A dynamical transition in urban systems

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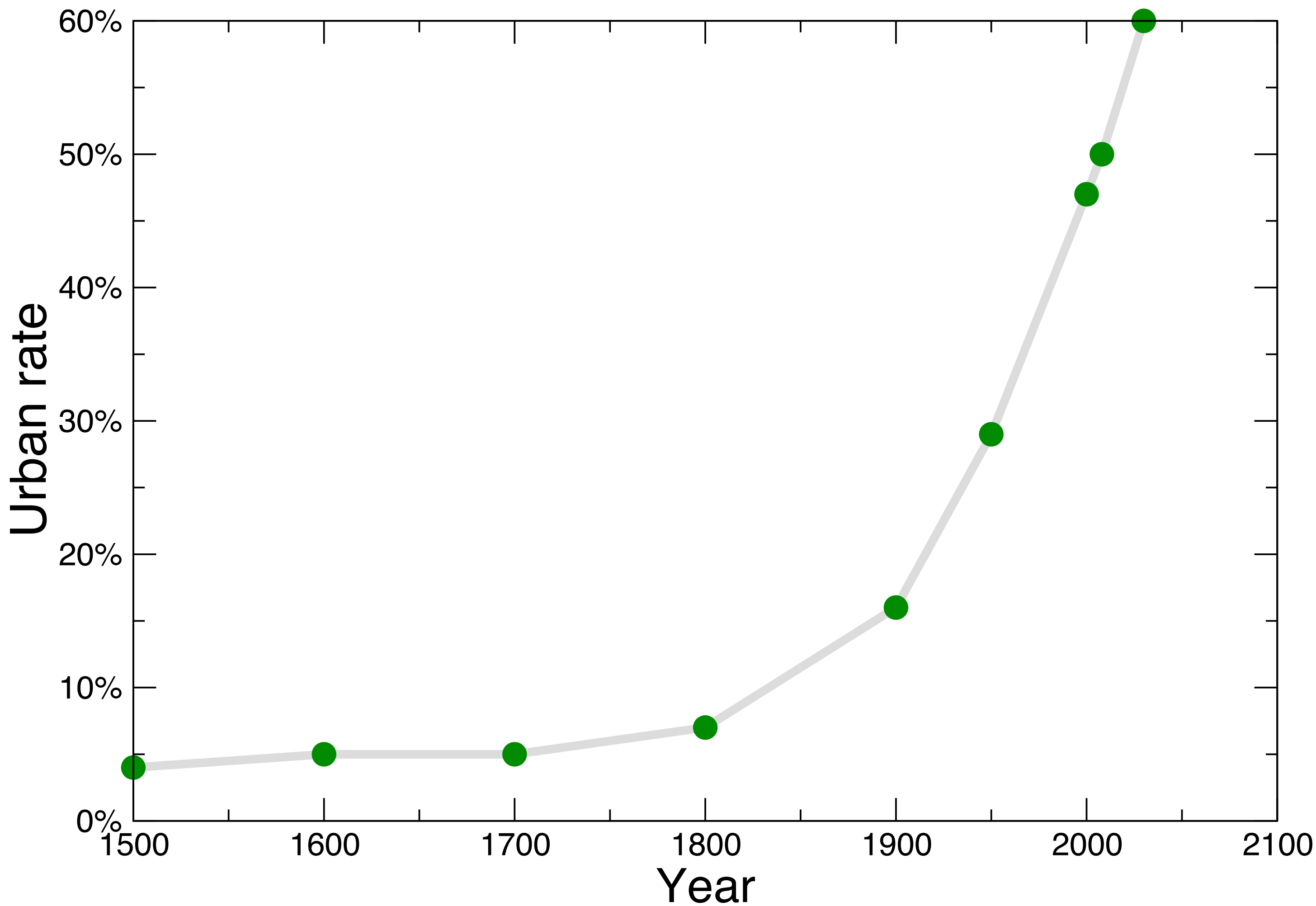
Outline

- Urban science: state of the art
- Polycentricity: empirical results
- Modeling: from urban economics to statistical physics
  - Krugman's model
  - The Fujita-Ogawa model
  - A physicist variant
- Discussion and perspectives

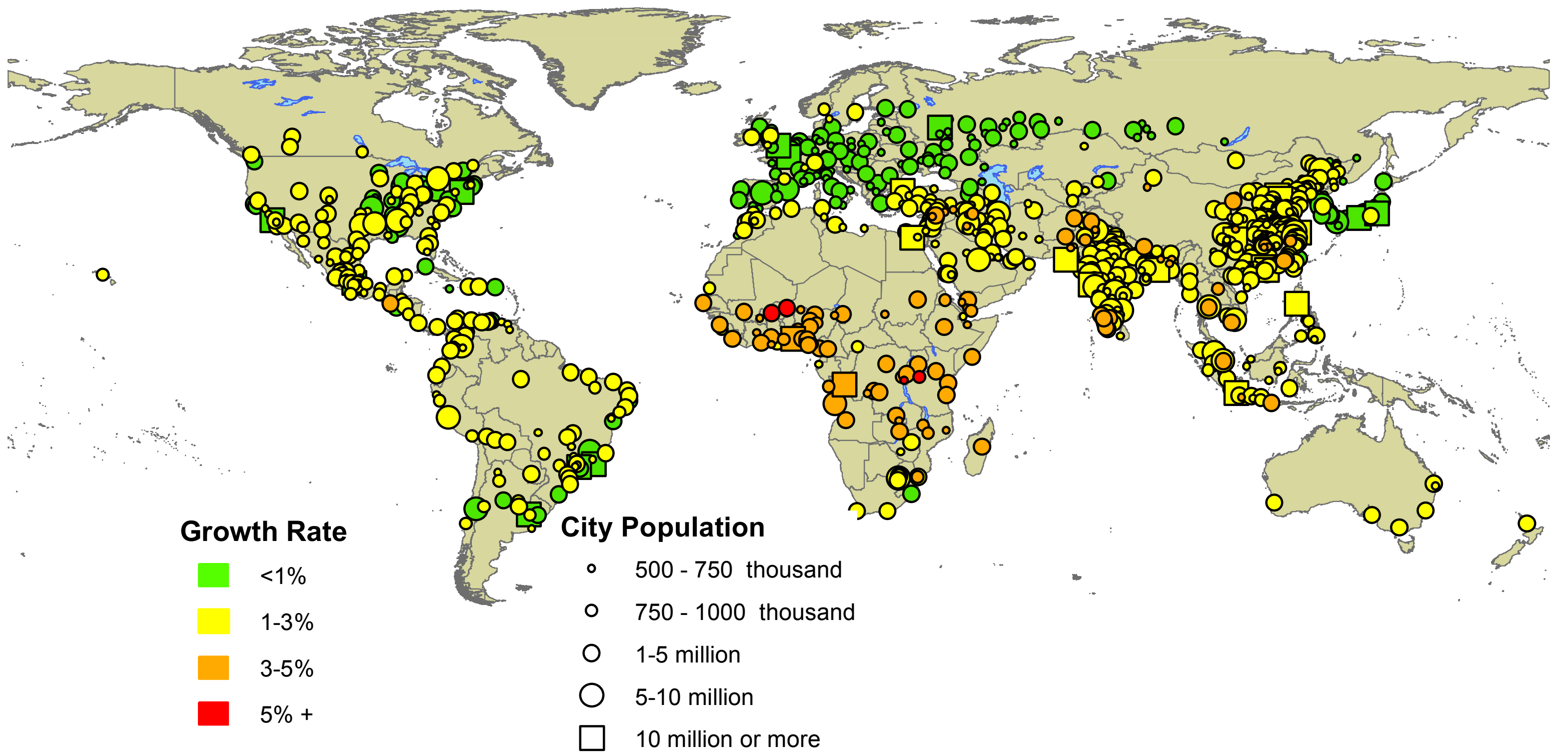
Importance of cities: urbanization rate



Projection: in 2050: 70% of the world population lives in cities

Data from: HYDE historical database

Importance of cities



Heterogeneous distribution of growth rates

Many 'theories' of urbanism but nevertheless, we observe a large number of problems !

- Social and economical problems (spatial income segregation, crime, accessibility, …)
- Traffic problems; pollution
- Sustainability of these structures ?

=> Necessity of understanding these phenomena and to achieve a science of cities and quantitative urbanism validated by data (in particular, for large-scale projects)

Science and cities: state of the art

Number of parameters

Urban economics:
Very abstract models, empirical tests ?

Complex simulations
(LUTI models):
Validity ?
Large perturbation ?

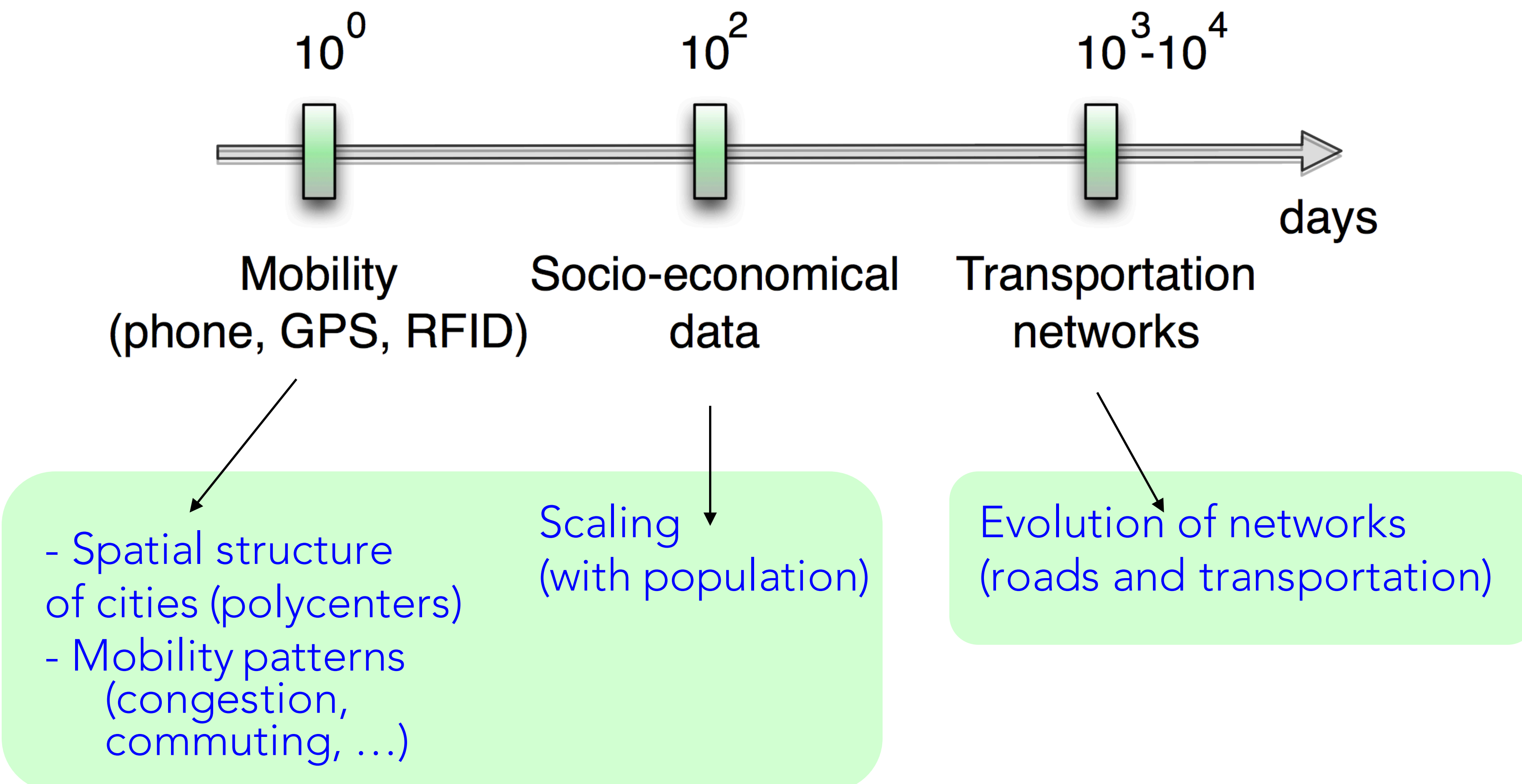
Minimal model: the smallest number of parameters and the largest number of verified predictions

Loop: theory-empirical data

- Open problem: Existence of (phase) transition in urban systems ??

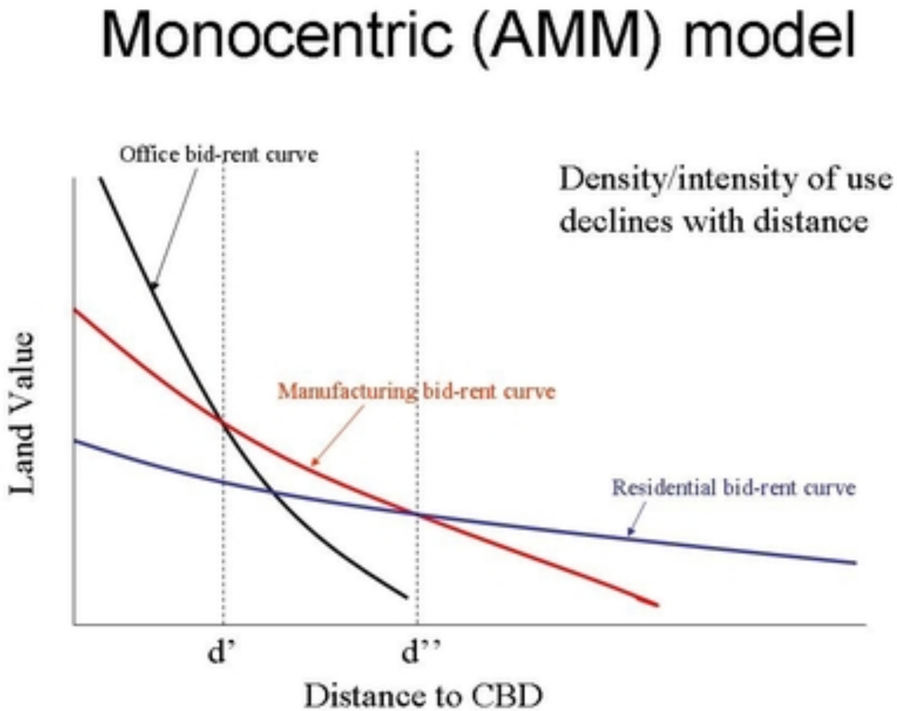
Towards a (new) science of cities

- Game changer ? Always more data about cities !
- Different scales, different phenomena



Spatial structure of cities

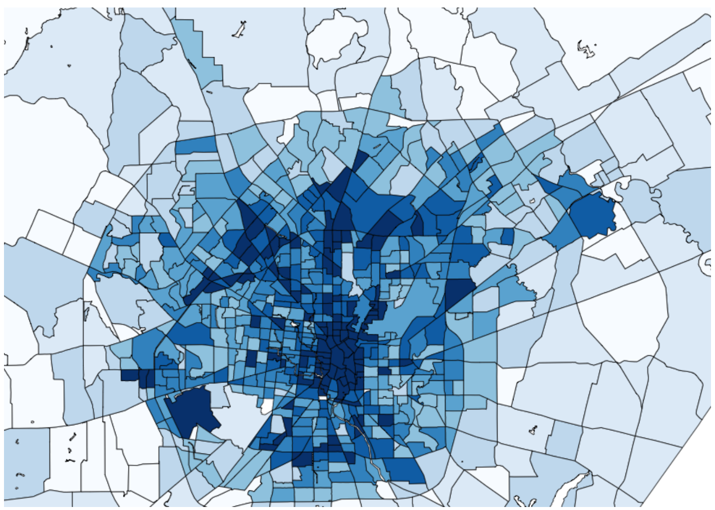
- Theoretical framework (Alonso-Muth-Mills):   -   - Monocentric organization: One center (the central business district)
  - The population density is decreasing with r (exact form depends on the utility !)



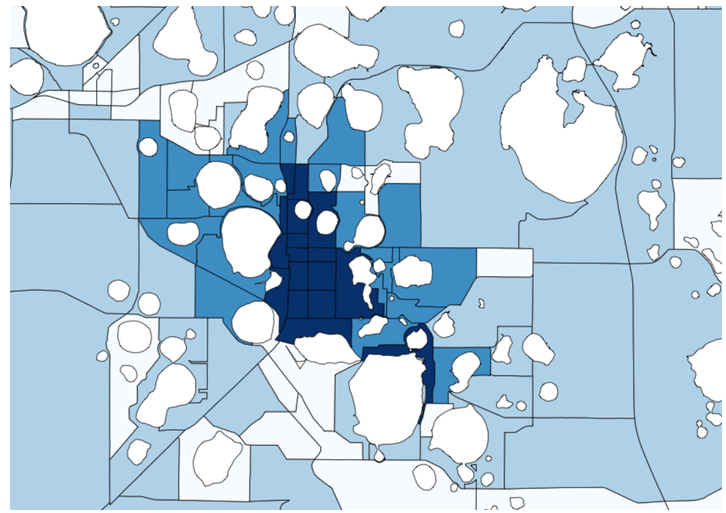
I. Polycentric structure: empirical results

Polycentric structure

- Activity centers (# of employees per zip code, USA)



San Antonio (TX), USA



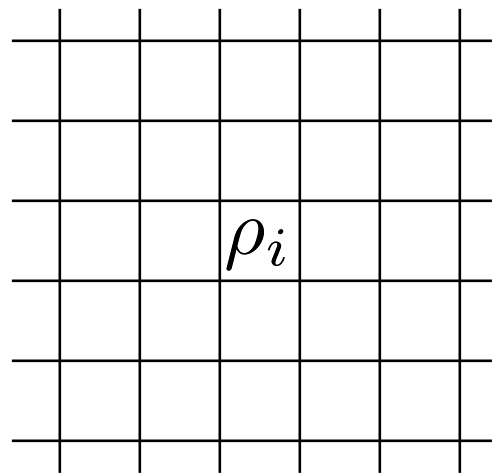
Winter Haven (FL), USA

- In general: existence of local maxima ('hotspots') of the density

Local maxima identification

- State of the art
  - No clear method
  - Density larger than a given threshold is a hotspot
  - Problem of the threshold choice ?

$$\rho_i > \rho_c \Rightarrow \; i \text{ is a Hotspot}$$



Louail, et al, Sci. Rep. 2014

Local maxima identification

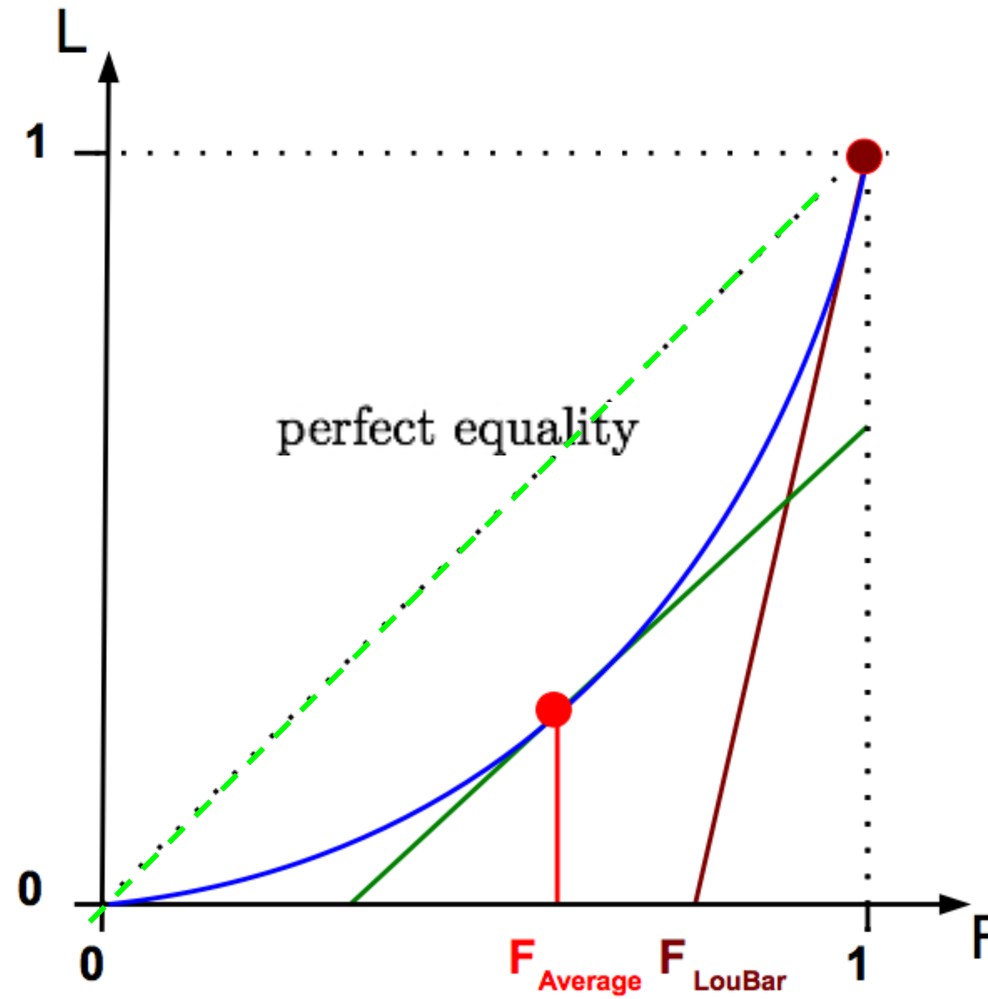
- Our proposal
  - Discussion on the Lorentz curve

$$F = i/N, \quad L = \sum_{j=1}^{i} \rho_j / \sum_{j=1}^{N} \rho_j$$

  - Identify a lower and upper threshold

$$F_{average} \ (\rightarrow \overline{\rho})$$

$$F_{LouBar} = 1 - \frac{\overline{\rho}}{\rho_{max}}$$

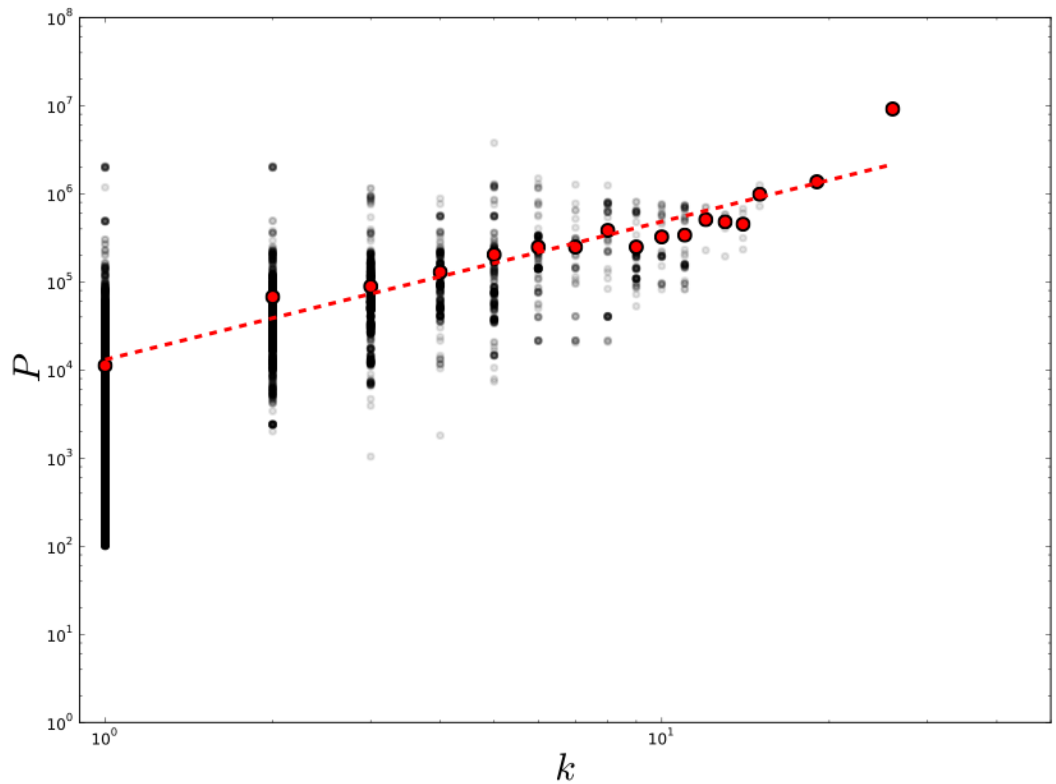


Louail, et al, Sci. Rep. 2014

Scaling for the number of centers

- We can count the number of hotspots (employment density data)
- The fit (9000 US cities, 1994-2010) gives

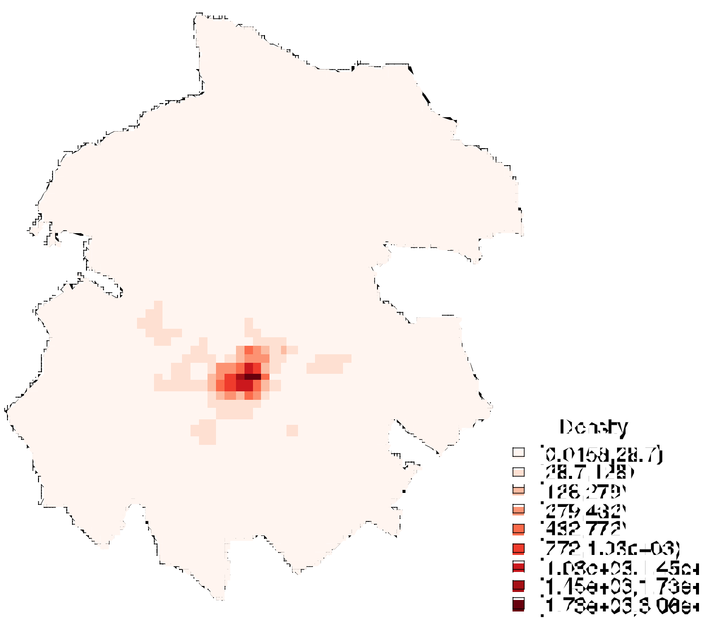
$$k \sim P^{\beta} \quad \beta \simeq 0.64$$



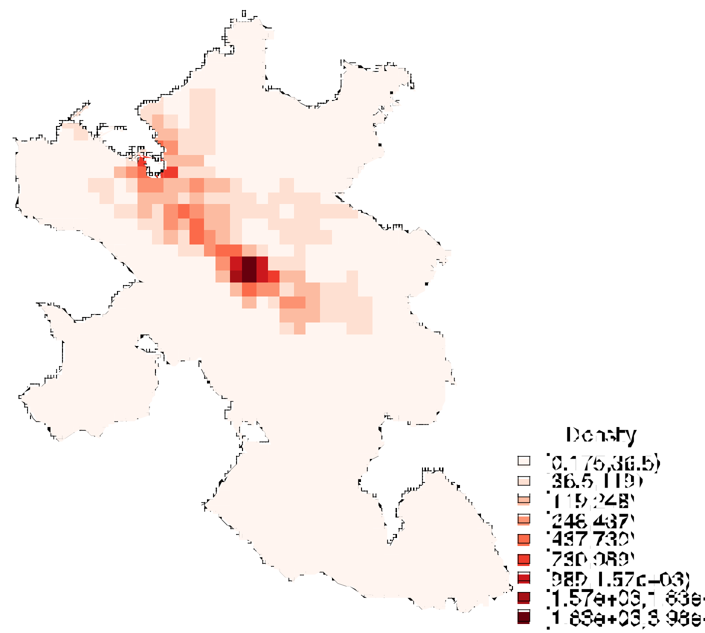
- Sublinear !

Mobile phone data: urban structures

Zaragoza



Bilbao

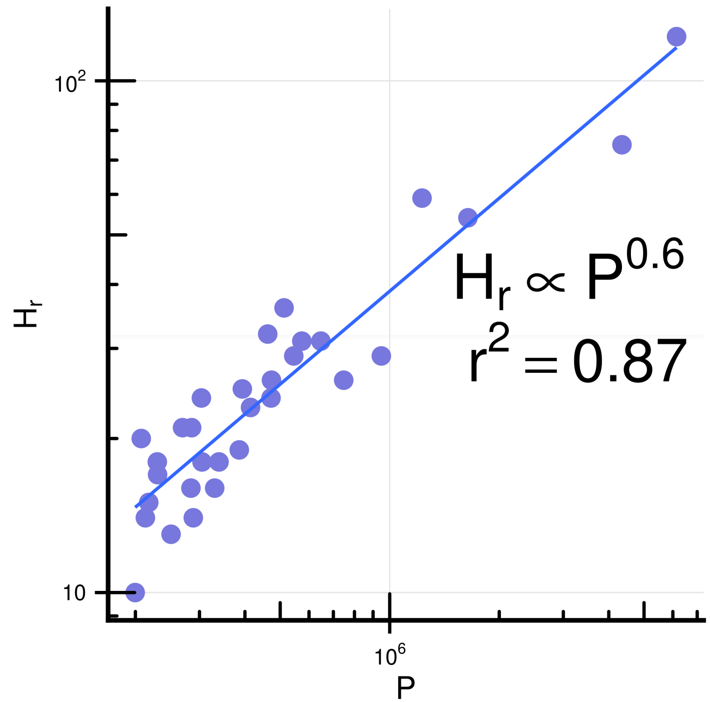


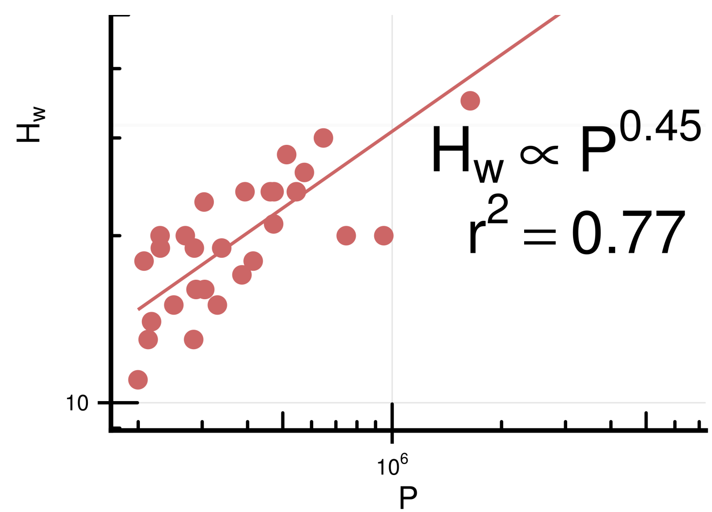
Scaling for the number of centers
(Urban areas -Spain)

Hotspots for residence density and 'activity' density

Exponent value is smaller for work/school/daily activity hotspots

→ The number of activity places grows slower than
the number of major residential places.





Summary: empirical results

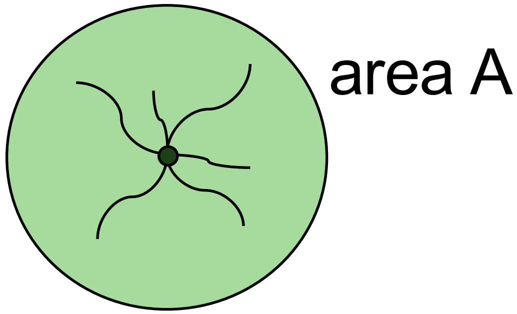
- We have a polycentric structure, evolving with P
- We can count the number H of centers

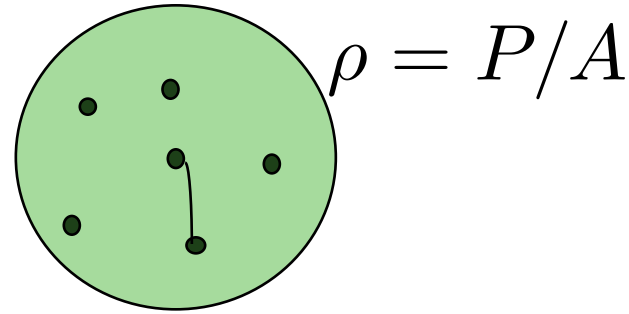
$$H \sim P^{\beta} \quad \beta \approx 0.5 - 0.6$$

- Mobility is the key: we need to model how individuals choose their home and work place
- Problem largely studied in geography, and in spatial economics: Edge City model (Krugman 1996), Fujita-Ogawa model (1982)
- Revisiting Fujita-Ogawa: predicting the value of β

II. Polycentric structure: Urban economics modeling

Naive scaling: Total commuting distance





Monocentric

$$\ell_1 \sim \sqrt{A}$$

$$L_{tot}/\sqrt{A} \sim P$$

Nearest neighbor

$$\ell_1 \sim 1/\sqrt{\rho} \sim \frac{\sqrt{A}}{\sqrt{P}}$$

$$L_{tot}/\sqrt{A} \sim P^{1/2}$$

- We obtain

$$\frac{L_{tot}}{\sqrt{A}} \sim P^{\beta} \quad \beta \in [0.5, 1]$$

$$\beta \simeq 0.66 \;\; (\mathrm{Samaniego,\ Moses,\ 2008})$$

What is wrong with the naive scaling

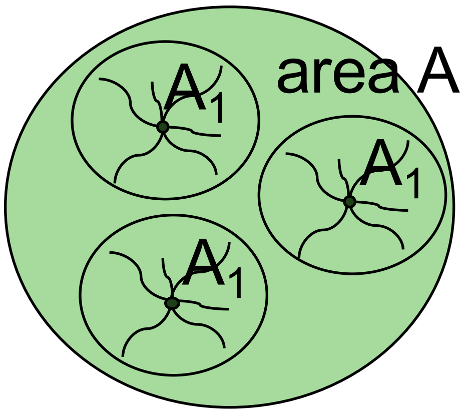
- Assume k secondary centers:

$$A = kA_1$$

Total commuting length:

$$L_{tot} = k\frac{P}{k}\sqrt{A_1}$$

$$\Rightarrow \frac{L_{tot}}{\sqrt{A}} = \frac{P}{\sqrt{k}}$$



- Can change scaling exponents if k varies with P !

- We have to understand the polycentric structure of cities

Spatial economics: the edge city model (Krugman 1996)

- The important ingredient is the 'market potential'

$$\Pi(x) = \int K(x-z)\rho_B(z)\mathrm{d}z$$

- Describes the spillovers due to the business density in z
- Specifically

$$K(x) = K_+(x) - K_-(x)$$

- The average market potential is

$$\overline{\Pi} = \frac{1}{A}\int \Pi(x)\rho_B(x)\mathrm{d}x$$

Spatial economics: the edge city model (Krugman 1996)

- The equation for the evolution of business density is

$$\frac{\mathrm{d}\rho_B(x,t)}{\mathrm{d}t} = \gamma\left(\Pi(x,t) - \overline{\Pi}\right)$$

- Linearize around flat situation $\rho_B(x) = \rho_0 + \delta\rho_B(x)$

$$\delta\tilde{\rho}_B(k) \sim \mathrm{e}^{\gamma\tilde{K}(k)t}$$

- At least one maximum at k=k*; the number of hotspots is then:

$$H \sim Ak^{*2}$$

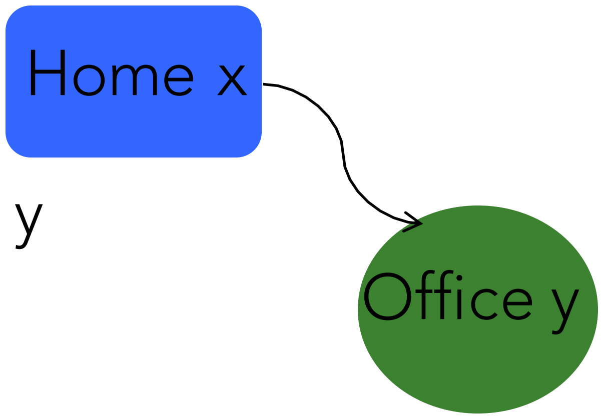
- Scaling with the population ? Individual's choices ?

Spatial economics: Fujita-Ogawa (1982)

- A model for the spatial structure of cities: an agent will choose to live in *x* and work in *y* such that

$$Z_0(x,y) = W(y) - C_R(x) - C_T(x,y)$$

is maximum



- $W(y)$ is the wage ('attractiveness') at y

- $C_R(x)$ is the rent at x

- $C_T(x,y)$ is the transportation cost from x to y

$$C_T(x,y) = td(x,y)$$

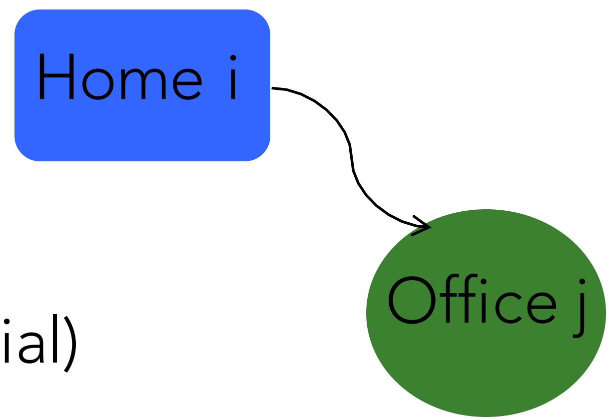
Spatial economics: Fujita-Ogawa (1982)

- And a similar equation for companies (maximum profit)

$$P(y) = \Pi(y) - C_R(y) - L(y)W(y)$$

- W(y) is the wage at y
- $C_R(y)$ is the rent at y
- L(y) number of workers ($N=ML_0$)
- $\Pi(y)$ is the benefit to come to y:

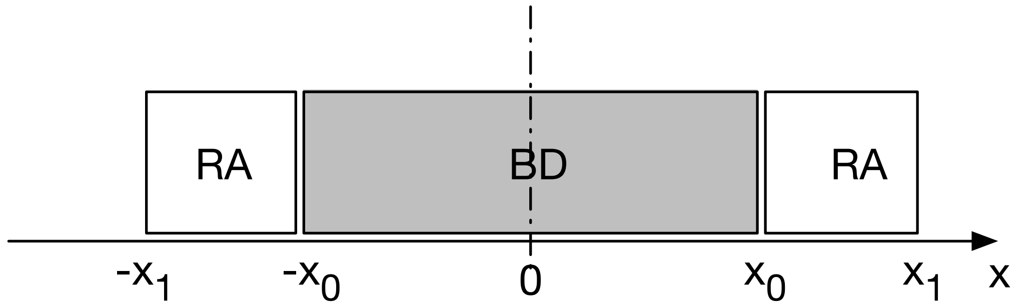
Agglomeration effect ! (market potential)



$$\Pi(y) = \int K(y-z)\rho_B(z)\mathrm{d}z$$

$$K(u) = k\mathrm{e}^{-\alpha|u|}$$

Spatial economics: Fujita-Ogawa (1982)



- Main result: monocentric configuration stable if

$$\frac{t}{k} \leq \alpha$$

- t: transport cost
- 1/α interaction distance between firms

- Effect of congestion: larger cost t

Spatial economics: Fujita-Ogawa (1982)

- This model is unable to predict the spatial structure and the number of activity centers….

- We have to simplify the problem !

Spatial economics: Fujita-Ogawa (1982)

- There are many problems with this model:
  - Not dynamical: optimization. We want an out-of-equilibrium model
  - No congestion (!) We want to include congestion (for car traffic)
  - No empirical test. Extract testable predictions (see the book: Spatial Economics, by Fujita, Krugman, Venables)

A physicist's variant of Fujita-Ogawa

- Assumptions and simplifications:
  - Assume that home is uniformly distributed (x): find a job !

$$Z_0(x,y) = W(y) - C_T(x,y)$$

  - We have now to discuss W and C_T

A physicist's variant of Fujita-Ogawa

- Assumptions and simplifications:
  - Add congestion (BPR function, t=cost/distance) and the generalized cost reads:

$$C_T(x,y) = td(x,y)\left[1 + \left(\frac{T(x,y)}{c}\right)^{\mu}\right]$$

  - Wages: a typical physicist assumption (s: typical salary)

$$W(y) = s\eta(y)$$

The 'attractivity' η is random (in [0,1]) (cf. Random Matrix Theory)
W can be seen as a the 'quality' of the job, encoding many factors

Summary: the model

- Every time step, add a new individual at a random i
- The individual will choose to work in *y* (among N_c possible centers) such that

$$Z(x,y) = \eta(y) - \frac{d(x,y)}{\ell}\left[1 + \left(\frac{T(y)}{c}\right)^{\mu}\right]$$

is maximum

- W(y) is the wage at y --> random

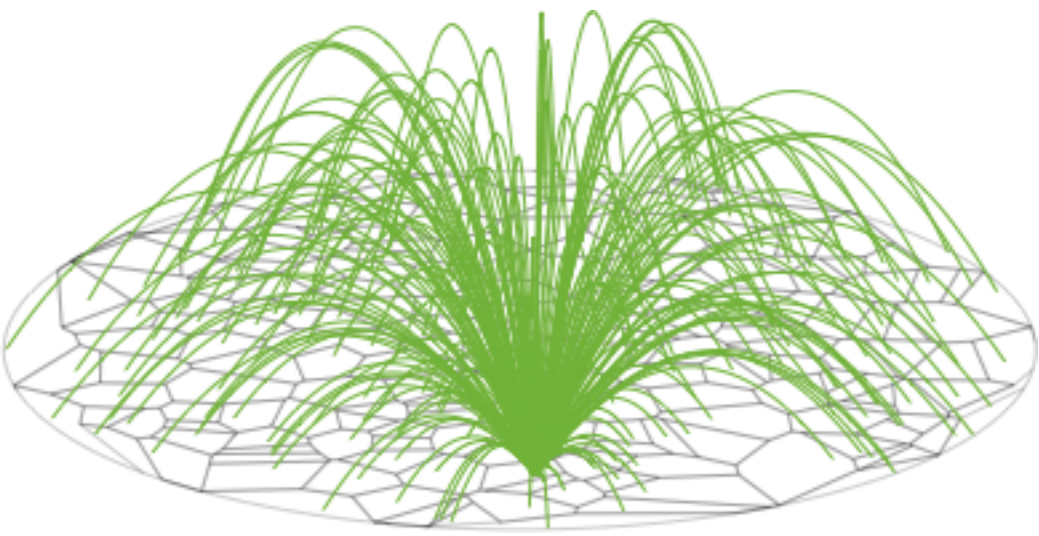
- $C_T(x,y)$ is the transportation cost from x to y: depends on the traffic from x to y --> congestion effects

Louf, MB, PRL 2013

Results

- Depending on the values of parameters, we see three type of mobility patterns:

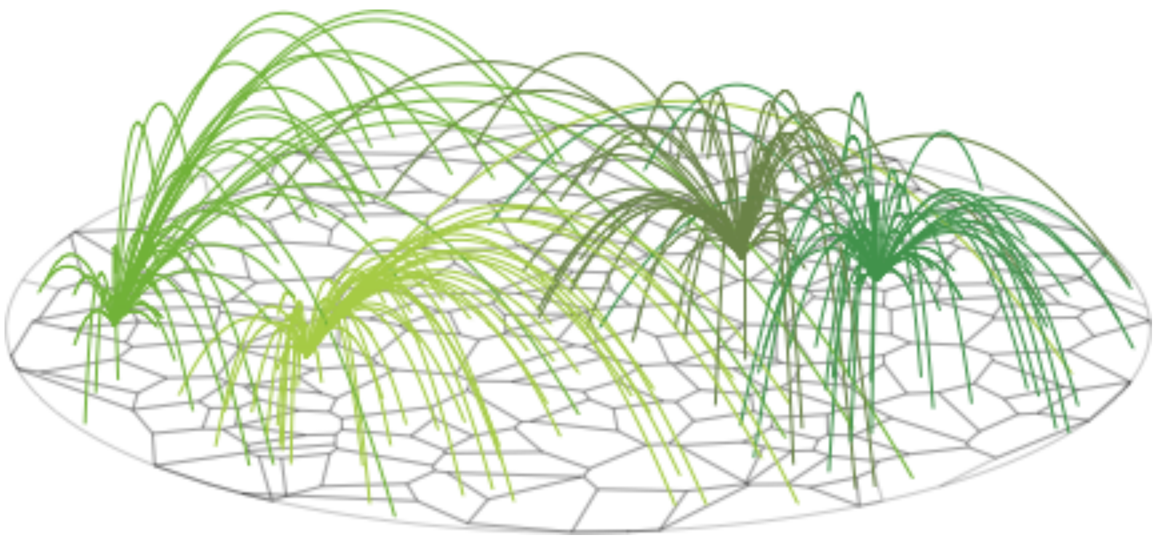
1. Monocentric: one activity center



Results

- Depending on the values of parameters, we see three type of mobility patterns:

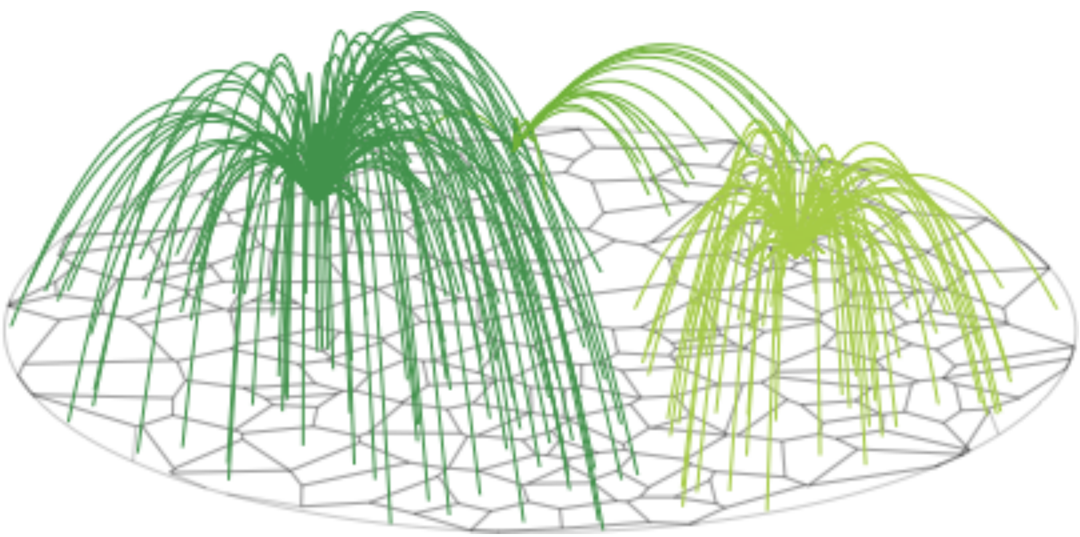
  2. Attractivity driven polycentrism: many activity centers, attractivity η dominates



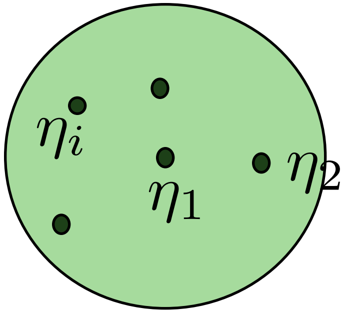
Results

- Depending on the values of parameters, we see three type of mobility patterns:

  3. Spatial polycentrism: many activity centers, basins spatially coherent



Monocentric-polycentric transition



- Start with one center $\eta_1 > \eta_2 > \cdots > \eta_{N_C}$
- T(1)>0 and all other subcenters have a zero traffic T(j)=0
- The number of individuals P increases, T(1) increases and for a new individual i, there is another center j such that:

$$Z(i,j) > Z(i,1)$$

Or:

$$\eta_j - \frac{d_{ij}}{\ell} > \eta_1 - \frac{d_{i1}}{\ell}\left[1 + \left(\frac{P}{c}\right)^{\mu}\right]$$

Monocentric-polycentric transition

$$\eta_j - \frac{d_{ij}}{\ell} > \eta_1 - \frac{d_{i1}}{\ell}\left[1 + \left(\frac{P}{c}\right)^{\mu}\right]$$

- Mean-field type argument
  - $d_{i1} \sim d_{ij} \sim \sqrt{A}$
  - The new subcenter has the second largest attractivity η_2
  - on average

$$\overline{\eta_1 - \eta_2} \simeq \frac{1}{N_c}$$

- We obtain a 'critical' value for the population

$$P > P^* = c\left(\frac{\ell}{\sqrt{A}N_c}\right)^{1/\mu}$$

Monocentric-polycentric transition

- Critical value for the population: effect of congestion !

$$P > P^* = c\left(\frac{\ell}{\sqrt{A}N_c}\right)^{1/\mu}$$

- c sets the scale

- If ℓ is too small, P*<1 and the monocentric regime is never stable

Monocentric-polycentric transition

- If the population continues to increase, other subcenters will appear. We assume that for P, we have k-1 subcenters:

$$\eta_1 \geq \eta_2 \geq \cdots \geq \eta_{k-1}$$

with traffic: $$T(1) \sim T(2) \sim \cdots \sim T(k-1) \sim \frac{P}{k-1}$$

- The next individual will choose a new subcenter k if:

$$Z(i,k) > \max_{j=1,\ldots,k-1} Z(i,j)$$

$$\eta_k - \frac{d_{ik}}{\ell} > \max_{1,\ldots,k-1} \{\eta_j - \frac{d_{ij}}{\ell}\left[1+\left(\frac{T(j)}{c}\right)^{\mu}\right]\}$$

- We assume: $d_{ik} \sim d_{ij} \sim L$

Results: scaling for the number of centers

- We obtain the average population for which a k[th] subcenter appears is:

$$\overline{P}_k = P^*(k-1)^{\frac{\mu+1}{\mu}}$$

- Which implies:

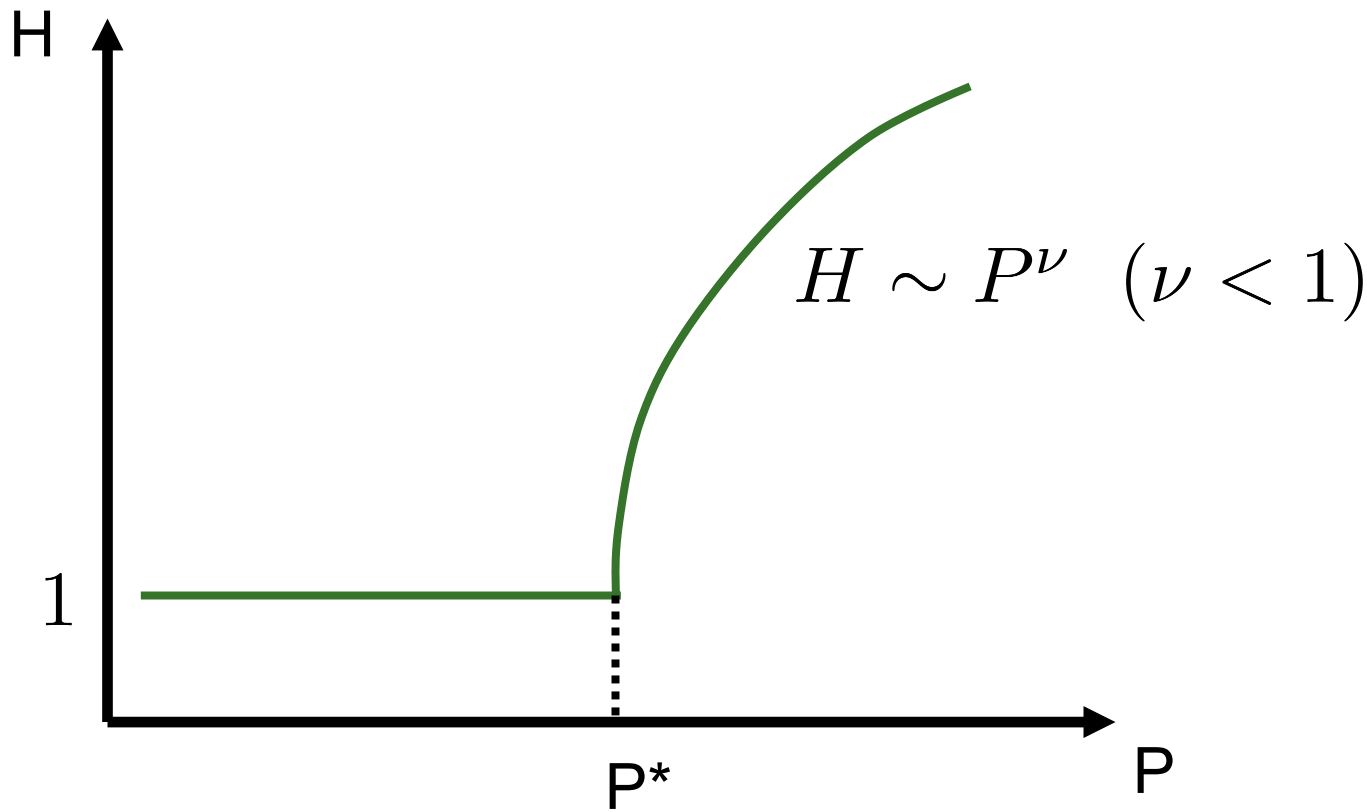
$$k \sim \left(\frac{P}{P^*}\right)^{\frac{\mu}{\mu+1}}$$

Sublinear relation !

- From US employment data (9000 cities)

$$k \sim P^{0.64} \qquad (\Rightarrow \mu \simeq 2)$$

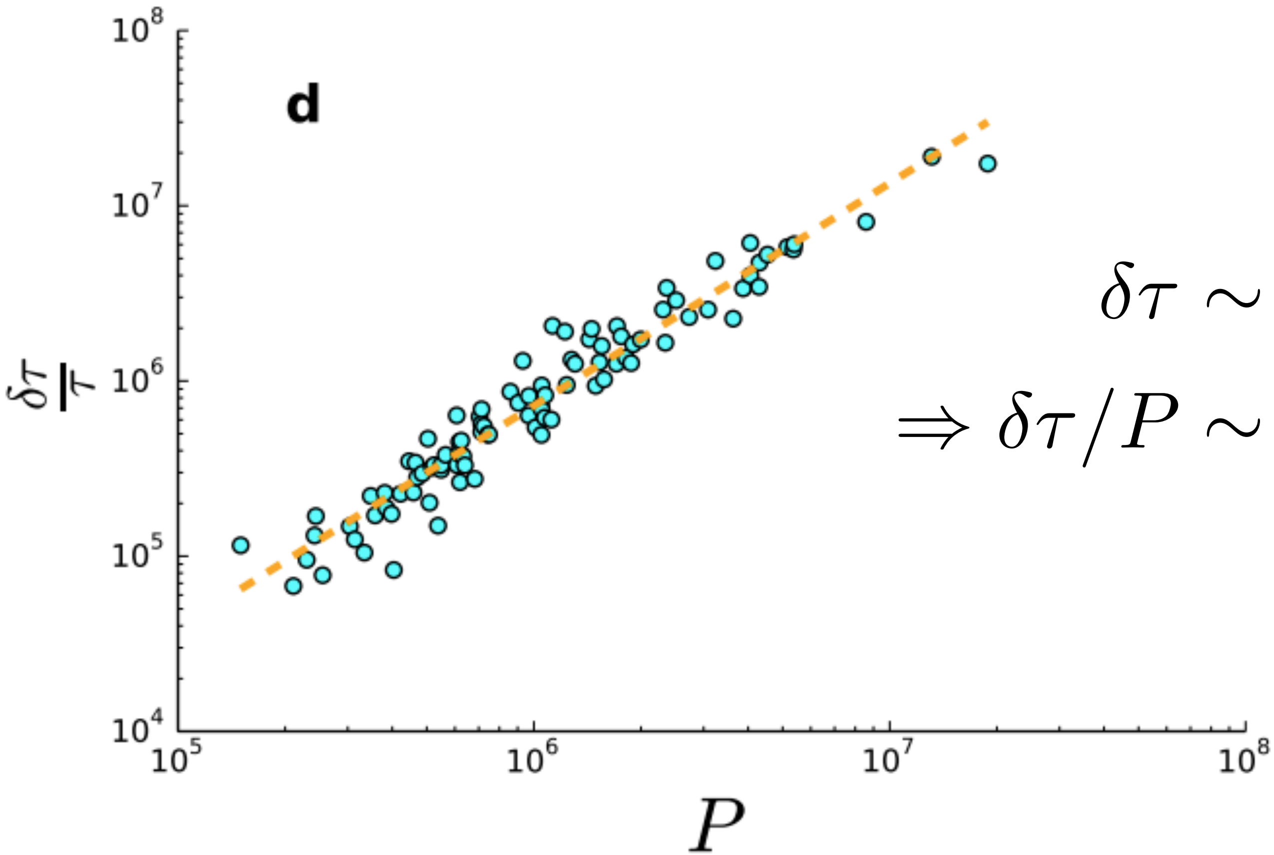
‘Phase diagram’



Number of hotspots H versus population P

Other quantities

- We know the location of home and office => we can compute other mobility-related quantities



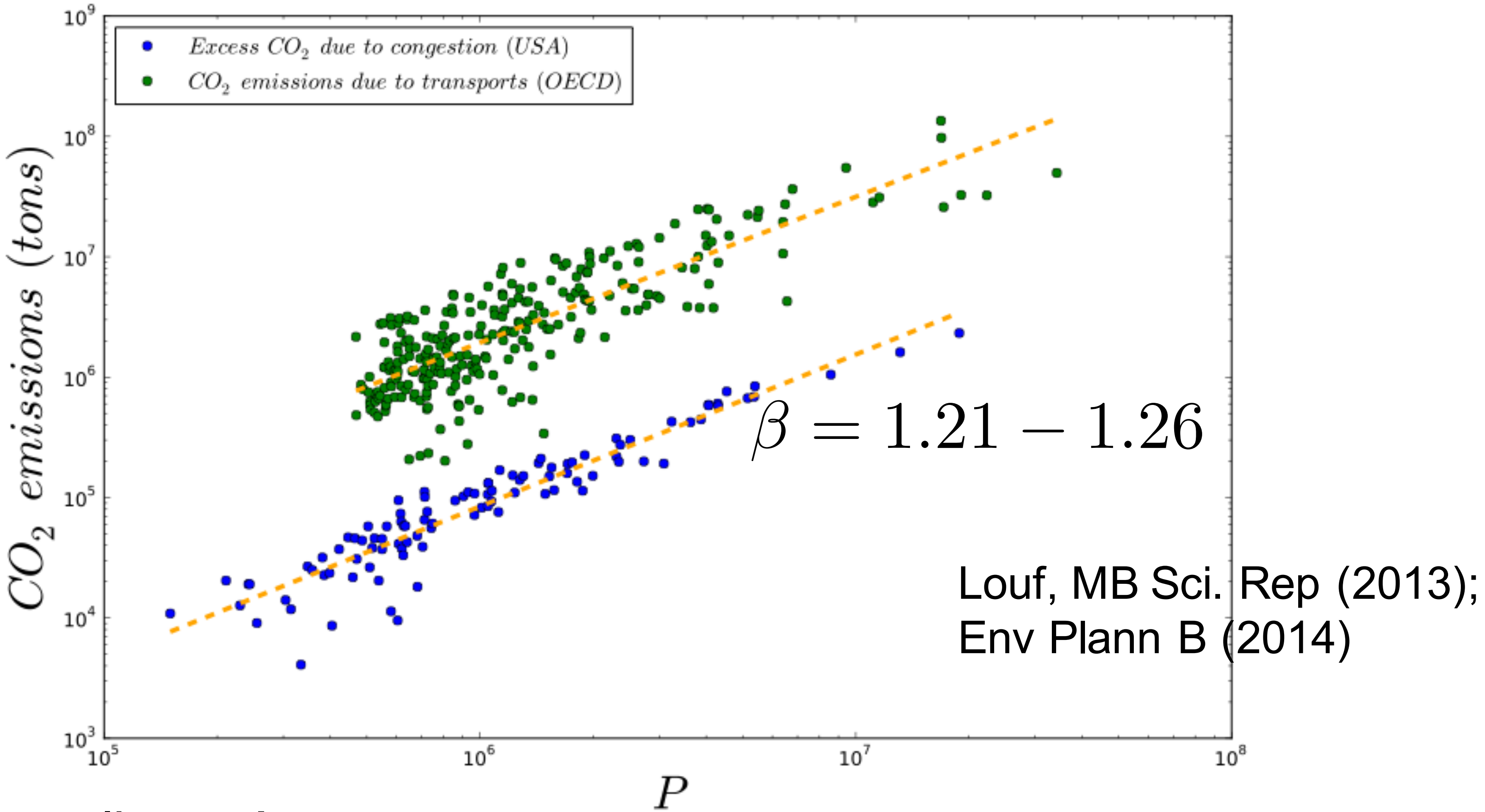
$$\delta\tau \sim P^{1.3}$$

$$\Rightarrow \delta\tau / P \sim P^{0.3}$$

- Scaling of delay due to traffic jams (US cities)

Scaling in cities

- Variation of socio-economical indicators with the population



- Superlinear !

Predicting the exponent values

Quantity	Theoretical dependence on P ($\delta = \alpha/\alpha + 1$)	Predicted value	Measured value
A/ℓ^2	$\left(\frac{P}{c}\right)^{2\delta}$	$2\delta = 0.78 \pm 0.20$	0.853 ± 0.011 ($r^2 = 0.93$) [USA]
L_N/ℓ	$\sqrt{P}\ \left(\frac{P}{c}\right)^{\delta}$	$\frac{1}{2} + \delta = 0.89 \pm 0.10$	0.765 ± 0.033 ($r^2 = 0.92$) [USA]
$\delta\tau/\tau$	$P\ \left(\frac{P}{c}\right)^{\delta}$	$1 + \delta = 1.39 \pm 0.10$	1.270 ± 0.067 ($r^2 = 0.97$) [USA]
$Q_{gas,CO_2}/\ell$	$P\left(\frac{P}{c}\right)^{\delta}$	$1 + \delta = 1.39 \pm 0.10$	1.262 ± 0.089 ($r^2 = 0.94$) [USA] 1.212 ± 0.098 ($r^2 = 0.83$) [OECD]

- Polycentrism is the natural response of cities to congestion, but not enough !

- For large P: Effect of congestion becomes very large
  => large cities based on individual cars are not economically sustainable !

Louf, MB (2013, 2014)

Discussion

- Pushing the models and compute predictions; testing predictions against data
- Goal: understand the hierarchy of mechanisms (and a model with a minimal number of parameters).
- Here: existence of a **dynamical transition** leading to a polycentric structure of activities
- Congestion: an important factor but not the only one
- End of story ? Integrating socio-economical factors: rent, other transportation modes, …

Thank you for your attention.

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