Open Problems in Mathematics

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Open Day Presentation 2009

Topic Outline

1. Some Thoughts about Mathematics
   - Why You Should Study Mathematics
   - What is Mathematics
2. Mathematical Problems
   - Some Million Dollar Problems
   - Examples of Solved and Open Problems
3. The 3n+1 Problem
   - Statement of the Problem
   - Some Examples
   - Why the Conjecture should be True
   - Extending the Problem
   - The Ulam Spiral

Outline

1. Some Thoughts about Mathematics
   - Why You Should Study Mathematics
   - What is Mathematics
2. Mathematical Problems
3. The 3n+1 Problem

Top 10 Reasons to Study Maths

10 So you'll know that a negative number isn't a number with an attitude problem.

9 When the teacher talks about an acute angle, you'll know she's not referring to how attractive the angle is.

8 So you'll realize that a factor tree is not the oak next to the school parking lot.

7 Because studying π is simply delicious!

6 To learn that irrational numbers really do make sense.

5 So you'll realize that place value doesn't refer to how close your desk is to the pencil sharpener.

4 So you'll understand that the distributive property has nothing to do with real estate.

3 Because solving word problems could lead to solving the world's problems.

2 To learn that Dewey didn't invent the decimal.

1 If you can't count to a million, how will you know if you've become a millionaire?

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- You are really good at maths
- You like problem solving
- You could get into business school
- You want to keep your options open

Bad Reasons for Studying Mathematics

- Your language skills are really weak
- You like memorising formulas
- You can't get into business school
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Mathematics is not

- just "doing things with numbers and letters and other symbols"
- just a collection of facts and rote recipes
- just computational and arithmetic skills

Mathematics is

- a way of thinking
- the language of science
- a creative discipline
- a source of pleasure and wonder
- a means of problem solving

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Seven Million Dollars Prize Money

7 Prize Problems, selected by Clay Mathematics Institute in 2000

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

These are hard problems (it might be easier to rob a bank...)

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Solved Problems in Mathematics

Some recently proved problems:

- Fermat's last theorem (1637, proved 1994): If an integer n is greater than 2, then the equation

  $$a^n + b^n = c^n$$

  has no solutions in non-zero integers a, b, and c.

  For $n = 2$, this is of course possible, for example

  $$3^2 + 4^2 = 5^2 \ .$$

- The four colour theorem (1852, proved 1976): Given any plane separated into regions, such as a political map of the states of a country, the regions may be coloured using no more than four colours.

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Open Problems in Mathematics

Some unsolved problems:

- Goldbach's conjecture (1742): every even integer greater than 2 can be written as the sum of two primes.

  $$\text{For example, } 18 = 5 + 13 = 7 + 11.$$

- The twin prime conjecture (300 BC): there are infinitely many primes p such that $p + 2$ is also prime.

  For example, 17 and 19 are twin primes.

- How many different Sudoku squares of size $n \times n$ are there? There are

  $$6,670,903,752,021,936,960$$

  valid 9×9 Sudoku squares. The problem is to find a formula for general n.

There are many more well-known open problems, see e.g.
http://en.wikipedia.org/wiki/Unsolved_problems_in_mathematics

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"The history of mathematics is a history of horrendously difficult problems being solved by young people too ignorant to know that they were impossible."

Freeman Dyson, "Birds and Frogs", AMS Einstein Lecture 2008

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   - The Ulam Spiral

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Some Examples

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- $n = 11$ produces the sequence

$$11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.$$

- $n = 27$ produces the sequence
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526,263,790,395,1186,593,1780,890,445,1336,668,334,167,502,
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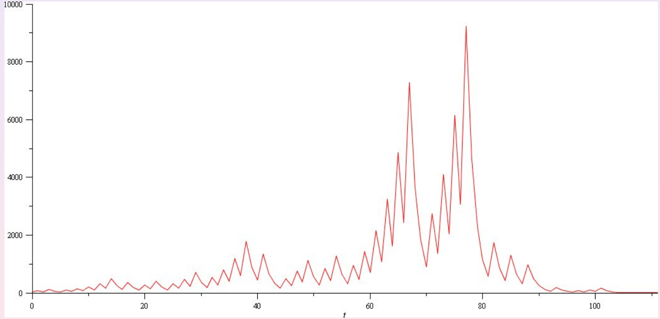
Graphing the Sequences

A graph of the sequence obtained from $n = 27$



This sequence takes 111 steps, climbing to over 9000 before descending to 1.

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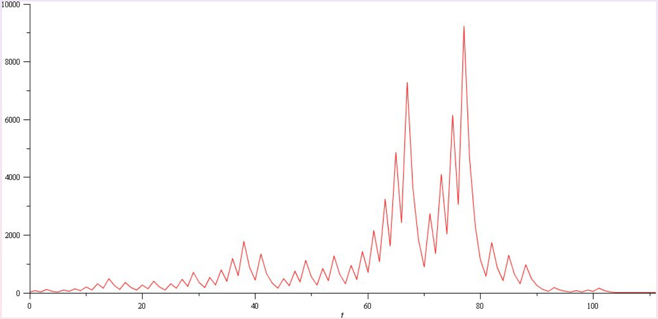

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Supporting Arguments for the Conjecture

- Experimental evidence:

  The conjecture has been checked by computer for all starting values up to $19 \times 2^{58} \approx 5.48 \times 10^{18}$.

- A probabilistic argument:

  One can show that each odd number in a sequence is on average 3/4 of the previous one, so every sequence should decrease in the long run.

  This not a proof because Collatz sequences are not produced by random events.

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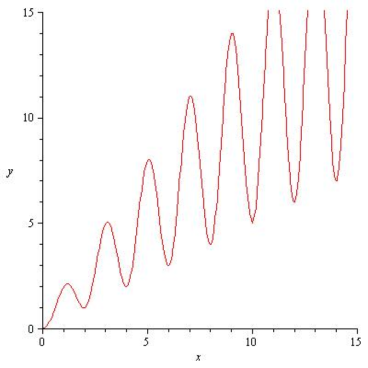
Iterating on Real Numbers



Reduce an orbit by replacing $3n+1$ with $(3n+1)/2$:

$$10, 5, 16, 8, 4, 2, 1, 4, 2, 1, \ldots$$

shortens to

$$10, 5, 8, 4, 2, 1, 2, 1, \ldots$$

The function graphed is given by

$$f(x) = \frac{x}{2}\cos^2\left(\frac{\pi}{2}x\right) + \frac{3x+1}{2}\sin^2\left(\frac{\pi}{2}x\right)$$

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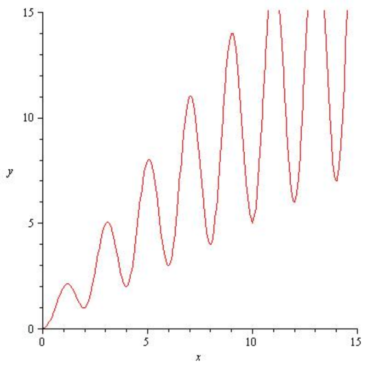

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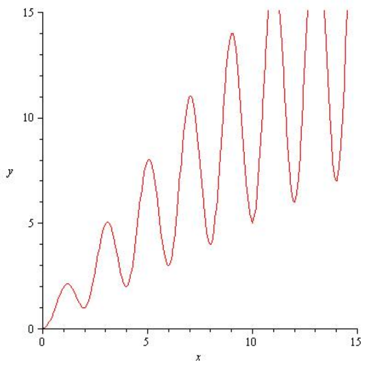

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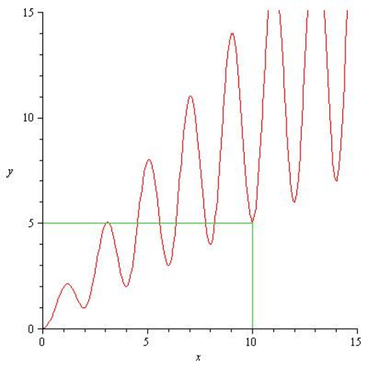

$10 \longrightarrow 5$

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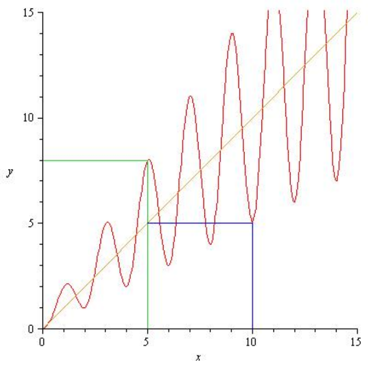

$10 \to 5 \to 8$

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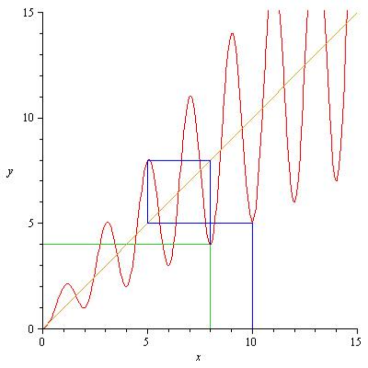

$10 \to 5 \to 8 \to 4$

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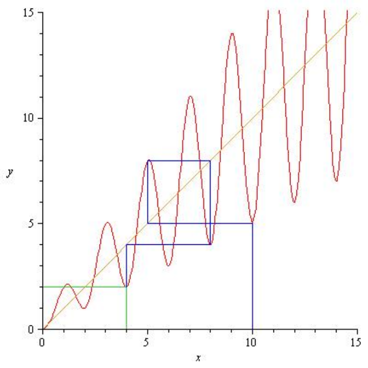

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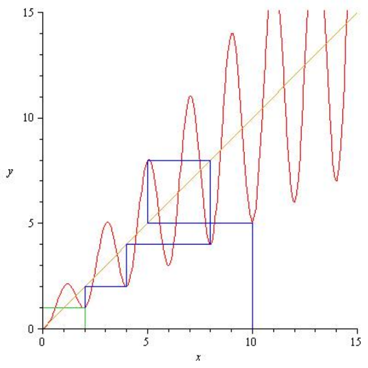

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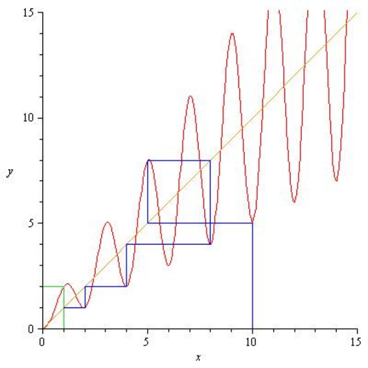

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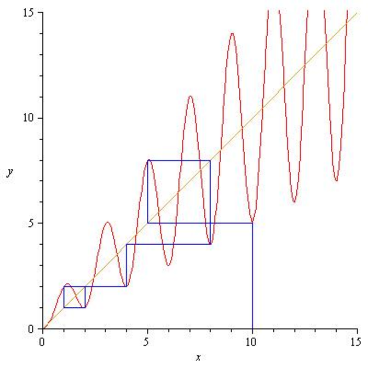

A "cobweb" plot

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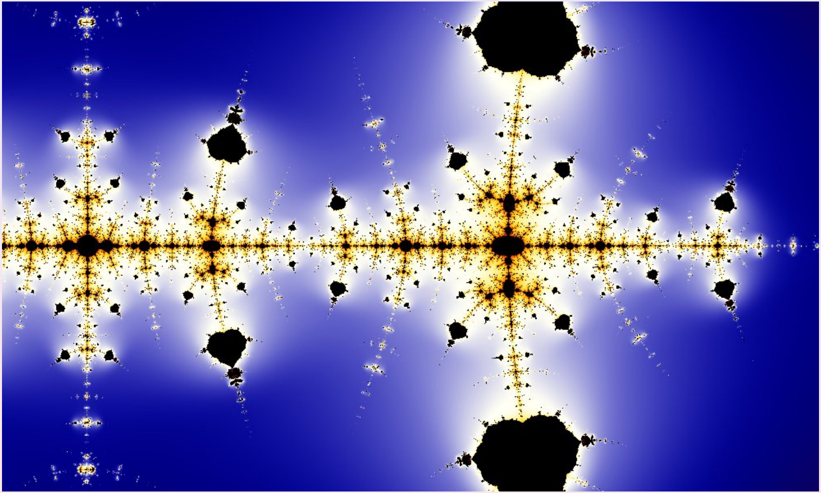
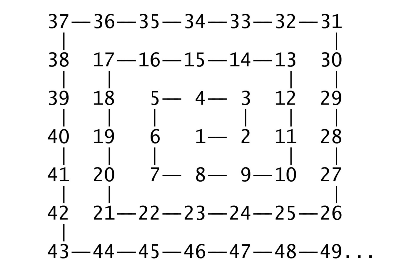


$3n + 1$ on the Ulam Spiral



The $3n + 1$ iteration on the Ulam spiral

> *"Mathematics is not yet ready for such problems."*

Paul Erdős, 1913 - 1996

The End