Dynamics of Instabilities and Intermittency

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We study the behavior of a finite classical system in the instability region. The equation of state of such a system resembles that of nuclear matter. Through a study of mass distributions, scaled factorial moments, and anomalous fractal dimensions, we provide evidence of the presence of critical behavior of our system. Such behavior can be understood by use of the droplet model of the liquid-gas phase transition.

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Recent experiments in heavy ion (HI) collisions at energies around the Fermi energy have revealed the creation of many fragments in the final stages of the reaction. These experiments were prompted by some theoretical speculations based on the present knowledge of the nuclear matter equation of state (EOS) that pointed out the presence of instabilities and the possibility of a liquid-gas phase transition. Of course one would like to know if the detected fragments have something to do with the predicted phase transition. Strictly speaking sharp phase transitions do not exist in a system with a small number of constituents, and fluctuations may play an important role [1]. In particular, in such small systems such as two colliding nuclei, the fluctuations can completely wash out the phase transition. Furthermore, assuming that a phase transition is possible, the problem is how to provide evidence of it from the large amount of experimental data. In this Letter we address both problems and we demonstrate that finite systems may in fact exhibit a critical behavior that can be revealed through an intermittency analysis.

An exact solution of the quantum many-body problem is presently out of sight, and this is especially true for the nuclear systems. The dynamical approaches available give the time evolution of the one-body distribution function as in time-dependent Hartree-Fock theory or the semiclassical analog, i.e., the Vlasov equation. This is clearly not sufficient when the system enters the spinodal region and long-range correlations become important to form blobs of matter. On the other hand, the exact classical many-body problem can be quite easily solved for a system made of about 100–400 particles. In classical molecular dynamics all correlations are present, therefore we can gain important information from a detailed dynamical study. The important problem is to try to understand what the role of quantum fluctuations would be. Naively one would expect that quantum effects smooth any sharp transition that can be present in the classical limit. However, if excitation energies are large and densities are small, then the classical limit may be a good approximation.

Let us assume that the nucleus is made up of A nucleons that behave classically. In this model, particles move under the influence of a two-body potential V given by [2]

$$\begin{aligned} V_{np}(r) &= V_r[\exp(-\mu_r r/r) - \exp(-\mu_r r_c/r_c)] \\ &\quad - V_a[\exp(-\mu_a r/r) - \exp(-\mu_a r_a/r_a)], \\ V_{nn}(r) &= V_{pp}(r) = V_0[\exp(-\mu_0 r/r) - \exp(-\mu_0 r_c/r_c)]. \end{aligned} \tag{1}$$

$r_c = 5.4$ fm is a cutoff radius. V_{np} is the potential acting between a neutron and a proton, while V_{nn} is the potential acting between two identical nucleons. The first potential is attractive at large r and repulsive at small r, while the latter is purely repulsive so no bound state of identical nucleons can exist. This is done in order to somehow mimic the Pauli principle. The values of the parameters entering the Yukawa potentials are given in Ref. [2] and give a corresponding EOS of classical matter having about 250 MeV of compressibility (set M in Ref. [2]). This EOS strikingly resembles that of nuclear matter [i.e., equilibrium density $\rho_0 = 0.16$ fm^{-3} and energy $E(\rho_0) = -16$ MeV/nucleon]. Furthermore, in Refs. [2,3] it is shown that many experimental data on HI collisions are reasonably explained by this classical model. Of course this is not accidental, but it is due to the accurate choice of the parameters of the two-body potentials.

The classical Hamilton equations of motion are solved using the Taylor method at the order $O((\delta t)^3)$, where δt is the integration time step [4]. The nucleus is initialized in its ground state by using the frictional cooling method [5]. After, it is excited at a temperature T giving a Maxwellian velocity distribution to its nucleons, by means of a Metropolis sampling [4]. We have studied the disassembly of a $(A = 100, Z = 50)$ nucleus starting from an initial density $\rho = 0.125$ fm^{-3} and with different values of the initial temperature. In our calculations, the Coulomb interaction is not taken into account.

In Fig. 1 we plot the time evolution of our system in the density-temperature plane. In the plot the full lines give the isothermal (ITS—isothermal spinodal) and

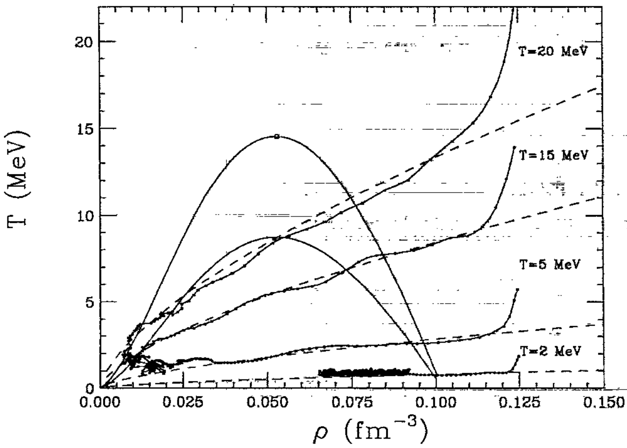

FIG. 1. Average temperature versus average density of the biggest fragment in the expansion of nuclei starting from temperatures $T = 2$, 5, 15, and 20 MeV. Solid curves are the ITS (top) and the AS (bottom) lines and dashed curves some of the isentropes of the system.

isentropic (AS—adiabatic spinodal) spinodal regions of infinite classical matter. The point at $T = 15$ MeV and $\rho = 0.05$ fm^{-3} gives the critical point for the liquid-gas phase transition in an infinite system. These curves are calculated from the polynomial fit to the resulting mean field for our system given in Ref. [2]. Note again the strong resemblance to the (predicted) EOS of nuclear matter. The dashed lines give the values for isentropic expansion.

In the calculations density and temperature are determined following Ref. [6]. Since in the initial stage the system is not perfectly equilibrated, the expansion turns out to be not isentropic. But quickly (after about 5 fm/c) entropy creation stops and the following expansion is isentropic. We discuss first the $T = 2$ MeV case. This is a typical case of evaporation; the system expands and emits particles. Quickly the expansion comes to a halt and the system oscillates back and forth while it cools down through particle emission. It is important to note that the system enters the region of instability of infinite matter. Finite size effects reduce such a region and shift it at lower densities.

At higher temperatures the system enters deeply into the instability region. In particular, for the highest temperatures, 15 and 20 MeV, the densities reached are sometimes outside the instability region from the gas side. This implies that there is a quick expansion which leads the system in the gas region, and then small drops start to form. This is approximately true for all the expansions at T larger than 5 MeV. For lower temperatures the system never hits the gas region and bubbles form from the liquid side.

The probability of formation of bubbles (droplets) can be estimated in the Fisher model [7] and results in a mass yield given by

$$Y = Y_0 A^{-\tau} \exp[-(f - \mu A)/T], \qquad (2)$$

where f is the Helmholtz free energy of the cluster, μ is the chemical potential, and τ is a critical exponent and is related to the curvature energy.

In the upper part of Fig. 2 we plot the mass yield corresponding to three different initial temperatures. At each temperature 10 000 events were performed in order to generate the mass distributions. Following Eq. (2) we

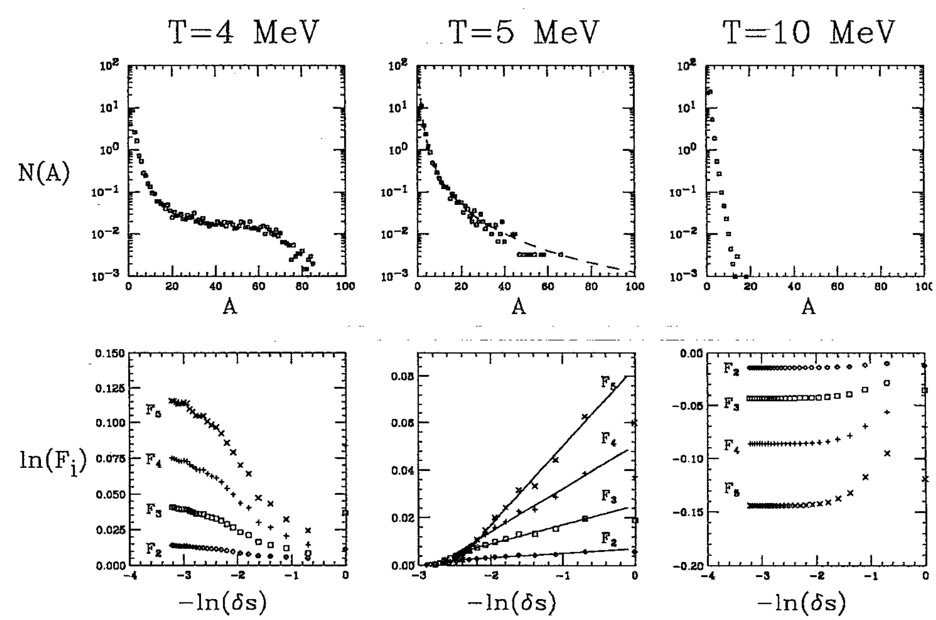

FIG. 2. Mass distributions and the corresponding scaled factorial moments $\ln(F_i)$ versus $-\ln(\delta s)$ for events with initial temperatures $T = 4$, 5, and 10 MeV.

fitted the mass distributions according to

$$Y(A) = Y_0 A^{-\tau} \exp\left[\frac{b(T)}{T} A^{2/3} + \frac{a(T)}{T} A\right],$$
$$Y(A) = Y_0 A^{-\tau} X^{A^{2/3}} Y^A, \quad (3)$$

where X, Y, and τ are fitting parameters. In the droplet model of Fisher [7], τ is related to the curvature energy [8] while $a(T)$ and $b(T)$ are related to volume and surface contributions. In this model $b(T) = b(0)(1 + 3T/2T_c)(1 - T/T_c)^{3/2}$ for $T < T_c$, and it becomes zero at temperatures larger than T_c. From the fits to the mass distributions we get $\tau = 2.23$, X very close to 1 for temperatures larger than 4 MeV while $Y = 1.0182$, 0.995, 0.867, 0.697, 0.431, and 0.3097 at $T = 4$, 5, 7, 10, 15, and 20 MeV, respectively. These values for the fitting parameters are obtained in the Fisher model around the critical point [1]. The point $X = 1$, $Y = 1$ is obtained at the critical temperature, i.e., $T_c \approx 5$ MeV. Note the large difference in the temperature with the value obtained in the mean field description of the infinite system.

A value $X = 1$ implies that the surface tension goes to zero. Also, the value of temperature for which the surface tension disappears in nuclei is approximately the same [9]. This again is due to the resemblance of the EOS in the two cases and the close analogies between finite nuclei and the classical approximation we are using. At $T = 3$ MeV, a fit to the mass yield gives a surface tension $\sigma_s(0) = b(0)/4\pi r_0^2 \approx 1$ MeV/fm^2.

Of course the mass yield shape alone cannot be considered as conclusive proof for a critical behavior that recalls a phase transition in an infinite system.

Many methods have been developed to analyze the fluctuations and the correlations for various physical quantities. In particular, one of the most powerful and promising possibilities seems to be the analysis of event by event data in terms of intermittency. Intermittency is a statistical concept initially developed to study turbulent flows [10], and now applied in many different fields. Bialas and Peschanski introduced this idea to study the dynamical fluctuations in rapidity distributions of particles produced in relativistic energy HI reactions [11], and more recently Ploszajczak and Tucholski have found intermittent charge patterns in nuclear multifragmentation at intermediate energy, both in data and in models [12].

Generally, the occurrence of intermittency corresponds to the existence of large nonstatistical fluctuations which have self-similarity over a broad range of scales. This signal can be deduced from the scaled factorial moments which measure the properties of dynamical fluctuations without the bias of statistical fluctuations [11]:

$$F_i(\delta s) = \frac{\sum_{k=1}^{X_{\max}/\delta s} \langle n_k(n_k - 1)\cdots(n_k - i + 1)\rangle}{\sum_{k=1}^{X_{\max}/\delta s} \langle n_k\rangle^i}. \quad (4)$$

Here $X_{\max}$ is an upper characteristic value of the system (i.e., total mass or charge, maximum transverse energy or momentum, etc.) and i is the order of the moment. The total interval $0 - X_{\max}$ is divided in $M = X_{\max}/\delta s$ bins of size δs, n_k is the number of particles in the kth bin for an event, and the brackets $\langle\,\rangle$ denote the average over many events. If self-similar fluctuations exist at all scales δs, the scaled factorial moments follow the power law $F_i(\delta s) \propto (\delta s)^{-\lambda_i}$, where λ_i are called intermittency exponents. So the intermittent behavior is defined as a linear rise in a plot of $\ln(F_i)$ versus $-\ln(\delta s)$.

An important quantity connected to the intermittency exponents is the anomalous fractal dimension [11,12,13]

$$d_i = \lambda_i/(i - 1). \quad (5)$$

Different processes seem to give a different behavior of these anomalous fractal dimensions d_i.

(i) $d_i =$ const corresponds to a monofractal, second order phase transition in the Ising model and in the Feynman-Wilson fluid [14,15]. It has also been demonstrated that in the case of a second order phase transition in the Ginzburg-Landau (GL) description one gets [15]

$$d_i = d_2(i - 1)^{\nu - 1}, \quad (6)$$

with $\nu = 1.304$. We note that the GL theory can also be applied to the mean field calculated for our system [2]. In such a case, it can be easily proven that the results obtained in Ref. [15] remain valid in our case when there is no external field [1,16].

(ii) $d_i \propto i$ corresponds to multifractal, cascading processes [11]. Therefore, a study of the anomalous fractal dimensions can give useful information about the evolution of the system [13].

In the lower part of Fig. 2 we plot the scaled factorial moments $\ln(F_i)$ versus $-\ln(\delta s)$. At $T = 10$ MeV, the system goes into complete vaporization, and the mass distribution has a rather steep slope. The logarithm of the scaled factorial moments $\ln(F_i)$ is always negative and independent of δs (i.e., variances are smaller than Poissonian [12]), and we have no intermittency signal.

The situation is different for the case $T = 5$ MeV. The logarithms of the scaled factorial moments are positive and almost linearly increasing versus $-\ln(\delta s)$, and the intermittency signal is observed. The presence of large fluctuations as indicated by the intermittency analysis plus the power law in the mass distribution for initial temperatures between 4 and 5 MeV indicate a self-similar behavior both for fluctuations and for averages. These features might be connected to a second order phase transition in an infinite system. But our system contains 100 constituents only. To better clarify this point we plot in Fig. 3 the anomalous fractal dimensions d_i versus i obtained at different temperatures. At $T = 4$ MeV, the d_i's are negative, while they are positive and almost on an increasing straight line for T larger than 4.5 MeV. We have fitted these curves according to Eq. (6) and found $\nu = 2.0$, 1.89, and 1.84 at $T = 4.5$, 4.75, and 5 MeV, respectively. A similar estimate but for a larger system $A = 400$ gives $\nu = 1.68$, 1.75, and 1.74 at $T = 4$, 5, and

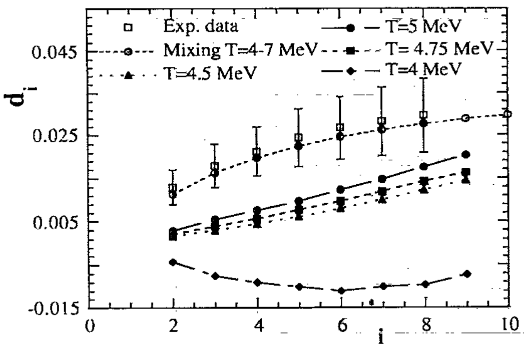

FIG. 3. Anomalous fractal dimensions d_i versus i for different temperatures T. Open squares represent the experimental data from Ref. [12], and open circles represent our results mixing the events having more than three fragments ($A > 4$) from $T = 4$, 5, 6, and 7 MeV calculations.

6 MeV, respectively. Recall that the value $\nu = 1.304$ was obtained in mean field theory and therefore it represents a rough estimate of its actual value. Our calculated values are larger than the GL estimate. Furthermore, increasing the mass of the systems results in ν values closer to the mean field estimate [1,16].

In the same figure we plot the experimental data obtained in Au fragmentation [12]. Our calculations have the same behavior of the data but are shifted down a factor of 2. In order to understand the difference between our results and the data, we have mixed the events with initial temperatures in the range of 4 to 7 MeV and we have chosen only those events having more than three fragments ($A > 4$) similar to the experimental cuts [12]. The results are shown in Fig. 3 by the open circles. They are in nice agreement with the data and exhibit the feature of being constant for large i values. In previous works [12,13] the apparent flattening of the data, which is in contrast to a cascade process, was related to the possibility of finite size effects. Our results instead indicate that the flattening is solely due to *the mixing of events and to the cuts in the multiplicities.*

In conclusion, we have shown in a dynamical model the critical behavior of a highly excited finite system. Under some initial conditions, the dynamical evolution creates a power law mass distribution of fragments with $\tau =$ 2.23 and develops an intermittent pattern of fluctuations. These features are reminiscent of a second order phase transition in an infinite system.

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