Banks on the verge of a crisis:
phase transitions and hysteresis in banking systems

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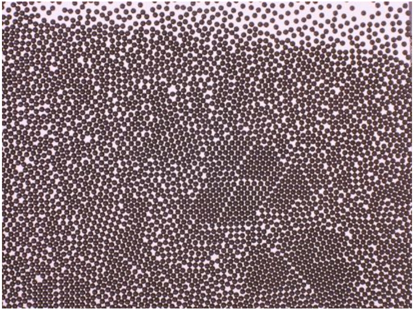
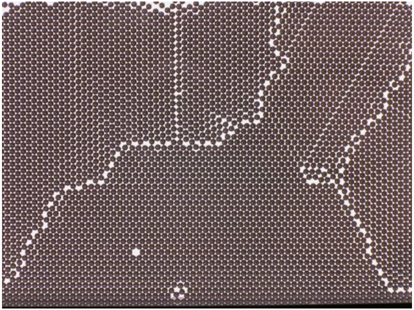
References:
Annika Birch and TA *"Systemic Losses Due to Counter Party Risk in a Stylized Banking System"* Journal of Statistical Physics **156** (2014) 998 - 1024
Annika Birch, Zijun Liu & **TA** *"A counterparty risk study for the UK banking system"* ssrn.com/abstract=2599891, under submission

Phase transition

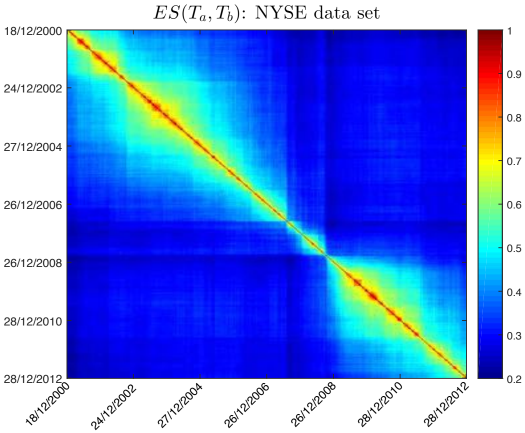
Change in the system state





Phase transition

Change in financial market structure across the banking crisis



N Musmeci, TA, Tiziana Di Matteo, arXiv:1605.08908 (2016)

What is a phase transition?

- **in Physics**
  - Change in the system state (liquid to solid) as consequence of a change in the parameters (temperature, pressure)
  - Change in the internal energy
  - Change in the system entropy
  - Emergence of collective properties
  - Order parameter becomes finite (symmetry breaking)
  - Appeerence of long-range correlations near the transition point
  - Appearance of “soft modes”
- **in General**
  - Change in the system state
  - ?

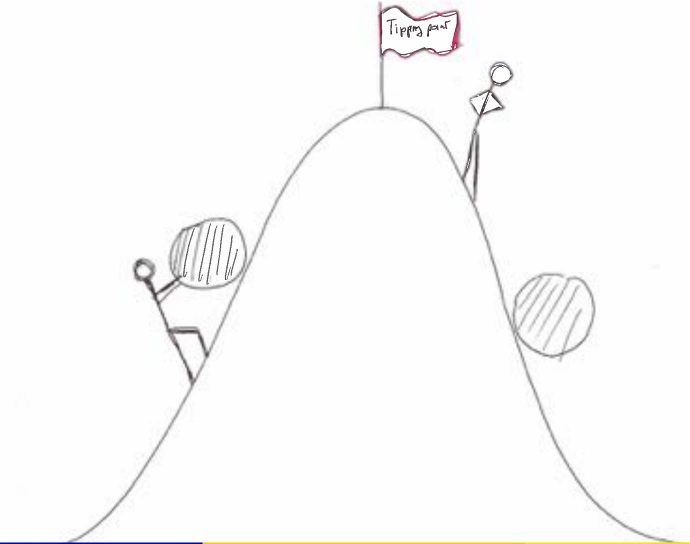
Tipping Point

the point at which small changes or incidents can cause large changes

- An object at a point of unstable equilibrium
- A rare phenomenon becoming rapidly more common
- The point at which a technology becomes dominant and the “winner takes all”
- A change happening as a consequence of a small cause that cannot be easly reverted

Tipping Point

the point at which small changes or incidents can cause large changes



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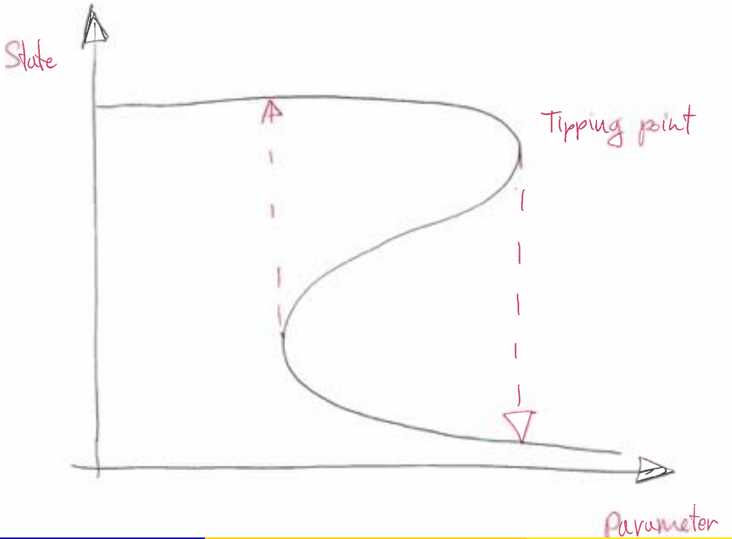

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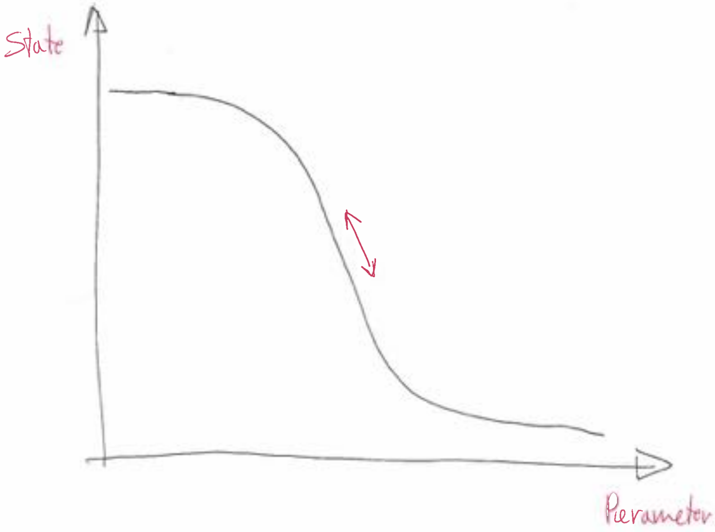


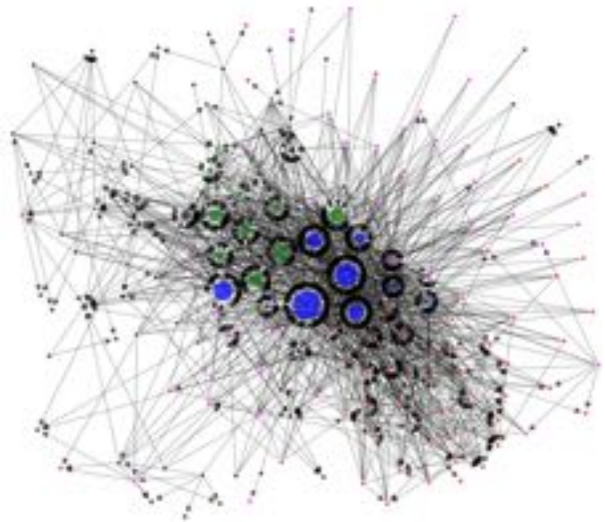
UK banking system data from BoE

We use form supervisory reports from Bank of England (BoE) for the years 2011, 2012 and 2013.

- Lending (unsecured, secured and undrawn)
- Holdings of equity and fixed-income securities (marketable securities) issued by banks;
- Credit default swaps (CDS) bought and sold
- Securities lending and borrowing (gross and net of collateral);
- Repo and reverse repo (gross and net of collateral);
- Derivatives exposures (with breakdown by type of derivative)

UK banks have to report their 20 largest counterparties to the BoE semi-annually. If the top 20 does not have at least six UK-based counterparties, banks report exposures to up to six UK-based counterparties in addition to the top 20. Branches of foreign banking groups in the UK are not included. There are 176 UK banks reporting to BoE.

UK Banking Network



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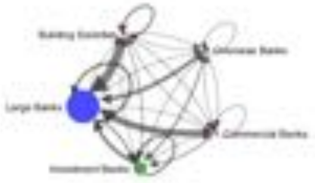
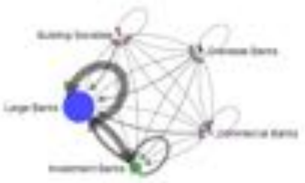
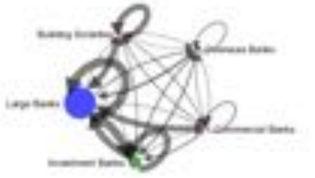


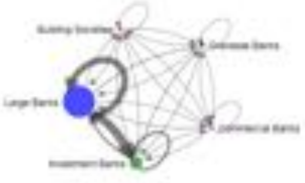
(a) In-degree



(b) In-weight



(c) Out-degree



(d) Out-weight

Phase transition in a Stylized Banking System

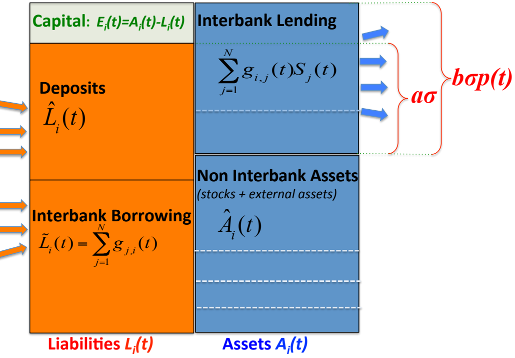
We propose a model combing the balance sheet based model[1][2], with the contagion model[3] creating a stylized banking system[4].
We distinguish between normally operating banks and distressed banks:

$$S_i(t) = \begin{cases} 1 & \text{if bank } i \text{ is } \textbf{operating normally} \\ 0 & \text{if bank } i \text{ is } \textbf{distressed} \end{cases} .$$

We consider a system of N banks that borrow and deposit money into each-other though an **interbank network**

[1] P. Gai et al. (2007). In: *Journal of Risk Finance* 8.2, pp. 156–165.
[2] E. Nier et al. (2007). In: *Journal of Economic Dynamics and Control* 31.6, pp. 2033–2060.
[3] J. P. Solorzano-Margain et al. (2013). In: *Computational Management Science*, pp. 1–31.
[4] S. Heise et al. (2012). In: *The European Physical Journal B* 85.4, pp. 1–19.

Balance Sheet

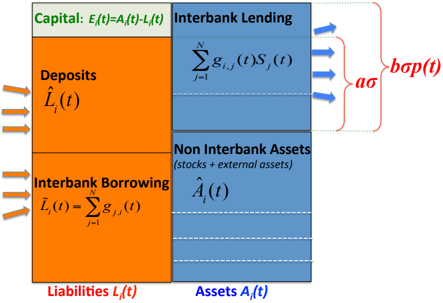


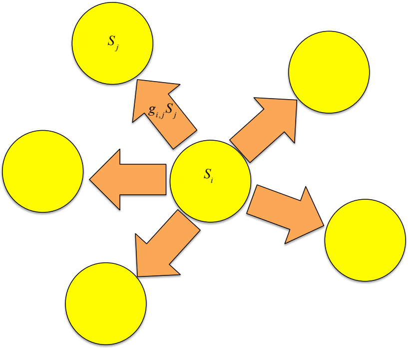
Liabilities:
$L_i(t)$ = sum of the bank's customer deposits, $\hat{L}_i(t)$, and interbank borrowings $\sum_j g_{j,i}$.

Assets:
$A_i(t)$ = sum of non-interbank assets, $\hat{A}_i(t)$, interbank lending to non distressed banks $\sum_j g_{i,j}(t) S_j(t)$

Bank's capital:
$E_i(t) = A_i(t) - L_i(t)$ (> 0)

Systemic risk





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Dynamics

Balance Sheet Equation: a bank operate normally if $A_i(t) \geq L_i(t)$, it is in distress otherwise. The state of a bank at time $t+1$ is

$$S_i(t+1) = \begin{cases} 1 & \text{if } A_i(t) - L_i(t) \geq 0 \\ 0 & \text{if } A_i(t) - L_i(t) < 0 \end{cases} .$$

The state of the system is associated with two main quantities:

Homogeneous system: *all banks same size and balance sheet quantities are randomly distributed with*

- **Non-Counterparty-dependent balance sheet**
  $\underbrace{L_i(t)}_{\textit{Liabilities}} - \underbrace{\hat{A}_i(t)}_{\textit{Non-Counterparty-dependent assets}}$

  mean: $a\sigma$; **Variance:** σ^2
- **Counterparty-dependent assets:**
  $\sum_j g_{i,j} S_j(t)$

  mean: $b\sigma p_t$

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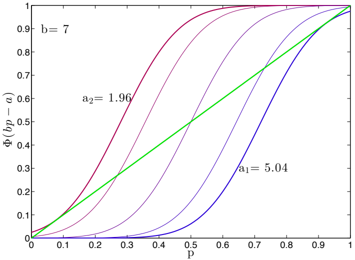
Fixed Point Solutions (Normal distribution)

$$p_{t+1} = \Phi(bp_t - a)$$

$$p = \Phi(bp - a)$$



Small a (large non-interbank assets):
only one solution $p = 1$
- *all banks functioning normally*

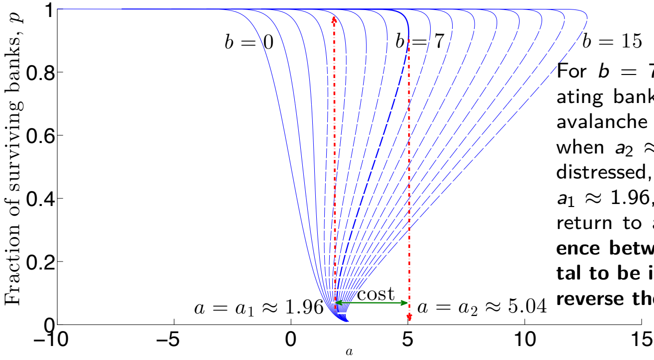
Large a (small non-interbank assets):
only one solution $p = 0$
- *all banks in distress*

Intermediate a: three solutions
- one unstable and 2 stable solutions

The central fixed point is unstable and forms a barrier.
The dynamics becomes irreversible.

Efficiency, Resistance, Failure and Recovery

Let us consider a banking system put to stress by decreasing non-interbank assets (i.e. increasing a) with interbank assets unchanged (i.e. b constant)



For $b = 7$, starting from fully operating banks (i.e. $p_0 = 1$) an infinite avalanche can bring down the system when $a_2 \approx 5.04$. Once all banks are distressed, a needs to be lowered to $a_1 \approx 1.96$, in order for the system to return to a stable state. **The difference between a_2 and a_1 is the capital to be injected into the system to reverse the state**

Fraction of surviving banks as a function of a given b. Solid lines indicate stable fixed points, dotted lines unstable fixed points.

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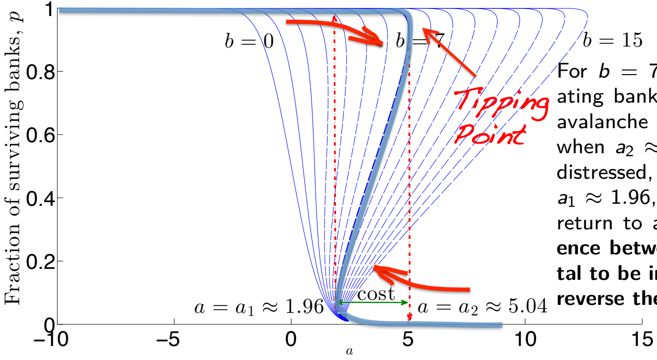

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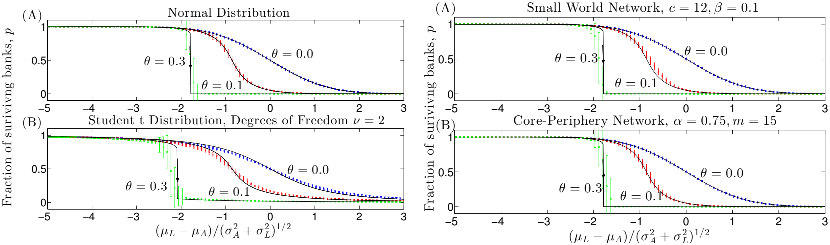
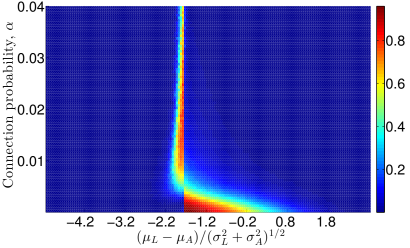
Interbank network effect: simulation results





Heterogeneous system simulations calibration

- The system consists of $N = 175$ banks.
- Assets $A_i(0)$ for each bank are drawn from a distribution with mean $\boldsymbol{\mu}_{A_i}$ retrieved from the data and variance $\boldsymbol{f}_A \boldsymbol{\mu}_{A_i}$ with $f_A = 0.001$
- Liabilities $L_i(0)$ for each bank are drawn from a distribution with mean $\boldsymbol{\mu}_{L_i}$ retrieved from the data but multiplied by a factor $\boldsymbol{f}_L \ \ \boldsymbol{\mu}_{L_i}$, variance is assumed $\boldsymbol{f}_A \boldsymbol{\mu}_{A_i}$
- Normal distributions are assumed
- Interbank assets are retrieved from the data
- Interbank lending structure is retrieved from the data
- recovery rate is $q = 0$

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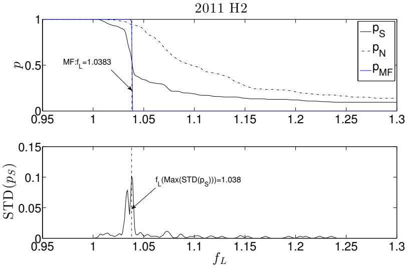
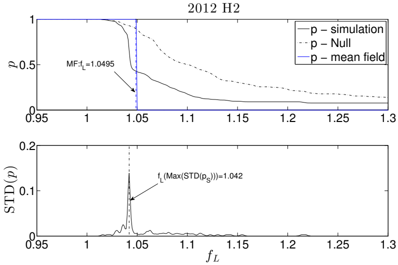
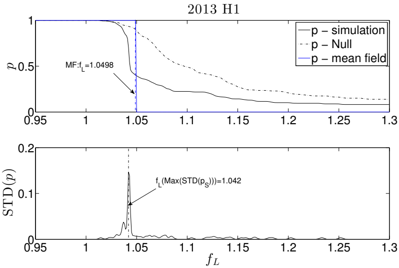
Results

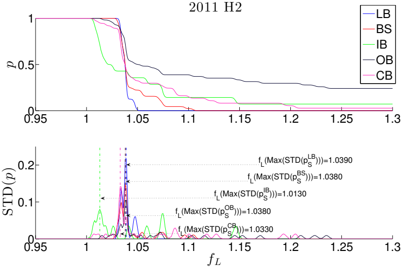
Fraction of surviving banks as function of mean liabilities. Comparison with homogeneous system model (mean field, MF) and system with no contagion (null, N)

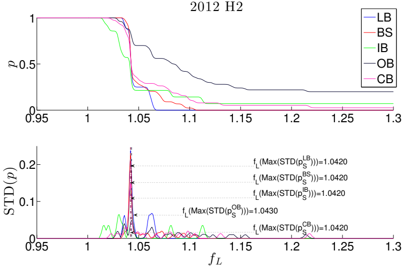


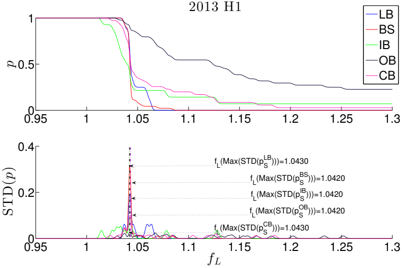




Fraction of surviving banks as function of mean liabilities Results for different bank types: “Large Banks" (LB), “Building Societies" (BS), “Investment Banks" (IB), “Oversea Banks" (OB) and “Other Commercial Banks" (CB)

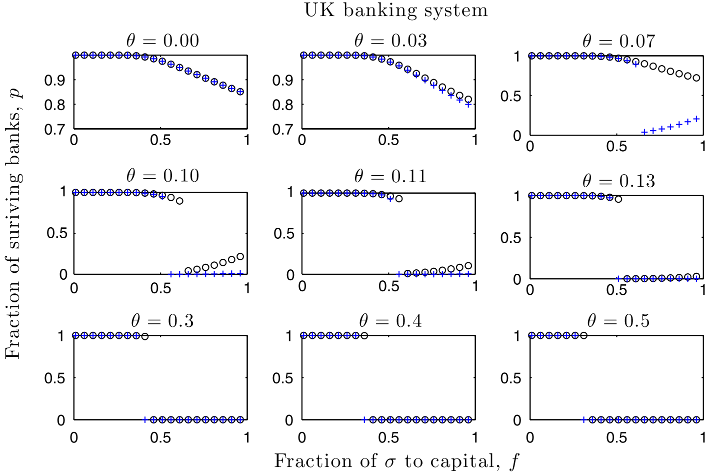






Emergence of systemic failure

Fraction of surviving banks against the fraction of capital



blue crosses - 2007; black circles - 2012
θ fraction of interbank assets to total assets
σ amplitude of fluctuations
p fraction of surviving banks

Conclusions

- a simple model from physics for a banking system where systemic fragility emerges
- in the fragile state a failure can trigger an avalanche that brings down the entire system
- Once the system fails it cannot go back to operating state without a recovery cost
- Numerical simulations show that the prediction of the homogeneous model are reproducing well the behavior of more realistic heterogeneous systems calibrated on BoE UK data
- tipping points
- phase transitions

Thank You

http://fincomp.cs.ucl.ac.uk/

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