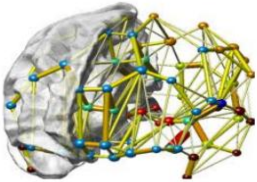
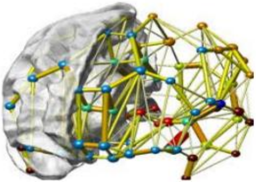
Many systems, one framework

adjacency matrix $A = \{a_{ij}\}$

Many systems, one framework

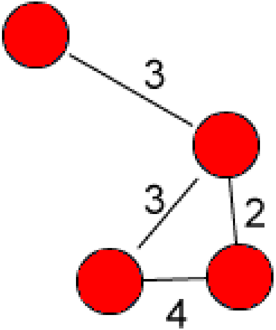




adjacency matrix $A = \{a_{ij}\}$

node degree $k_i = \sum_j a_{ij}$

Towards a richer architecture: weighted networks



Weighted adjacency matrix $W = \{w_{ij}\}$

Weights are used to represent strength, distance, cost, time, ...

General formalism for multiplex networks

A multiplex is a system whose basic units are connected through a variety of different relationships. Links of different kind are embedded in different layers.

- Node index $i = 1, \dots, N$
- Layer index $\alpha = 1, \dots, M$

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For each layer α:

- adjacency matrix $A^{[\alpha]} = \{a_{ij}^{[\alpha]}\}$
- node degree $k_i^{[\alpha]} = \sum_j a_{ij}^{[\alpha]}$

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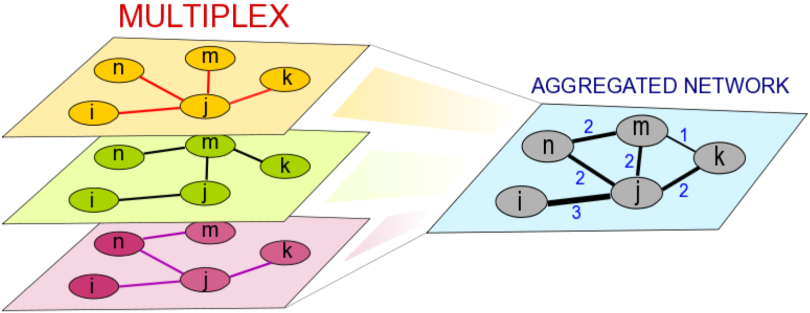
For the multiplex:

- vector of adjacency matrices $\mathbf{A} = \{A^{[1]}, ..., A^{[M]}\}$.
- vector of degrees $\mathbf{k}_i = (k_i^{[1]}, ..., k_i^{[M]})$.

Do we really need to preserve all this information?.

What are we losing collapsing all the information into a single network?



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Multilayer networks

Mikko Kivelä
Oxford Centre for Industrial and Applied Mathematics, Mathematical Institute, University of Oxford, Oxford OX2 6GG, UK

Alex Arenas
Departament d'Enginyeria Informática i Matemátiques, Universitat Rovira I Virgili, 43007 Tarragona, Spain

Marc Barthelemy
Institut de Physique Théorique, CEA, CNRS-URA 2306, F-91191, Gif-sur-Yvette, France and Centre d'Analyse et de Mathématiques Sociales, EHESS, 190-198 avenue de France, 75244 Paris, France

James P. Gleeson
MACSI, Department of Mathematics & Statistics, University of Limerick, Limerick, Ireland

Yamir Moreno
Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, Zaragoza 50018, Spain and Department of Theoretical Physics, University of Zaragoza, Zaragoza 50009, Spain

and

Mason A. Porter†
Oxford Centre for Industrial and Applied Mathematics, Mathematical Institute, University of Oxford, Oxford OX2 6GG, UK and CABDyN Complexity Centre, University of Oxford, Oxford OX1 1HP, UK

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ELSEVIER

The structure and dynamics of multilayer networks

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S. Boccaletti [a,b,*], G. Bianconi [c], R. Criado [d,e], C.I. del Genio [f,g,h], J. Gómez-Gardeñes [i], M. Romance [d,e], I. Sendiña-Nadal [j,e], Z. Wang [k,l], M. Zanin [m,n]

[a] CNR - Institute of Complex Systems, Via Madonna del Piano, 10, 50019 Sesto Fiorentino, Florence, Italy
[b] The Italian Embassy in Israel, 25 Hamered st., 68125 Tel Aviv, Israel
[c] School of Mathematical Sciences, Queen Mary University of London, London, United Kingdom
[d] Departamento de Matemática Aplicada, Universidad Rey Juan Carlos, 28933 Móstoles, Madrid, Spain
[e] Center for Biomedical Technology, Universidad Politécnica de Madrid, 28223 Pozuelo de Alarcón, Madrid, Spain
[f] Warwick Mathematics Institute, University of Warwick, Gibbet Hill Road, Coventry CV4 7AL, United Kingdom
[g] Centre for Complexity Science, University of Warwick, Gibbet Hill Road, Coventry CV4 7AL, United Kingdom
[h] Warwick Infectious Disease Epidemiology Research (WIDER) Centre, University of Warwick, Gibbet Hill Road, Coventry CV4 7AL, United Kingdom
[i] Institute for Biocomputation and Physics of Complex Systems, University of Zaragoza, Zaragoza, Spain
[j] Complex Systems Group, Universidad Rey Juan Carlos, 28933 Móstoles, Madrid, Spain
[k] Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong Special Administrative Region
[l] Center for Nonlinear Studies, Beijing–Hong Kong–Singapore Joint Center for Nonlinear and Complex Systems (Hong Kong) and Institute of Computational and Theoretical Studies, Hong Kong Baptist University, Kowloon Tong, Hong Kong Special Administrative Region
[m] Innaxis Foundation & Research Institute, José Ortega y Gasset 20, 28006 Madrid, Spain
[n] Faculdade de Ciências e Tecnologia, Departamento de Engenharia Electrotécnica, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal

MULTIPLEX NETWORKS

STRUCTURE

Basic measures

Community structure

Core-periphery structure

DYNAMICS

Random walks

Opinion formation

Cultural dynamics

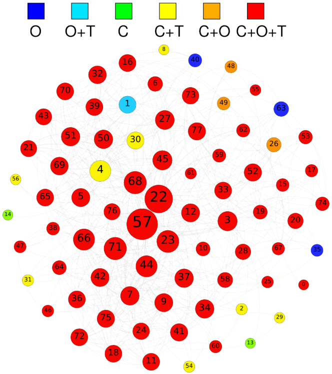
Evolutionary game theory

APPLICATIONS

The human brain

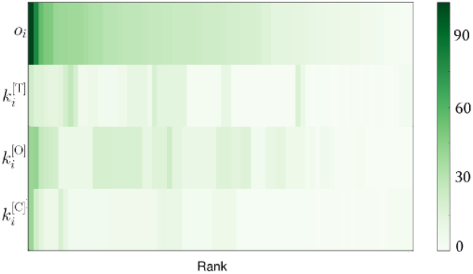
The multi-layer network of Indonesian terrorists

LAYER	CODE	N	K
MULTIPLEX	M	78	911
Trust	T	70	259
Operations	O	68	437
Communications	C	74	200
Businness	B	13	15



A layer-by-layer exploration of node properties: the case of the degree distribution.

overlapping degree: $o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}$



Different layers show different patterns.

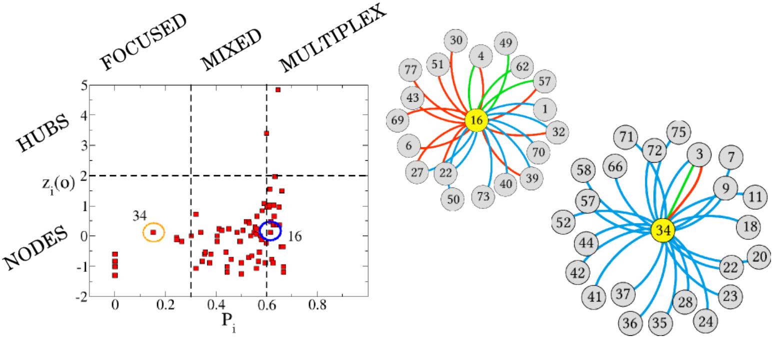
Basic node properties: cartography of a multiplex

Z-score of the overlapping degree: $z_i(o) = \frac{o_i - <o>}{\sigma_o}$ $\quad o_i = \sum_{\alpha=1}^{M} k_i^{[\alpha]}$

1. Simple nodes $\quad -2 \leq z_i(o) \leq 2$
2. Hubs $\quad z_i(o) > 2$

Participation coefficient: $P_i = \frac{M}{M-1}\left[1 - \sum_{\alpha=1}^{M}\left(\frac{k_i^{[\alpha]}}{o_i}\right)^2\right]$

1. Focused nodes $\quad 0 \leq P_i \leq 1/3$
2. Mixed-pattern nodes $\quad 1/3 < P_i \leq 2/3$
3. Truly multiplex nodes $\quad 2/3 < P_i \leq 1$



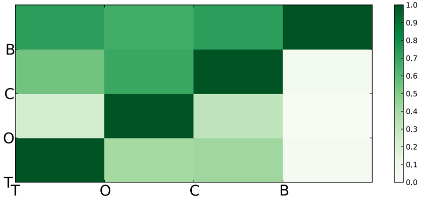
Multiplex analysis successfully distinguishes node 16 from node 34.

F. Battiston, V. Nicosia, V. Latora (2014)

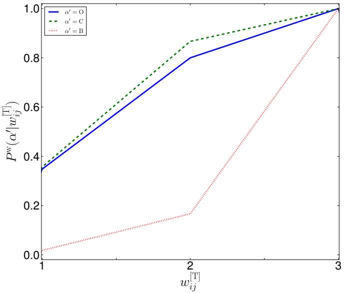
o_{ij}	Percentage of edges (%)
1	46
2	27
3	23
4	4

Conditional probability to have overlap:

$$P(a_{ij}^{[\alpha']}|a_{ij}^{[\alpha]}) = \frac{\sum_{ij} a_{ij}^{[\alpha']} a_{ij}^{[\alpha]}}{\sum_{ij} a_{ij}^{[\alpha]}} \tag{1}$$

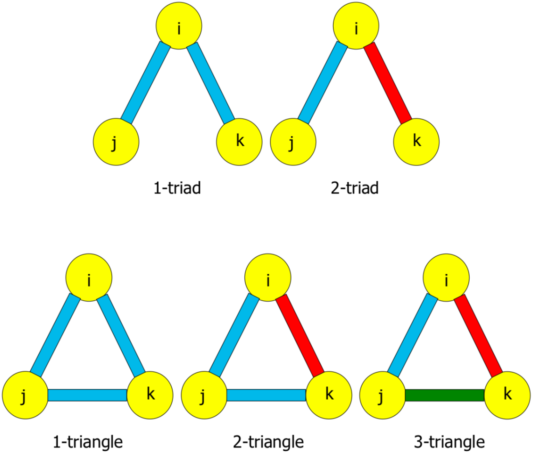


$$P(a_{ij}^{[\alpha']}|a_{ij}^{[\alpha]}) \rightarrow P^{\mathrm{w}}(a_{ij}^{[\alpha']}|w_{ij}^{[\alpha]})$$



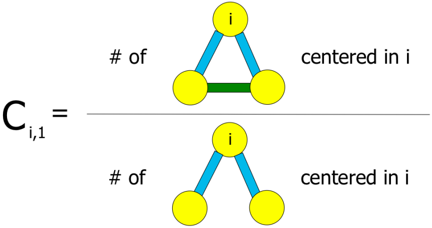
The existence of strong connections in the Trust layer, which represents the strongest relationships between two people, actually fosters the creation of links in other layers.

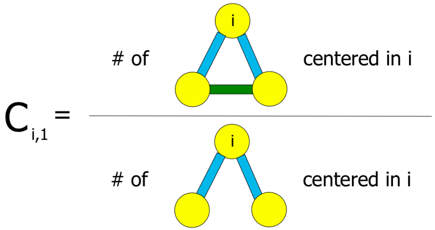
Triads and triangles

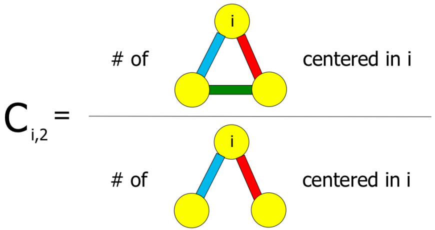


F. Battiston, V. Nicosia, V. Latora (2014)

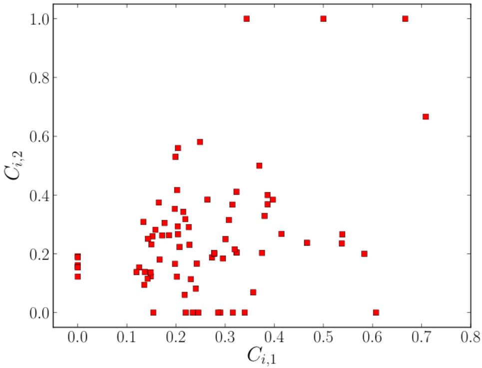
Clustering

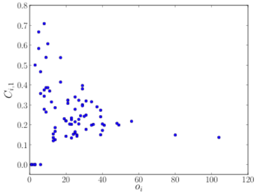


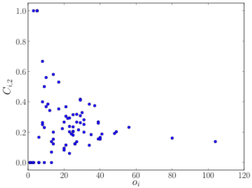

C
i,1
=
of
i
centered in i
of
i
centered in i


C
i,2
=
of
i
centered in i
of
i
centered in i

$C_{i,1}$ and $C_{i,2}$ show different patterns of multi-clustering and are not correlated with o_i.

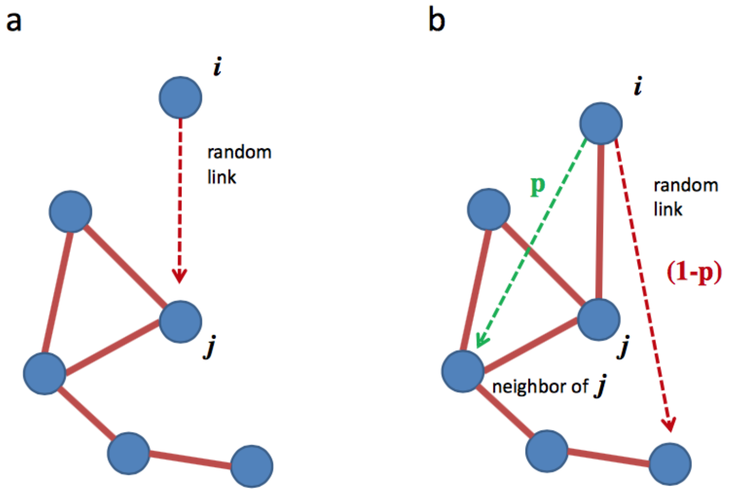






Communities and triadic closure

At each time step a new node attaches with 2 links:
a) the first link is at random
b) the second link closes a triangle with probability p



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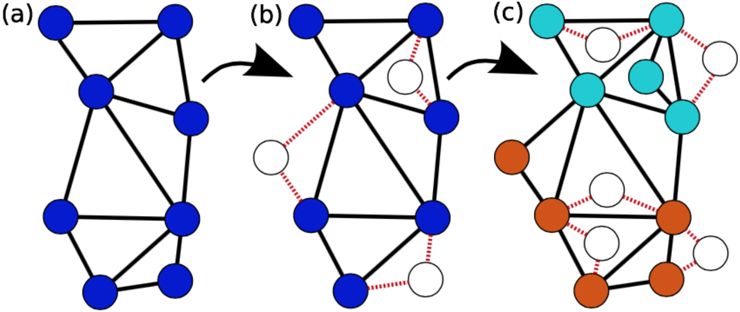
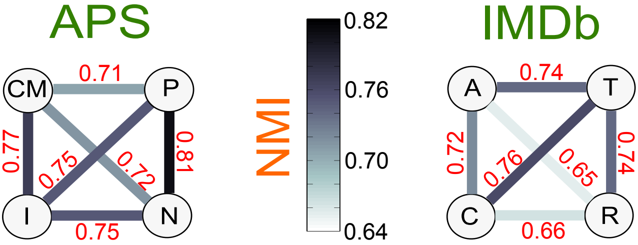


G. Bianconi et al., *Physical Review E* (2014)

APS: Particle (P), Nuclear (N), Condensed Matter (CM) and Interdisciplinary (I) physics

IMDb: Action (A), Crime (C), Thriller (T) and Romance (R) genres



Different layers may have more or less similar community structure

APS: Particle (P), Nuclear (N), Condensed Matter (CM) and Interdisciplinary (I) physics

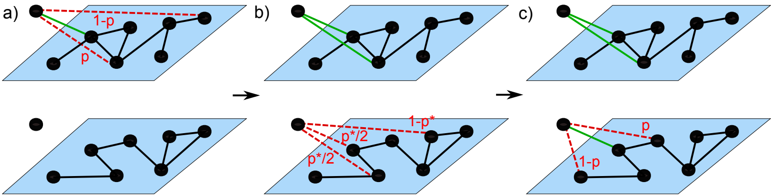
IMDb: Action (A), Crime (C), Thriller (T) and Romance (R) genres

$$NMI(\mathcal{P}_\alpha, \mathcal{P}_\beta) = \frac{-2\sum_{m=1}^{M_\alpha}\sum_{m'=1}^{M_\beta} N_{mm'} \log\left(\frac{N_{mm'}N}{N_m N_{m'}}\right)}{\sum_{m=1}^{M_\alpha} N_m \log\left(\frac{N_m}{N}\right) + \sum_{m'=1}^{M_\beta} N_{m'} \log\left(\frac{N_{m'}}{N}\right)}$$

L. Danon et al., *Journal of Statistical Mechanics: Theory and Applications* (2015)

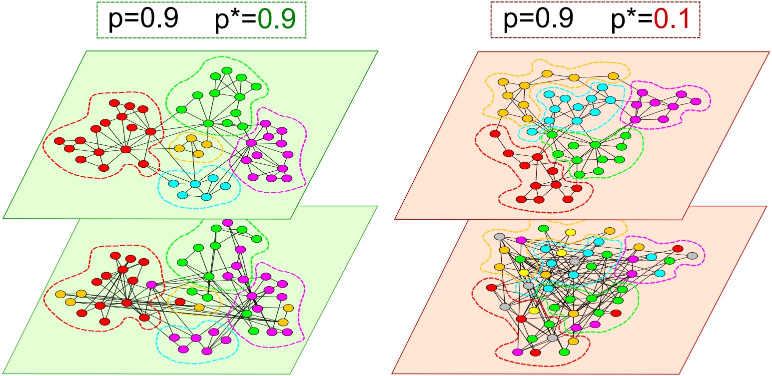
Growing models for multiplexes with communities

Real mechanisms by which collaborations grow:
1) 'intra-layer' triadic closure (with prob. p)
2) 'inter-layer' proximity bias (with prob. p^*)

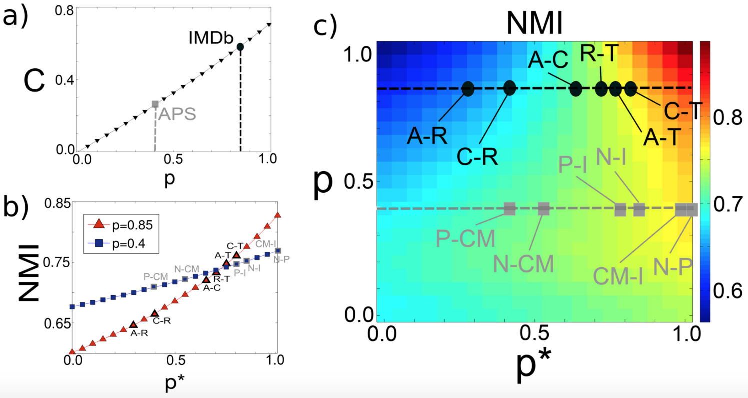


F. Battiston, J. Iacovacci et al., (2016)

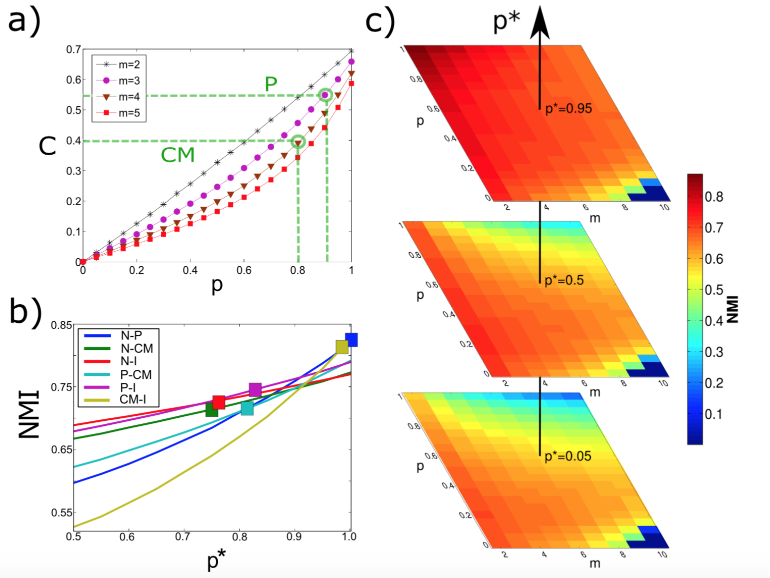
Growing models for multiplexes with communities

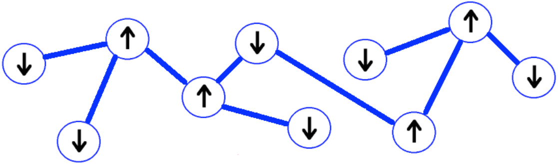


By tuning the strength of the 'inter-layer' proximity bias mechanism we can obtain similar ($p^* = 0.9$) or different ($p^* = 0.1$) community structures


a)
C
0.8
0.4
0.0
IMDb
APS
0.0
0.5
1.0
p
b)
NMI
0.85
0.75
0.65
p=0.85
p=0.4
C-T
CM-I
A-T
N-P
N-CM
P-CM
R-T
P-I
N-I
A-C
C-R
A-R
0.0
0.5
1.0
p*
c)
NMI
1.0
0.5
0.0
p
A-C
R-T
C-T
A-R
A-T
C-R
P-I
N-I
P-CM
N-CM
CM-I
N-P
0.8
0.7
0.6
0.0
0.5
1.0
p*

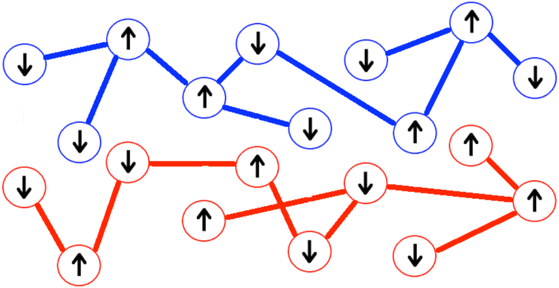
General model





TOPIC 1

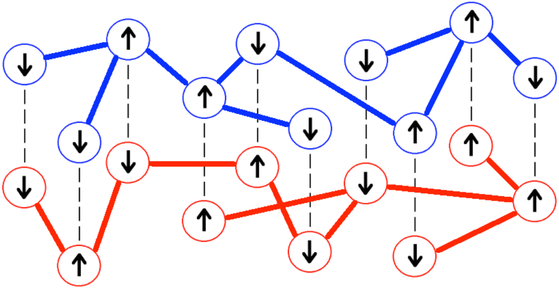
F. Battiston, A. Cairoli, et al. (2016)



TOPIC 1

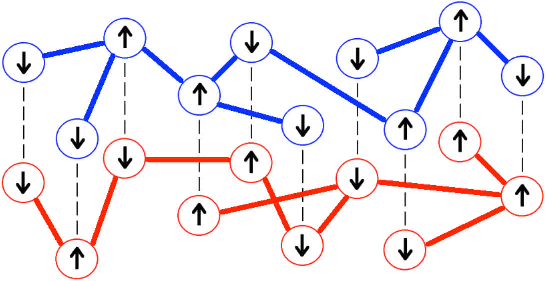
TOPIC 2

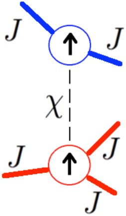
F. Battiston, A. Cairoli, et al. (2016)



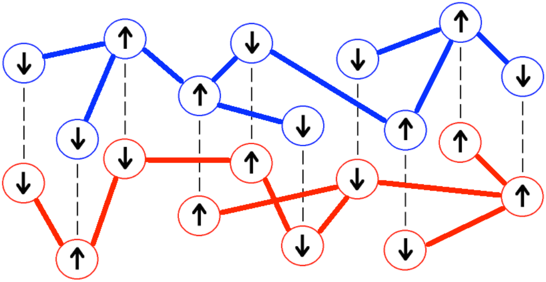
TOPIC 1

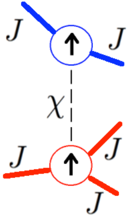
TOPIC 2



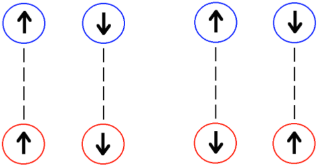


J
J
χ
J
J
J

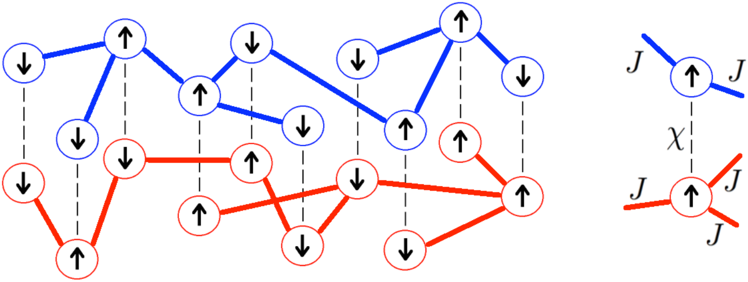




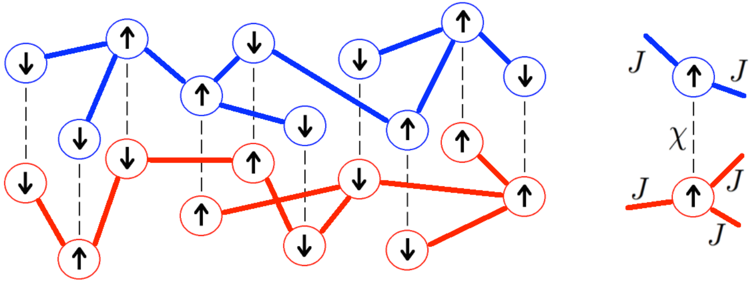
$$\chi > 0$$



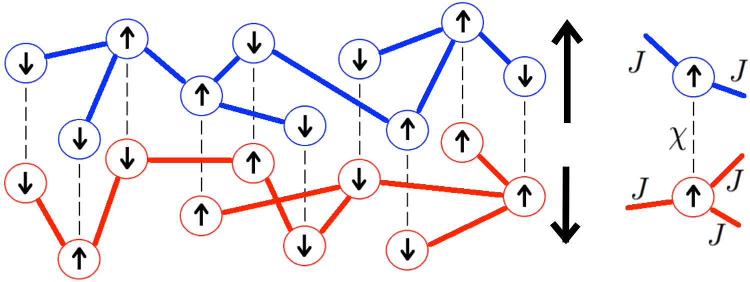
coherent incoherent



$$f_i^{[\alpha]} = J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma \frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{M} s_i^{[\beta]}$$



$$f_i^{[\alpha]} = \underbrace{J\sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]}}_{\text{peer pressure}} + h^{[\alpha]} + \gamma\frac{\chi_i}{J}\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M} s_i^{[\beta]}$$



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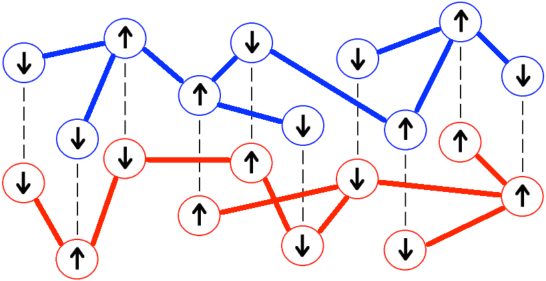
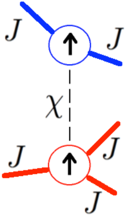
peer pressure

media

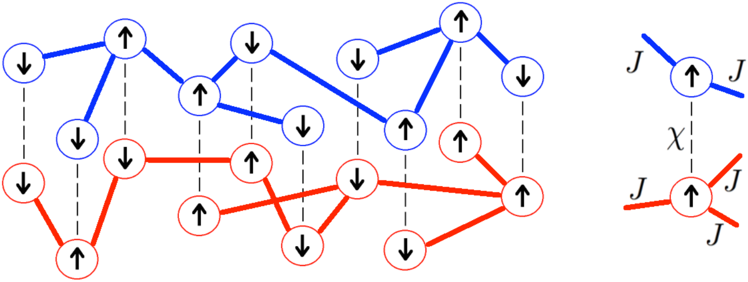
(intra-layer)





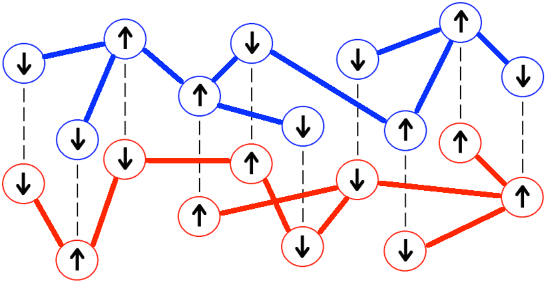
$$f_i^{[\alpha]} = \underbrace{J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]}}_{\text{peer pressure}} + \underbrace{h^{[\alpha]}}_{\text{media}} + \underbrace{\gamma \frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta\neq\alpha}}^{M} s_i^{[\beta]}}_{\text{coupling}}$$

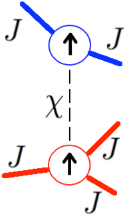
(intra-layer) (inter-layer)



$$f_i^{[\alpha]} = J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma \cancel{\frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta\neq\alpha}}^{M} s_i^{[\beta]}}$$

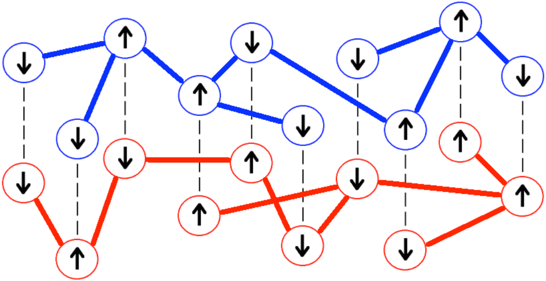
$$\gamma \simeq 0$$

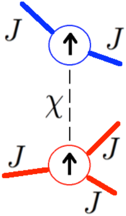




$$f_i^{[\alpha]} = \cancel{J\sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]}} + h^{[\alpha]} + \gamma\frac{\chi_i}{J}\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M} s_i^{[\beta]}$$

$$\gamma \to \infty$$

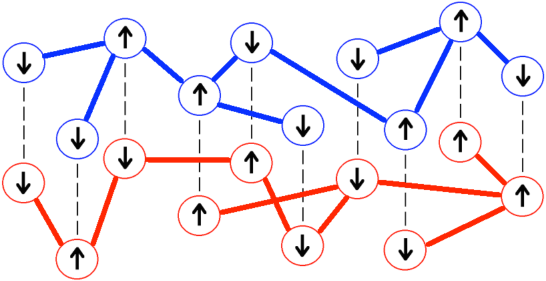


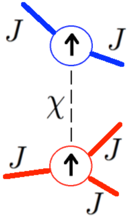


$$f_i^{[\alpha]} = J\sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma\frac{\chi_i}{J}\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M} s_i^{[\beta]}$$

maximize

$$F_i^{[\alpha]} = s_i^{[\alpha]} f_i^{[\alpha]}$$





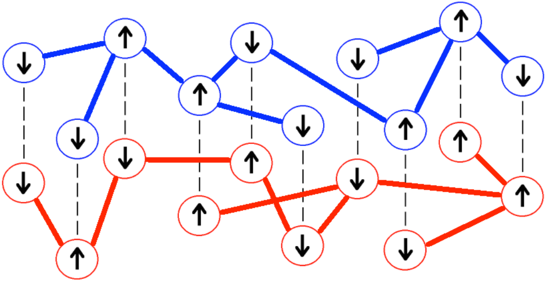
$$f_i^{[\alpha]} = J\sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma\frac{\chi_i}{J}\sum_{\substack{\beta=1\\ \beta\neq\alpha}}^{M} s_i^{[\beta]}$$

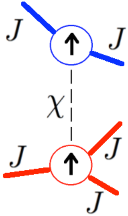
maximize

$$F_i^{[\alpha]} = s_i^{[\alpha]} f_i^{[\alpha]}$$

$$H = -\sum_{\alpha=1}^{M}\sum_{i=1}^{N} F_i^{[\alpha]}$$

two coupled
Ising models





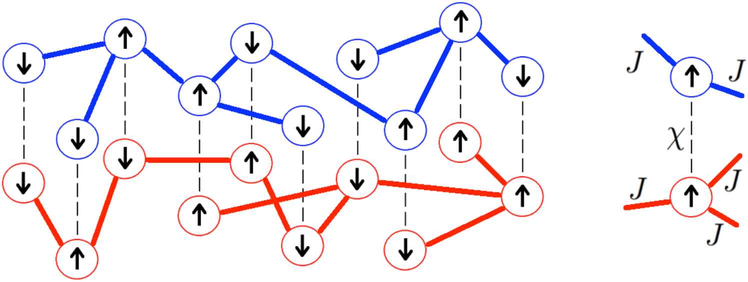
$$f_i^{[\alpha]} = J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma \frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{M} s_i^{[\beta]}$$

maximize

$$F_i^{[\alpha]} = s_i^{[\alpha]} f_i^{[\alpha]}$$

$$M^{[1]} = \frac{1}{N} \sum_{i=1}^{N} s_i^{[1]} \quad M^{[2]} = \frac{1}{N} \sum_{i=1}^{N} s_i^{[2]}$$

$$C = \frac{1}{N} \sum_{i}^{N} \mathrm{sgn}(\chi_i)\, s_i^{[1]} s_i^{[2]}$$



maximize

$$f_i^{[\alpha]} = J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma \frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{M} s_i^{[\beta]}$$

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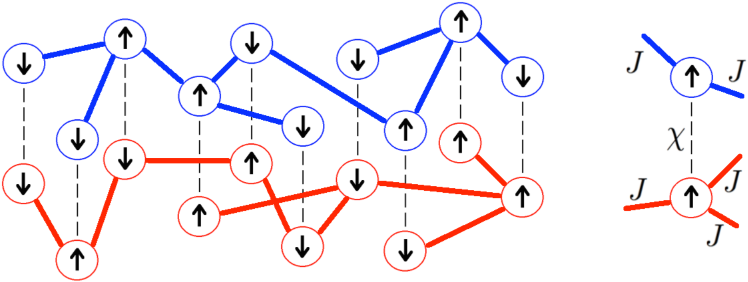
consensus



$$f_i^{[\alpha]} = J \sum_{j=1}^{N} a_{ij}^{[\alpha]} s_j^{[\alpha]} + h^{[\alpha]} + \gamma \frac{\chi_i}{J} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{M} s_i^{[\beta]}$$

maximize

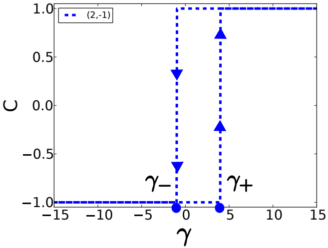
$$F_i^{[\alpha]} = s_i^{[\alpha]} f_i^{[\alpha]}$$

$$M^{[1]} = \frac{1}{N} \sum_{i=1}^{N} s_i^{[1]} \quad M^{[2]} = \frac{1}{N} \sum_{i=1}^{N} s_i^{[2]}$$

$$C = \frac{1}{N} \sum_{i}^{N} \mathrm{sgn}(\chi_i)\, s_i^{[1]} s_i^{[2]}$$

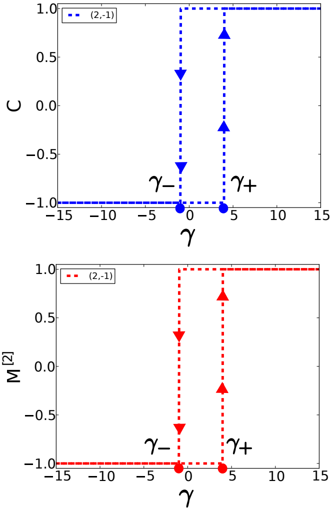
consensus

coherence



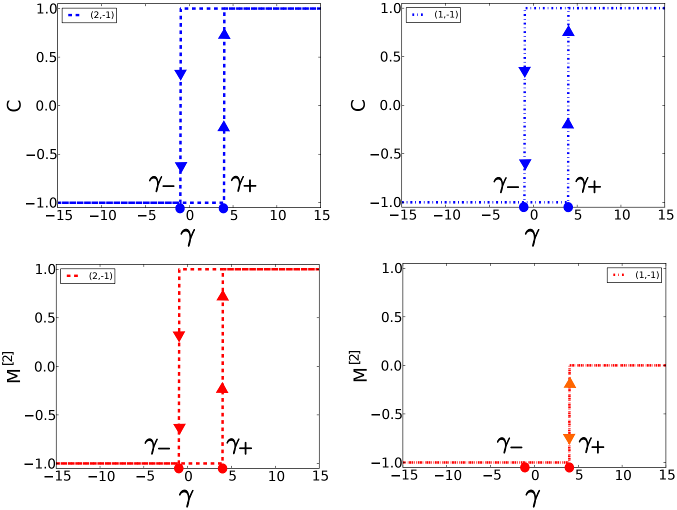
ABRUPT
TRANSITION

media with different intensities

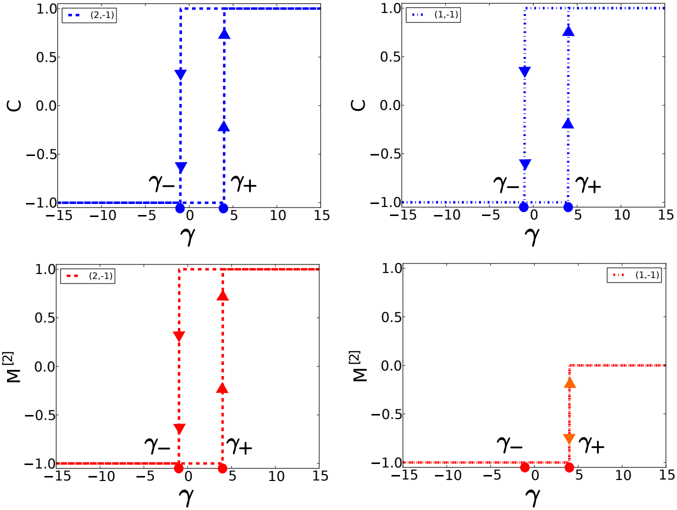


media with
different intensities

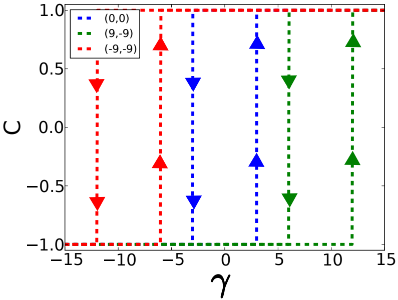
media with
same intensities

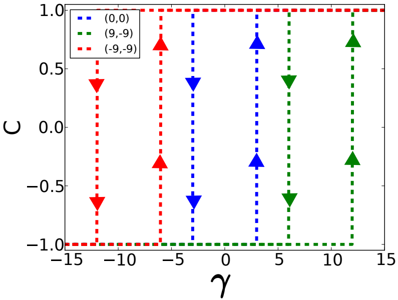


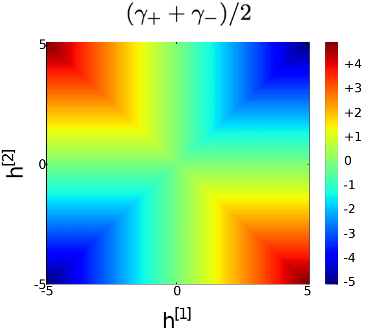
media with **different** intensities | media with **same** intensities



full consensus | **no** consensus

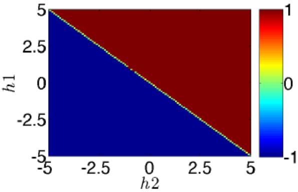

(0,0)
(9,-9)
(-9,-9)
C
1.0
0.5
0.0
−0.5
−1.0
−15
−10
−5
0
5
10
15
γ

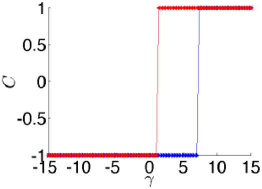




$$\gamma_{\pm} = \pm \langle k \rangle /2 - \mathrm{sign}(h^{[1]} h^{[2]}) \min \left(|h^{[1]}|, |h^{[2]}| \right)$$

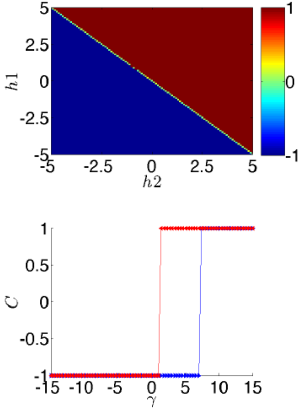
$M^{[2]}$

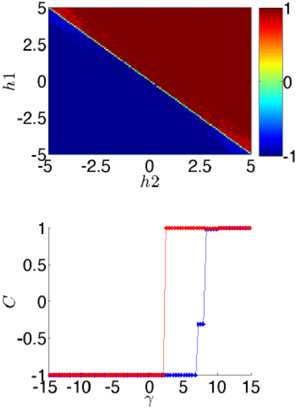




$$\chi_i = 1$$

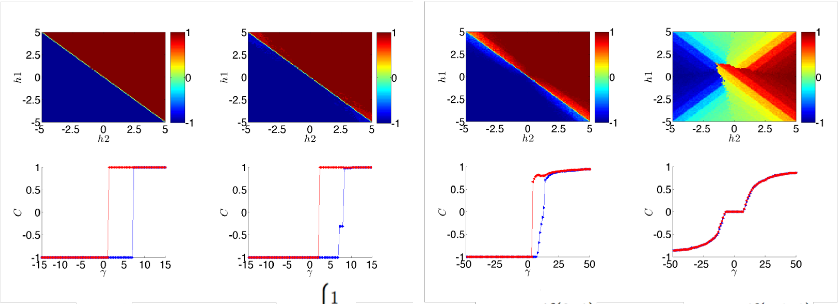
$M^{[2]}$ $M^{[2]}$





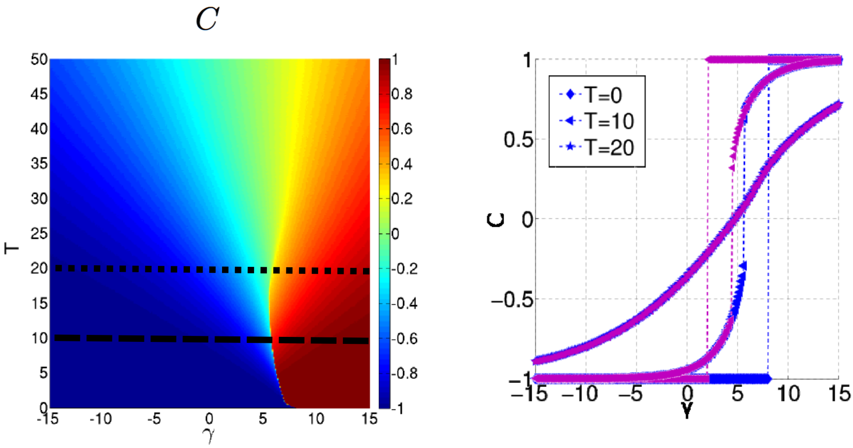
$$\chi_i = 1$$

$$\chi_i = \begin{cases} 1 \\ 1/2 \end{cases}$$



$\chi_i = 1$ $\quad$ $\chi_i = \begin{cases} 1 \\ 1/2 \end{cases}$ $\quad$ $\chi_i = \mathrm{unif}(0,1)$ $\quad$ $\chi_i = \mathrm{unif}(-1,1)$

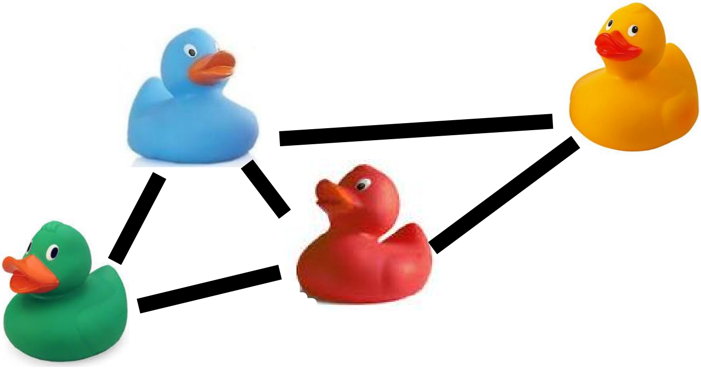
heterogeneous agents are needed to obtain states of partial consensus

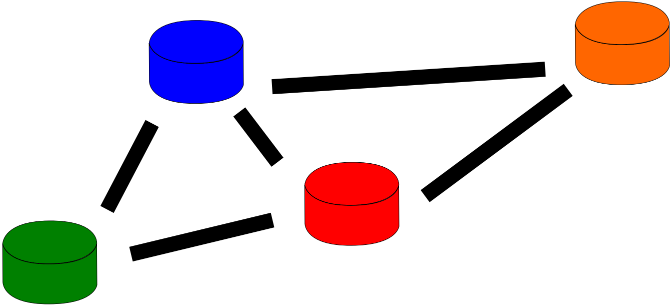


qualitatively same behavior below a critical noise

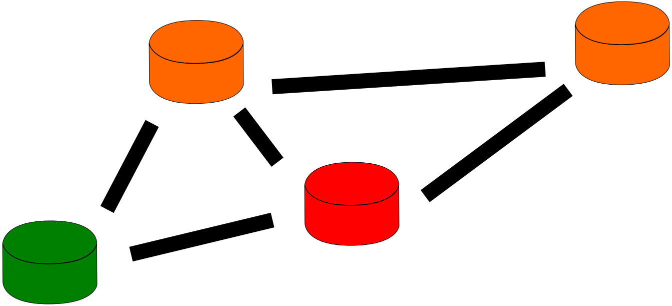
- Abrupt transition and hysteresis loop for the coherence C as a function of γ
- Empirical formula for the critical points γ_+ and γ_-.
- Heteroneous agents (values of χ_i) are needed to obtain non-trivial consensus
- Media are responsible for the level of consensus of the system
- Results are robust up to a critical level of noise

Why are **societies** inherently **multicultural**?

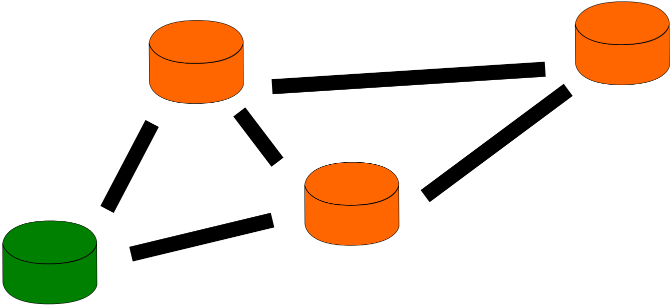


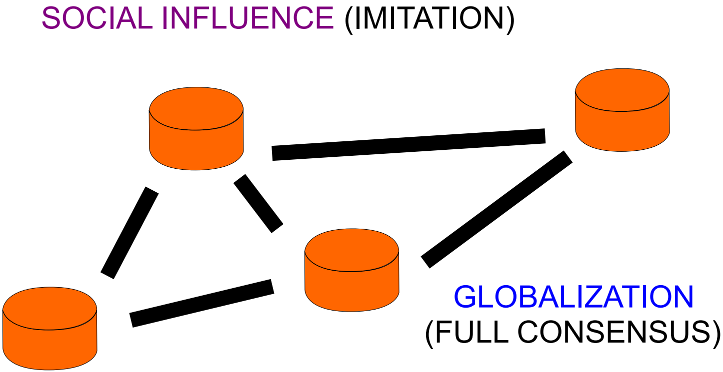


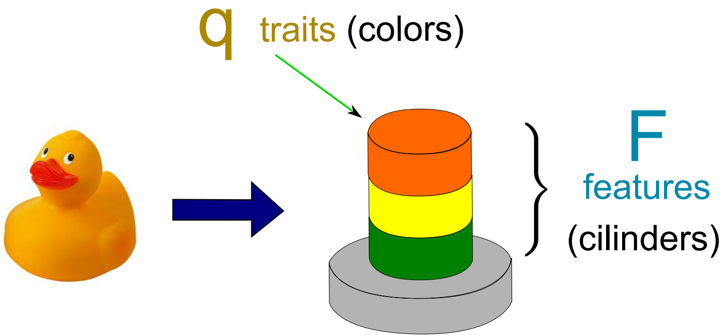
SOCIAL INFLUENCE (IMITATION)



SOCIAL INFLUENCE (IMITATION)



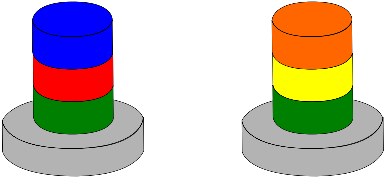

SOCIAL INFLUENCE (IMITATION)
GLOBALIZATION
(FULL CONSENSUS)



R. Axelrod, *Journal of Conflict Resolutions* (1997)

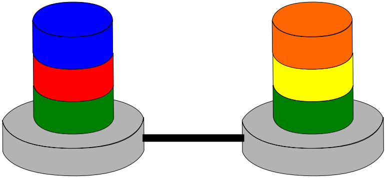
cultural overlap

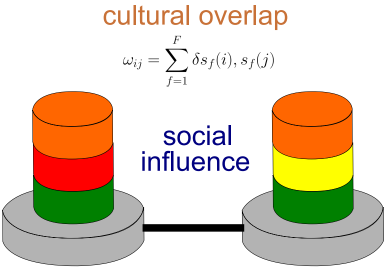
$$\omega_{ij} = \sum_{f=1}^{F} \delta s_f(i), s_f(j)$$



cultural overlap

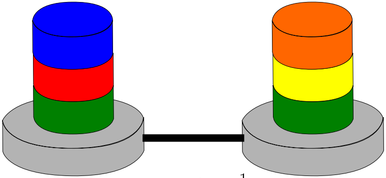
$$\omega_{ij} = \sum_{f=1}^{F} \delta s_f(i), s_f(j)$$




cultural overlap
$$\omega_{ij} = \sum_{f=1}^{F} \delta s_f(i), s_f(j)$$
social influence

cultural overlap

$$\omega_{ij} = \sum_{f=1}^{F} \delta s_f(i), s_f(j)$$

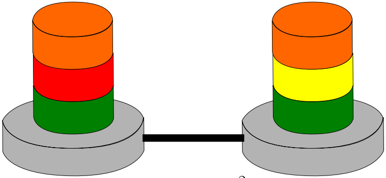


$$p_{ij} = \frac{\omega_{ij}}{F} = \frac{1}{3}$$

homophily

cultural overlap

$$\omega_{ij} = \sum_{f=1}^{F} \delta s_f(i), s_f(j)$$

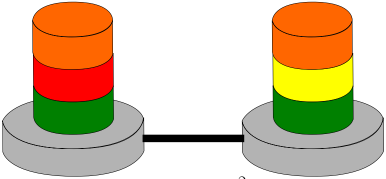


$$p_{ij} = \frac{\omega_{ij}}{F} = \frac{2}{3}$$

homophily

cultural overlap

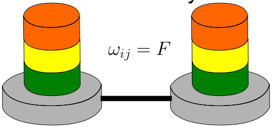
$$\omega_{ij} = \sum_{f=1}^{F} \delta s_f(i), s_f(j)$$



$$p_{ij} = \frac{\omega_{ij}}{F} = \frac{2}{3}$$

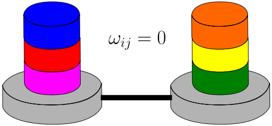
homophily

full similarity



$$\omega_{ij} = F$$

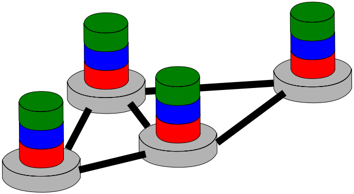
frozen bonds



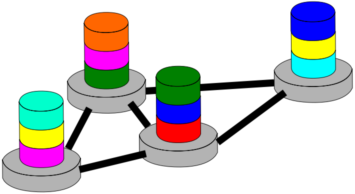
$$\omega_{ij} = 0$$

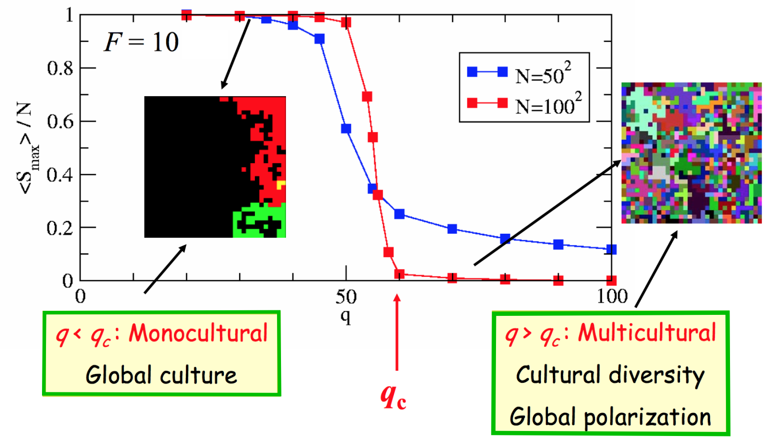
full diversity

globalization



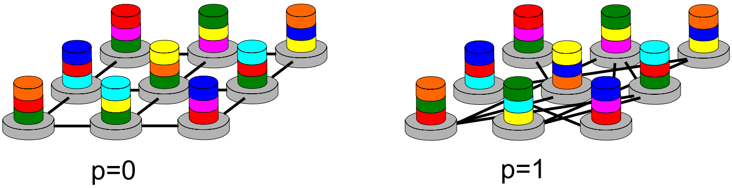
fragmentation

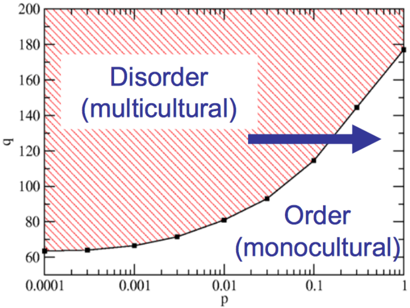




C. Castellano, M. Marsili, A. Vespignani, *PRL* (2000)

small-world networks



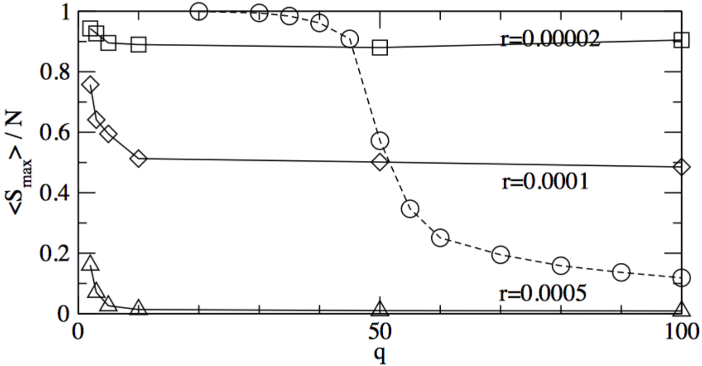


small-world connectivity
promotes
globalization

K. Klemm et al., *Physical Review E* (2003a)

drift: spontaneous mutation of cultural traits

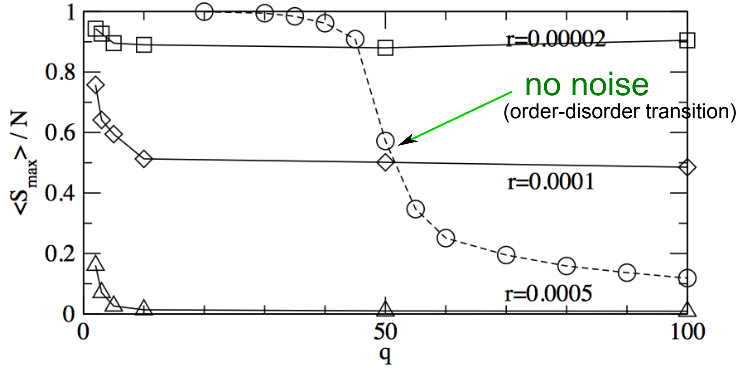
constant noise with rate r



K. Klemm et al., *Physical Review E* (2003b)

drift: spontaneous mutation of cultural traits

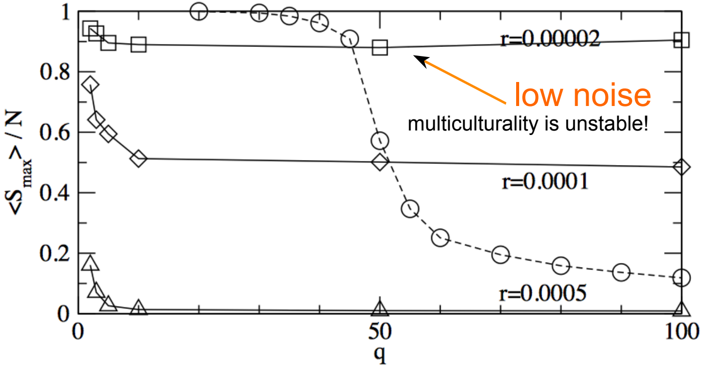
constant noise with rate r



K. Klemm et al., *Physical Review E* (2003b)

drift: spontaneous mutation of cultural traits

constant noise with rate r

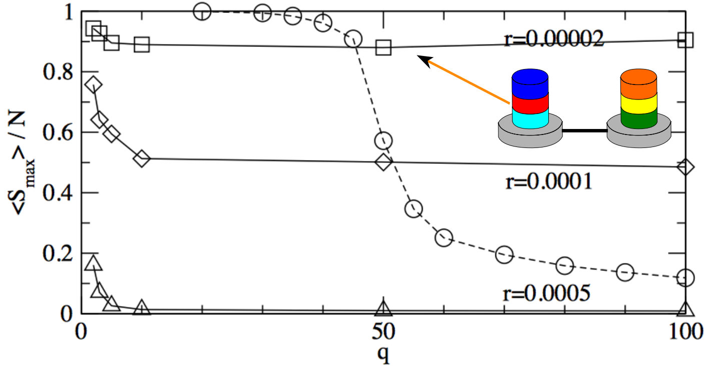


multiculturality is unstable under cultural drift!

K. Klemm et al., *Physical Review E* (2003b)

drift: spontaneous mutation of cultural traits

constant noise with rate r



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K. Klemm et al., *Physical Review E* (2003b)

drift: spontaneous mutation of cultural traits

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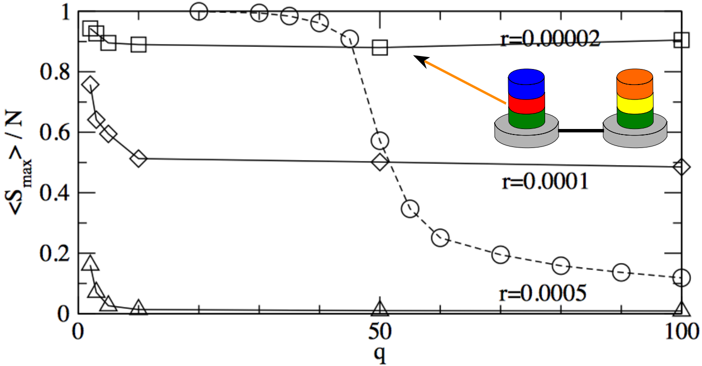

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K. Klemm et al., *Physical Review E* (2003b)

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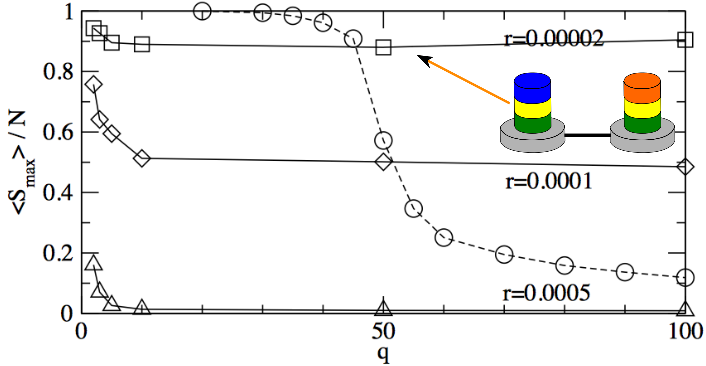

multiculturality is unstable under cultural drift!

K. Klemm et al., *Physical Review E* (2003b)

drift: spontaneous mutation of cultural traits

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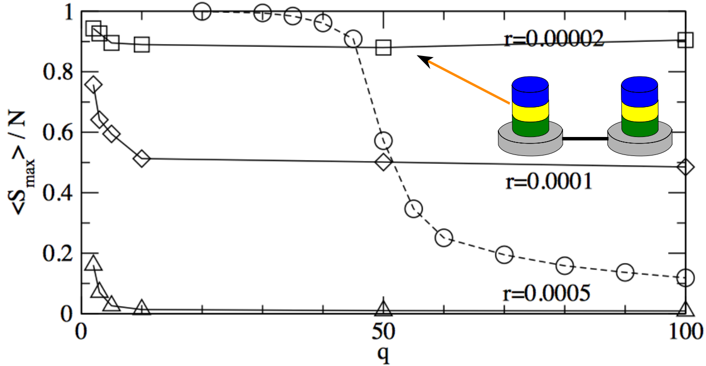

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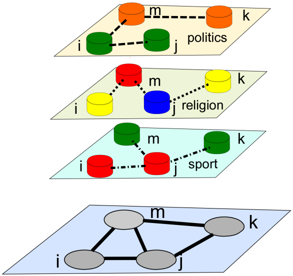
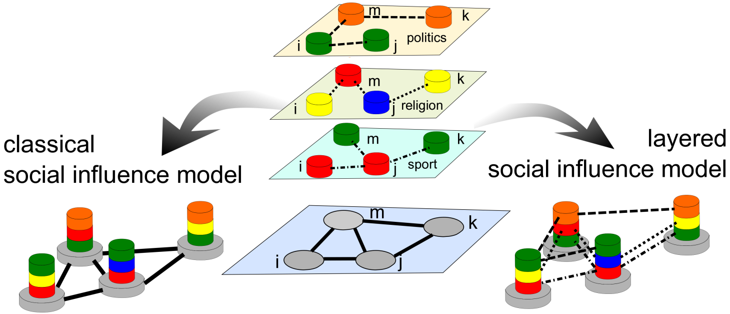
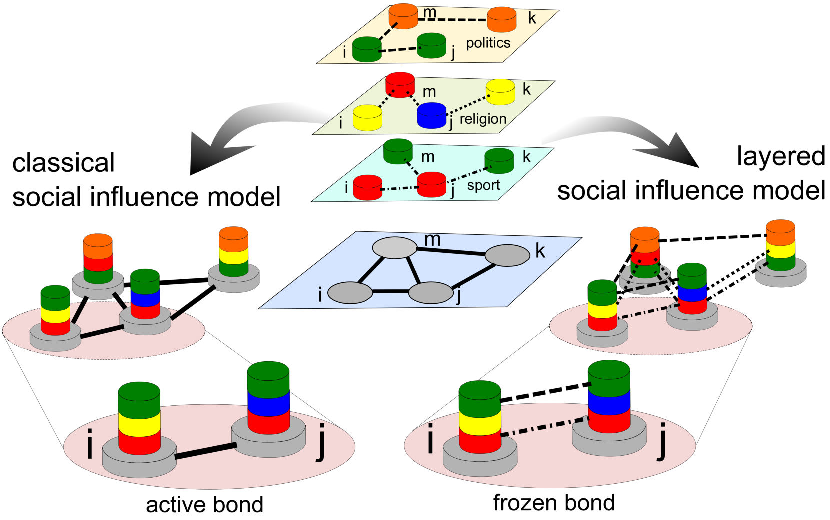
K. Klemm et al., *Physical Review E* (2003b)



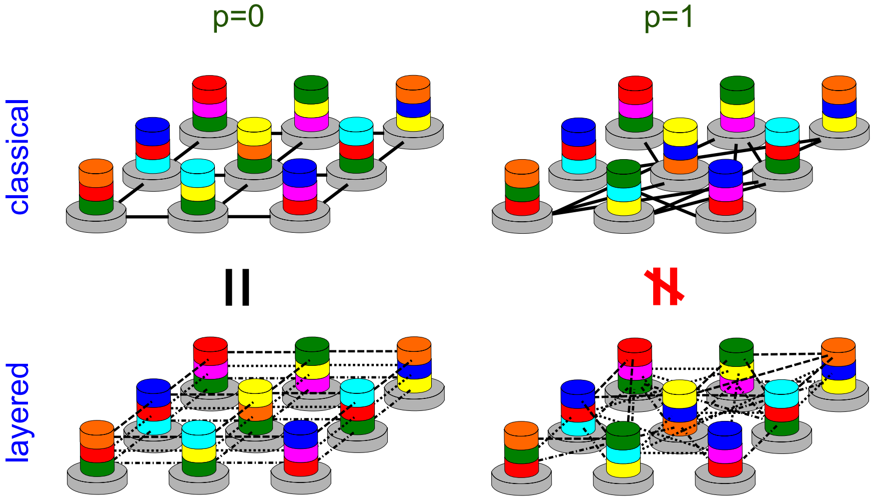
F. Battiston et al., (2015)



F. Battiston et al., (2015)

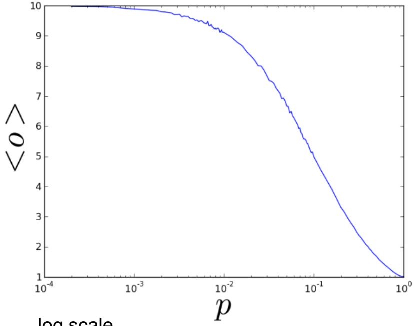


F. Battiston et al., (2015)

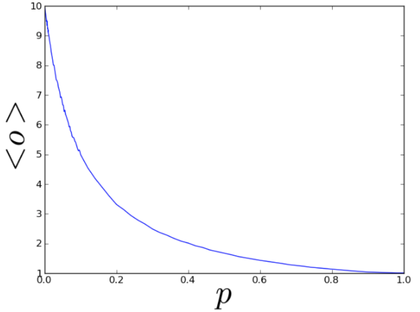

p=0
p=1
classical
layered

Rewire tunes the edge overlap

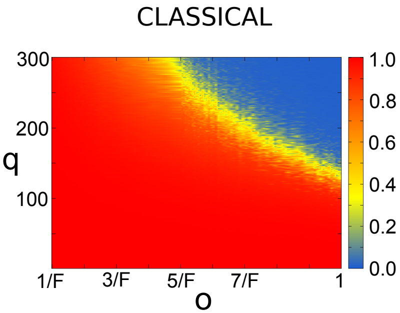
$$< o >= \frac{F}{\sum_{m=0}^{F} \binom{F}{m} (1-p)^m p^{F-m} (1 - \delta_{0,m} + F - m)}$$

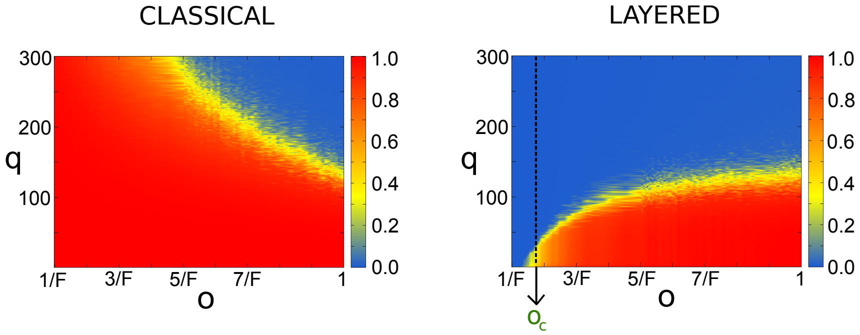


log scale



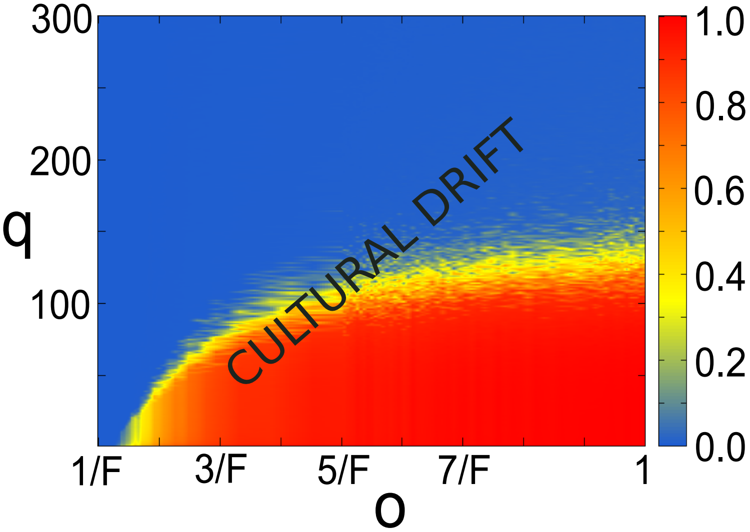
linear scale


CLASSICAL
LAYERED
300
200
100
q
1/F
3/F
5/F
7/F
1
o
1.0
0.8
0.6
0.4
0.2
0.0

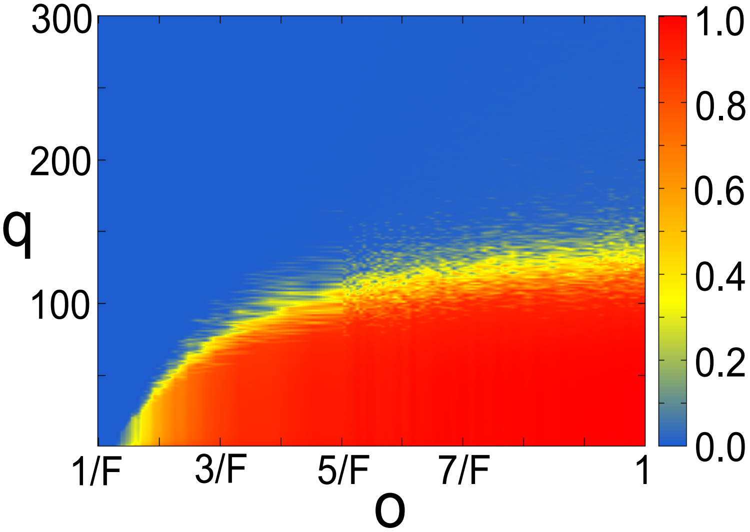


CLASSICAL
LAYERED
300
200
100
q
1/F
3/F
5/F
7/F
1
o
1.0
0.8
0.6
0.4
0.2
0.0
o_c

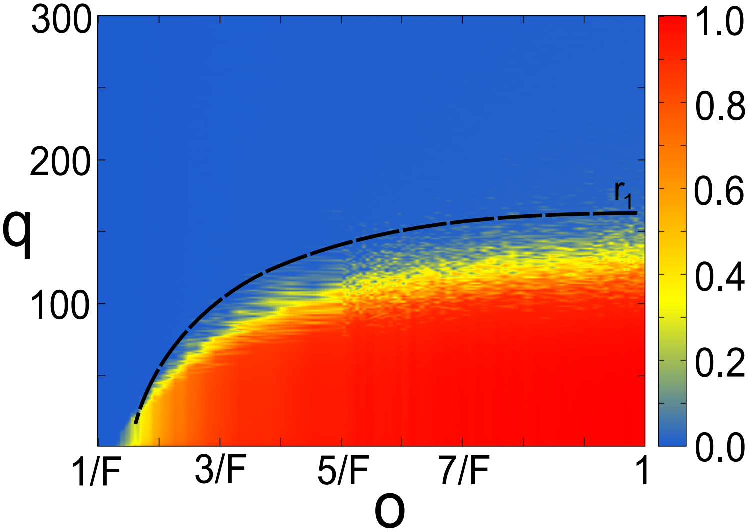
LAYERED



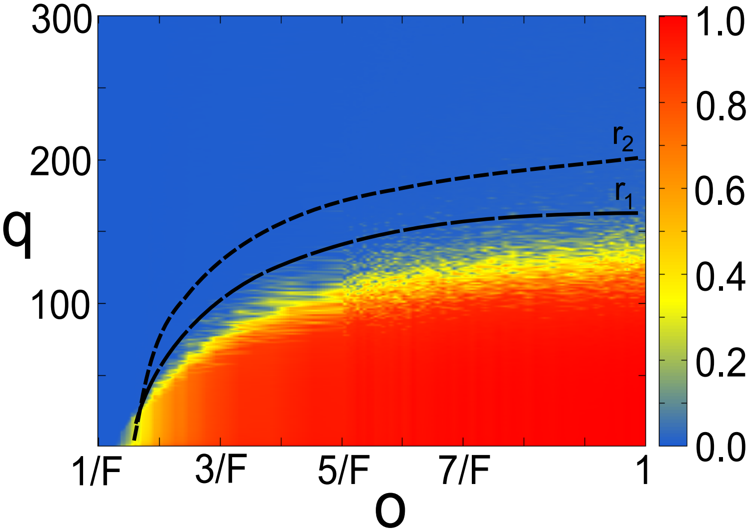
LAYERED



LAYERED



LAYERED



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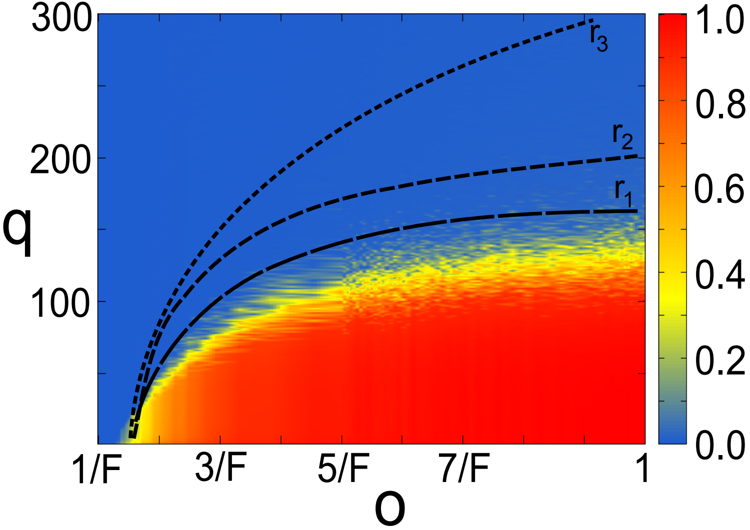

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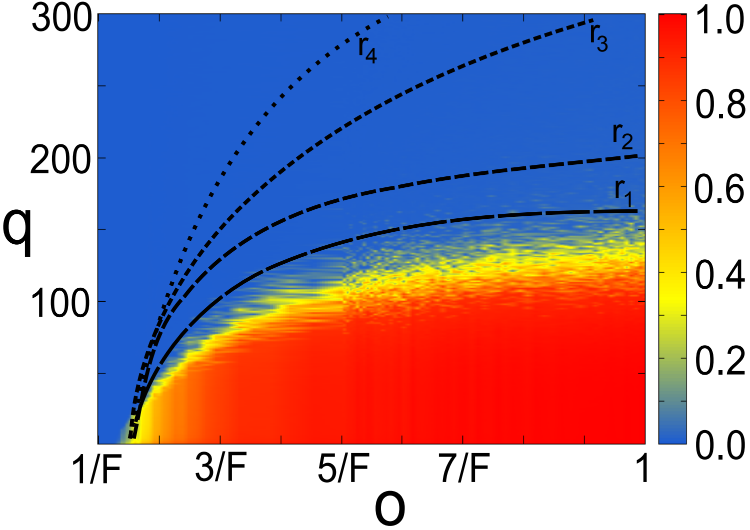

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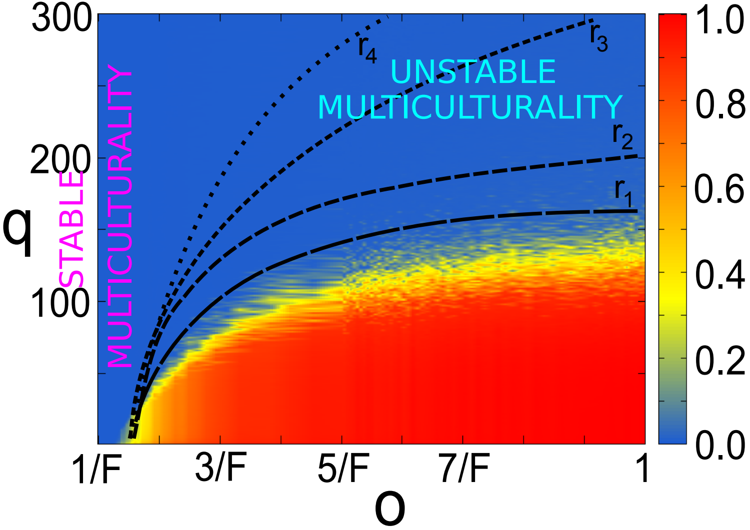

300
200
100
q
1/F
3/F
5/F
7/F
1
o
1.0
0.8
0.6
0.4
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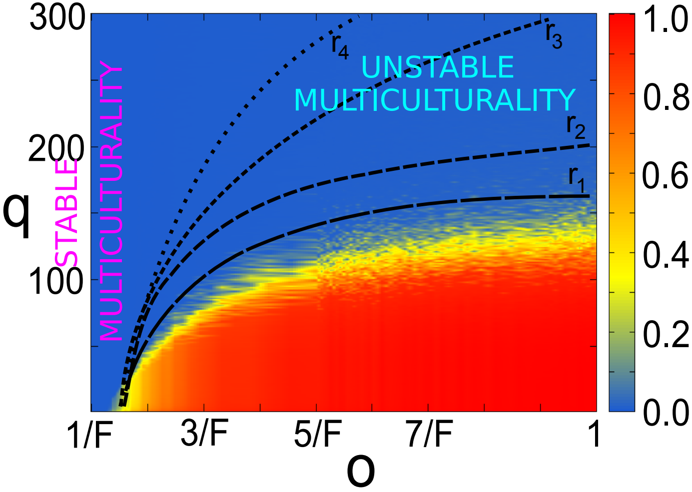
0.0
r4
r3
r2
r1
UNSTABLE
MULTICULTURALITY
STABLE
MULTICULTURALITY

LAYERED



multiculturality as a natural consequence
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F. Battiston, V. Nicosia, V. Latora and M. San Miguel

Email: f.battiston@qmul.ac.uk

Twitter: @fede7j

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