Depinning in Driven Viscoelastic Media with Disorder: a Dynamical Phase Transition

Thomas Prellberg
Technische Universität Clausthal

with

M. Cristina Marchetti and A. Alan Middleton
Syracuse University

"Viscoelastic Depinning of Driven Systems: Mean-Field Plastic Scallops," Phys. Rev. Lett. **85** (2000), 1104-1107

The problem

Drops of rain on a dirty windshield

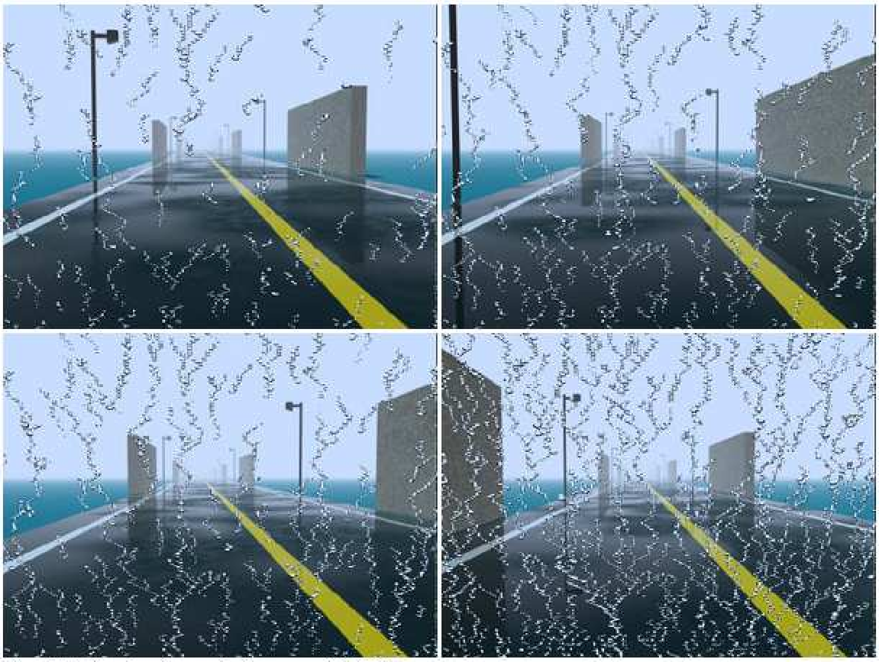
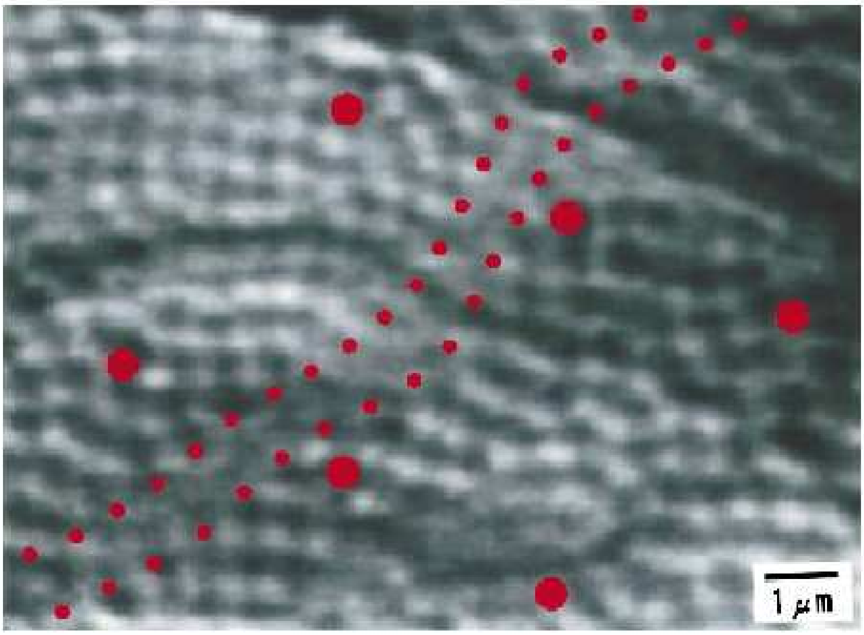


Figure 7: Animation of water droplets on a windshield.

(K. Kaneda, 1999: computer simulation)

Motion of flux lines in superconductors



(A. Tonomura, 1999: *Nb* film)

Viscoelastic depinning

Extension of a model for depinning in elastic media

- **Phenomenon:** plastic reaction of driven systems in the presence of frozen ("quenched") disorder.
- **Model:** coarse grained displacement field with viscoelasitic coupling in a disordered pinning potential.
- **Systems:** *flux lines in type-II superconductors*, charge density waves (CDW), sliding friction, ...
- **Properties:** model shows both *continuous* and *hysteretic* depinning, possibility of *"slip-stick instabilities"*.
- **Relevance:** modelling of experimentally observable phenomena.

H. Fukuyama and P.A. Lee, Phys. Rev. B **17**, 535 (1977)

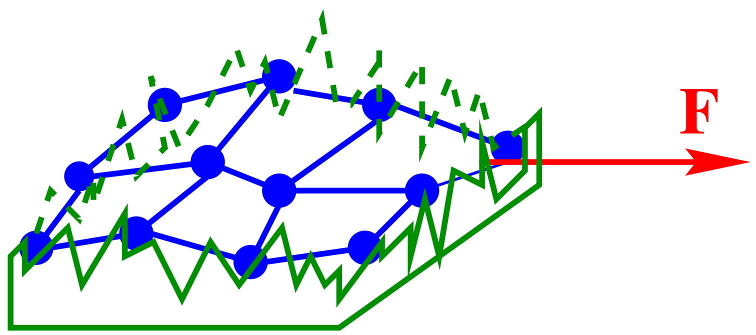
D.S. Fisher, Phys. Rev. B **31**, 1396 (1985)

A.A. Middleton, Phys. Rev. Lett. **68**, 670 (1992)

M.C. Marchetti *et. al.*, Phys. Rev. Lett. **85** 1104 (2000)

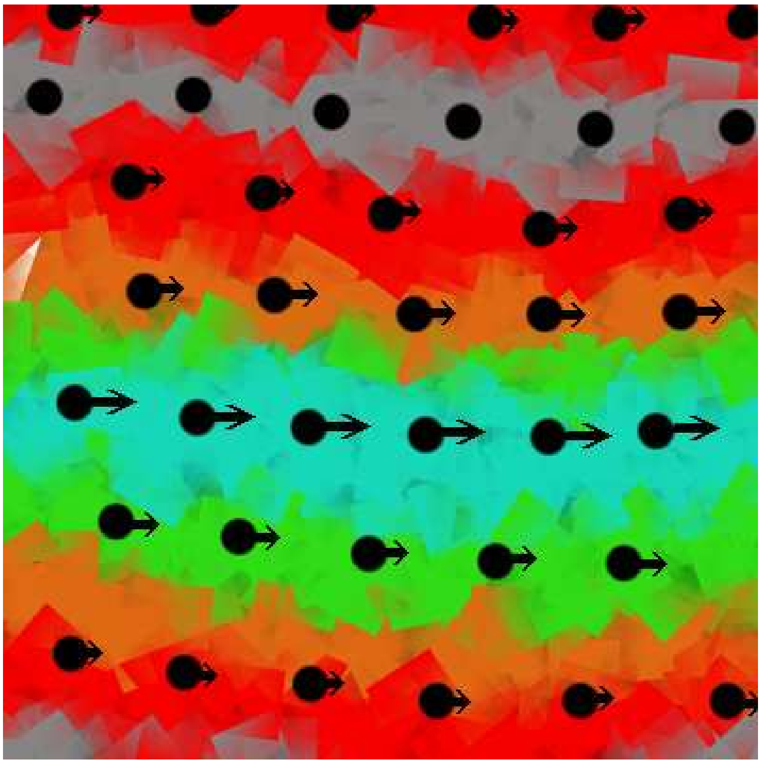
Flow through frozen disorder

- Many classes of simulations
  - Flux line lattices (FLL) in type-II superconductors
  - Adsorbate layers, friction phenomena
  - Atoms in real materials
  - Flow networks
- Superconductors, e.g.: melting of driven FLL driven channel flow
- Scant *general* theoretical understanding
  - Which kind of models should be used?

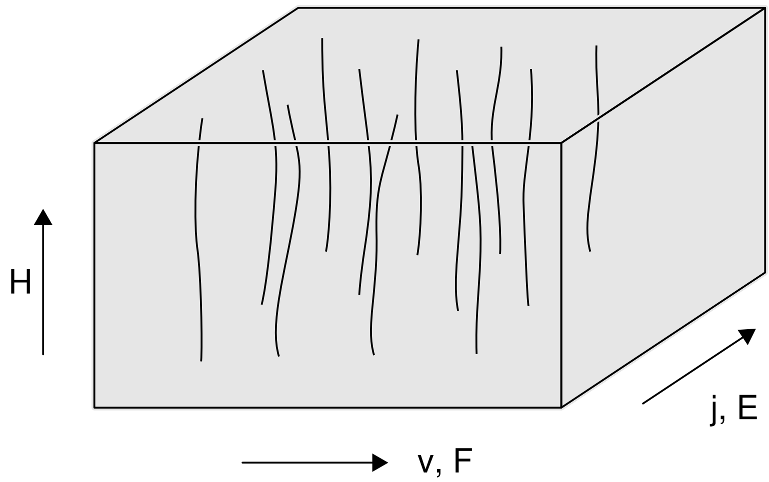


Flux lines in type-II superconductors

Driven flux line lattice

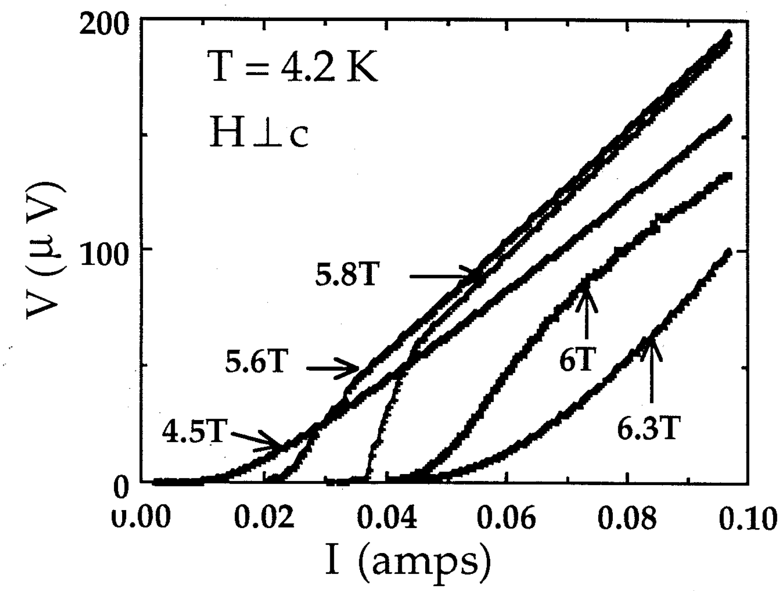


What is measured?



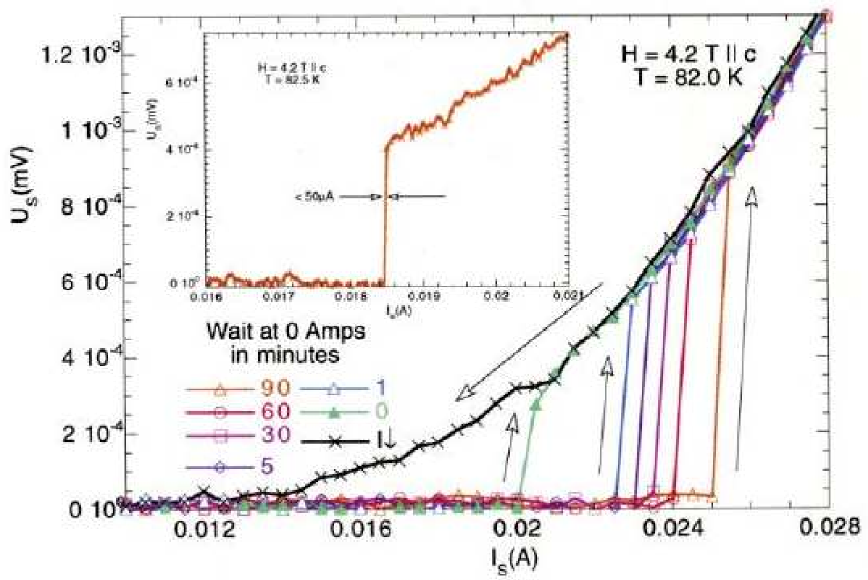
Depinning - experiment

S. Bhattacharya and M.J. Higgins, 1993: $NbSe_2$



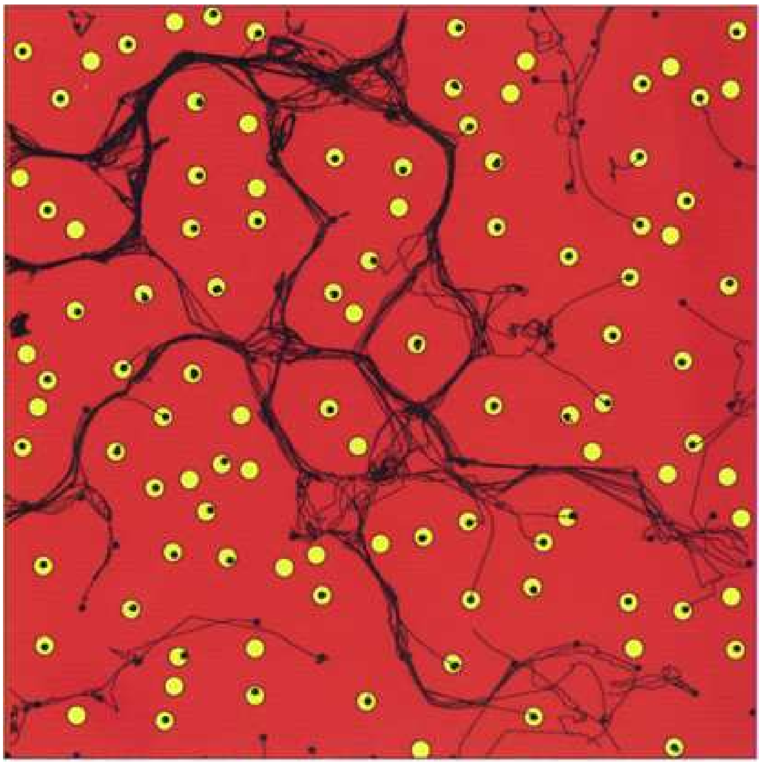
"We attribute this to *a crossover between a coherent motion of an elastic medium for a relatively stiff FLL to a primarily defective ("plastic") flow of a soft FLL.*"

J.A. Fendrich *et. al.*, 1996: $YBa_3Cu_2O_{7-\delta}$

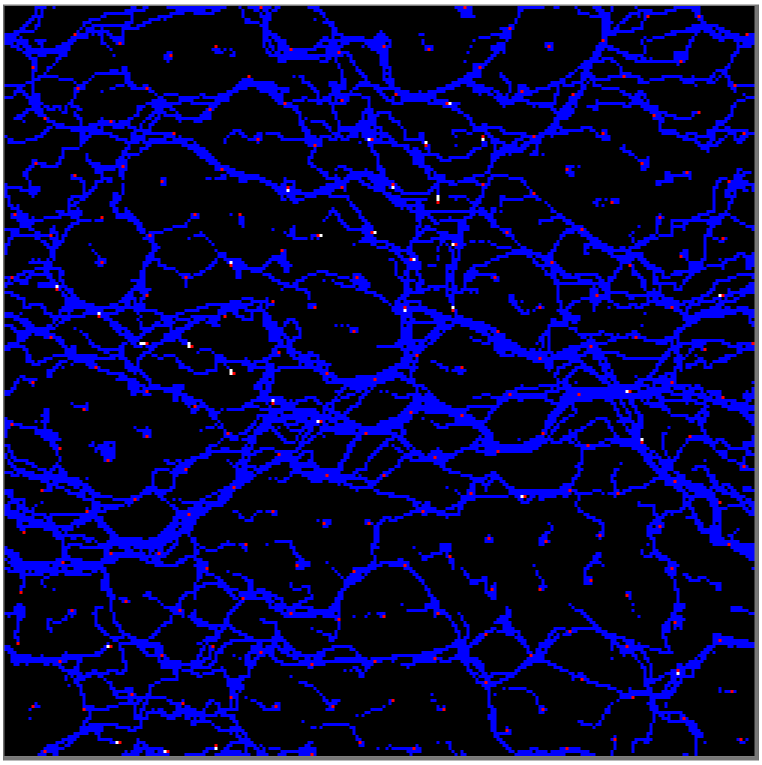


Depinning - simulation

C. Reichhardt *et. al.*, 1996

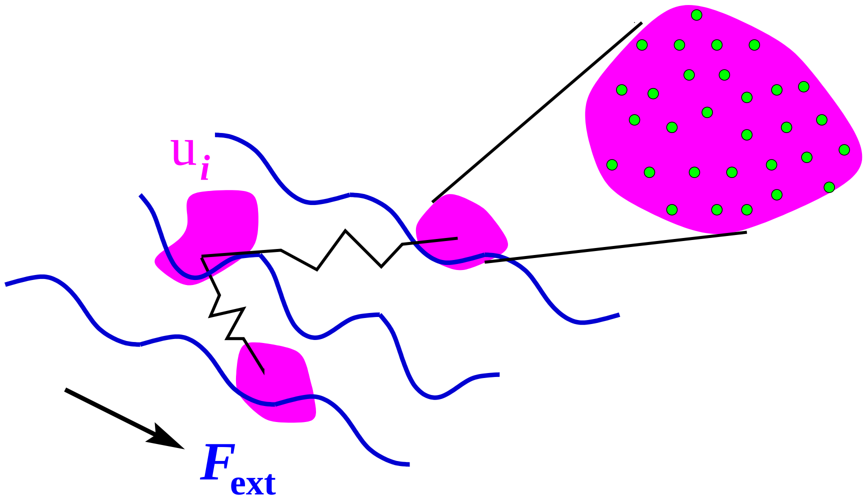


M. Faleski *et. al.*, 1997



Modelling of elastic depinning

Coarse grained description (Fukuyama and Lee, 1977):



$$\dot{u}_i = \frac{\mu}{|\Omega_i|} \sum_{j \in \Omega_i} (u_j - u_i) + F_{\text{ext}} - \frac{\partial V_i(u_i)}{\partial u_i}$$

- Overdamped case, no inertial term
- Scalar displacement field $u_i = u(\vec{r}_i)$
- *Elastic coupling* between neighbours $\sim (u_j - u_i)$
- Driving via external force F_{ext}
- *Frozen disorder* given by $V_i(u_i)$.

Pinning potential $V_i(u_i)$ periodic in u_i, random in i, e.g.

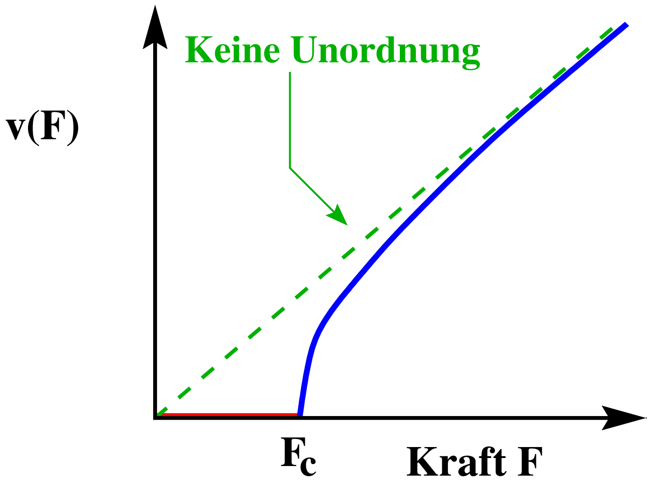
$$V_i(u_i) = h_i \cos(u_i - \beta_i)$$

with h_i and β_i random variables

$$\boxed{\text{Mean velocity: } v(F_{\text{ext}}) = \langle \dot{u}_i \rangle}$$

Properties of elastic depinning

- *Continuous* transition between resting ($v = 0$) state and sliding ($v > 0$) state at critical force F_c

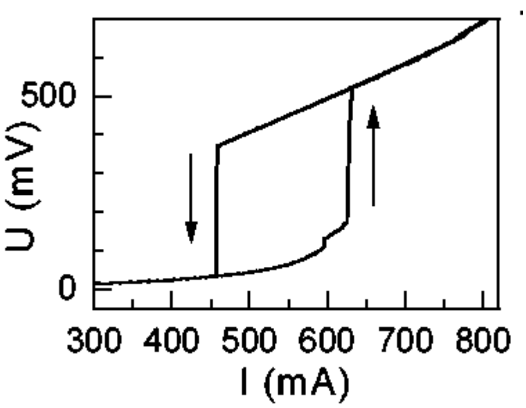


- Universal critical behaviour (D.S. Fisher, 1985):
  $v \sim (F_{\text{ext}} - F_c)^\beta$

- *Unique* sliding state (A.A. Middleton, 1992)
  $\Rightarrow$ no hysteresis

But: **experiments show hysteresis**

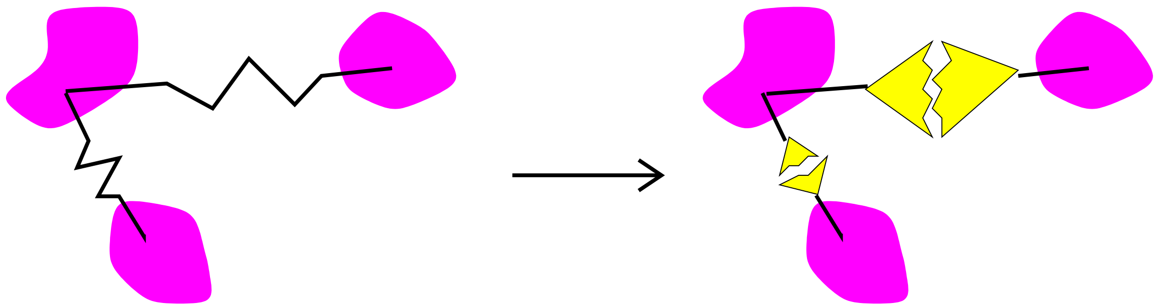
M. Danckwerts *et. al.*, 2000: $PbIn$ film, $T = 4.2K$, $H = 71mT$



Interpretation: *tearing of flux line lattice*

Modelling of plastic depinning

Plasticity via viscoelastic coupling:



$$\mu(u_j - u_i) \to \int_{-\infty}^{t} C(t-s)\left[\dot{u}_j(s) - \dot{u}_i(s)\right] ds$$

with $C(t)$ autocorrelation function of the local tension.

- Viscosity $\eta = \int_0^\infty C(t)dt$, viscous limit: $C(t) = \eta\delta(t)$.
- Elasticity $\mu = C(0)$, elastic limit: $C(t) = \mu$.
- Coupling between velocities allows for *sliding* $\Rightarrow$ *coexisting* resting and sliding u_i.

The equations of motion are now:

$$\dot{u}_i = \frac{1}{|\Omega_i|}\sum_{j\in\Omega_i}\int_{-\infty}^{t} C(t-s)\left[\dot{u}_j(s) - \dot{u}_i(s)\right] ds + F_{\mathrm{ext}} - \frac{\partial V_i(u_i)}{\partial u_i}$$

Modifications: Inertial term (models for friction), non-convex interactions (CDW), non-monotonic elastic coupling (crack propagation).

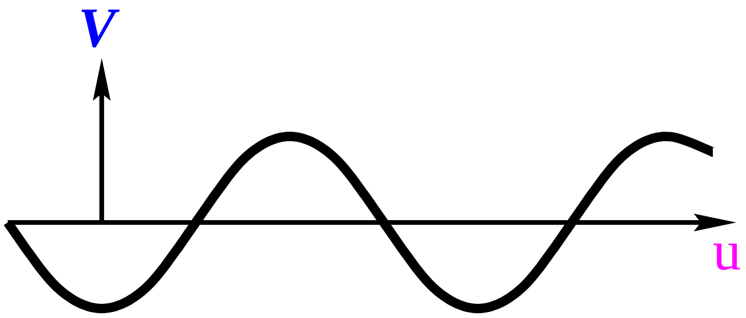
Which pinning potential?

Fisher, 1985:

$$v \sim (F_{\text{ext}} - F_c)^{\beta}$$

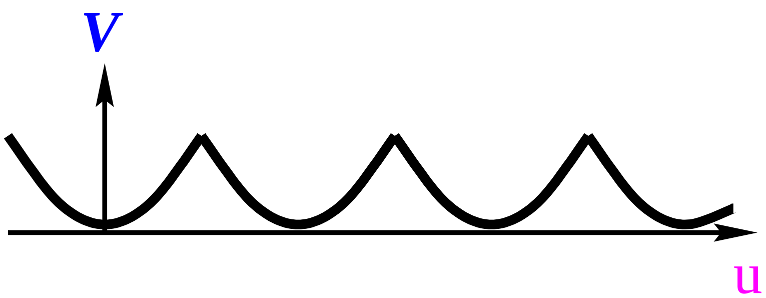
Mean field theory ($d = \infty$):

for a smooth potential $V_i(u_i) = h_i \cos(u_i - \beta_i)$:



$\beta = 3/2$

for a piecewise quadratic potential $V_i(u_i) = h_i(u_i - \beta_i)^2$:



$\beta = 1$

Which β ist generic?

$d \neq \infty$:

the *effective* potential for u_i changes rapidly when a neighbour "hops".

$\Rightarrow$ the "correct" MFT is given by a potential with a discontinuous derivative (jumps in the force).

Mean field analysis

Coupling to mean velocity $v = \langle \dot{u}_j \rangle$:

$$\frac{1}{|\Omega_i|} \sum_{j \in \Omega_i} [\dot{u}_j(s) - \dot{u}_i(s)] \to v - \dot{u}_i(s)$$

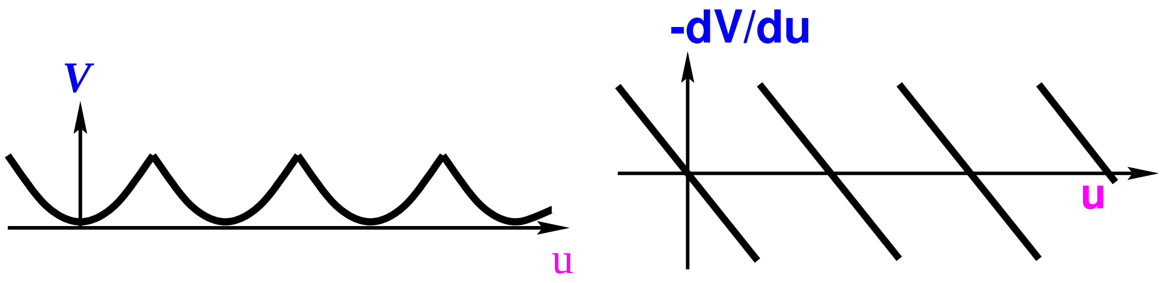
and therefore

$$\dot{u}_i = \eta v - \int_{-\infty}^{t} C(t-s)\dot{u}_i(s)ds + F_{\text{ext}} - \frac{\partial V_i(u_i)}{\partial u_i}$$

With the Maxwell model $C(t) = \mu e^{-t/\tau}$, $\eta = \mu\tau$ one obtains

$$\tau \ddot{u}_i + \left[1 + \eta + \tau \frac{\partial^2 V_i(u_i)}{\partial {u_i}^2}\right] \dot{u}_i = F_{\text{ext}} + \eta v - \frac{\partial V_i(u_i)}{\partial u_i}$$

Solve MFT with piecewise parabolic potential $V_i(u_i) = h_i {u_i}^2$ *under the assumption of constant* v.



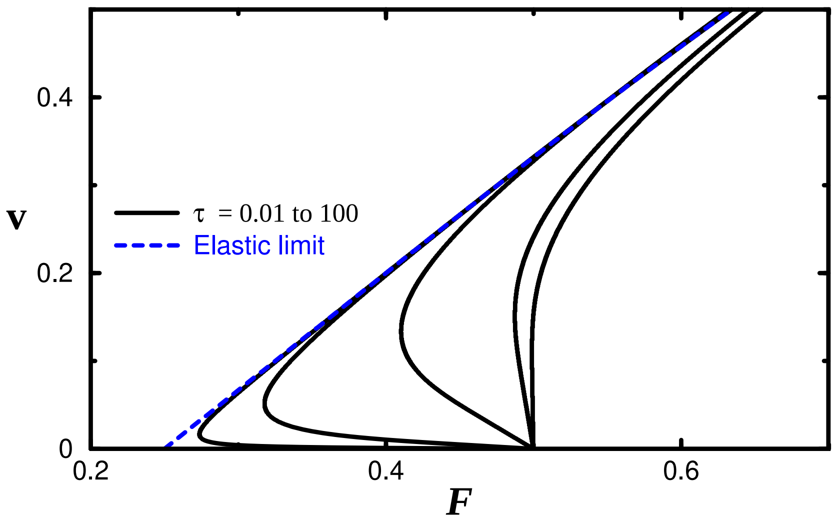
- Solve linear 2nd order ODE.
- Determine $T_i(\eta v + F_{\text{ext}}, h_i, \eta, \tau)$.
- Self-consistency condition: $\left\langle T_i^{-1} \right\rangle_{h_i} = v$.

Parameters: τ, η $(\eta = \mu\tau)$, and distribution $\rho(h_i)$.

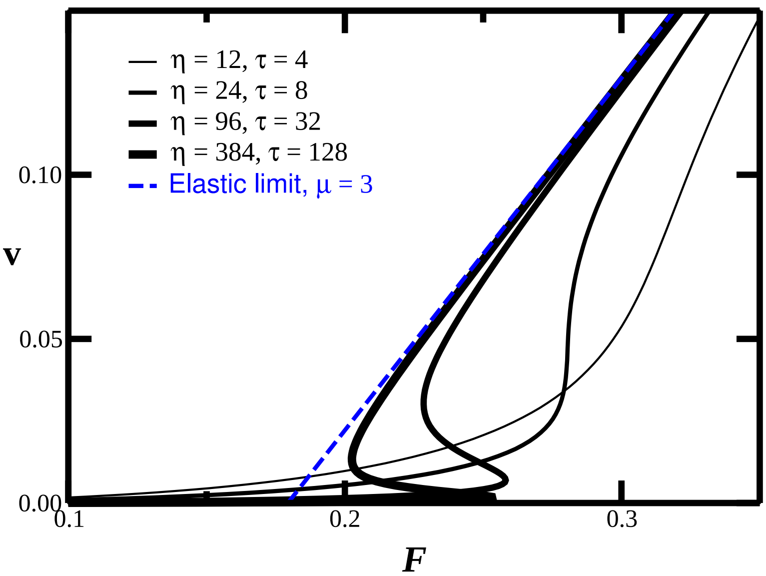
Discussion of the mean field results

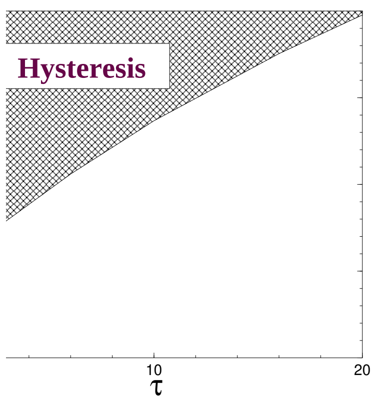
Parameters: τ, η ($\eta = \mu\tau$), and distribution $\rho(h)$.

- Fixed h, $\rho(h) = \delta(h-1)$, $\mu = 1.0$:



- Distribution $\rho(h) = \exp(-h)$, $\mu = 3.0$:

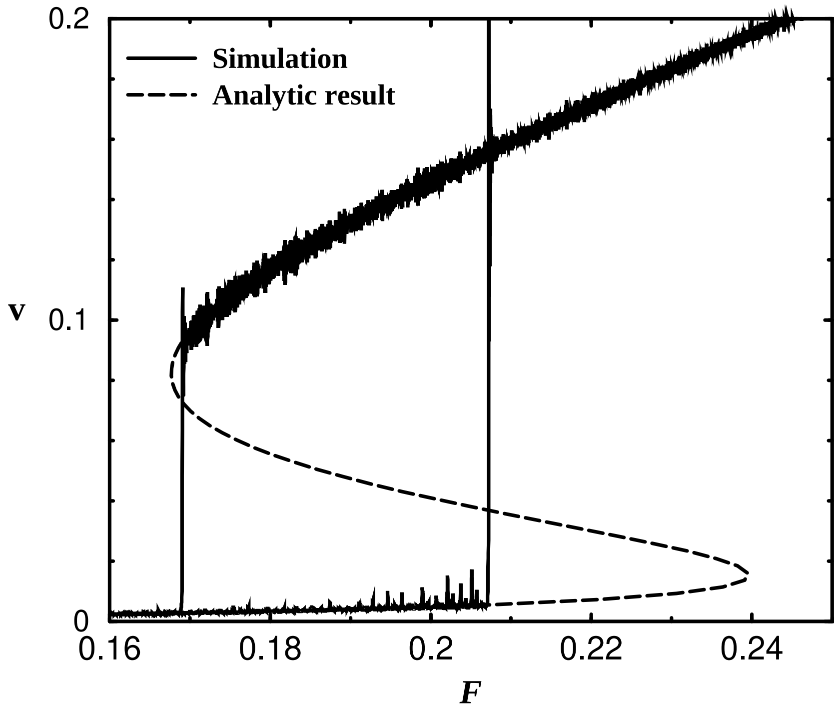


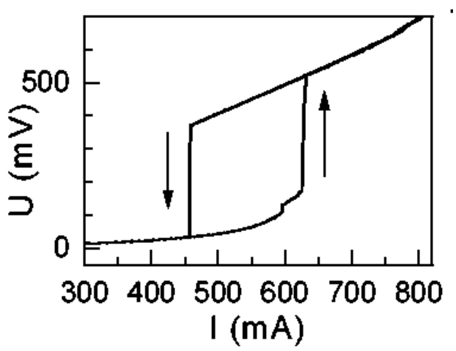


Both *continuous* and *hysteretic* depinning.

Simulation of the mean field model

$\eta = 32.0$, $\tau = 0.8$, $N = 16384$:



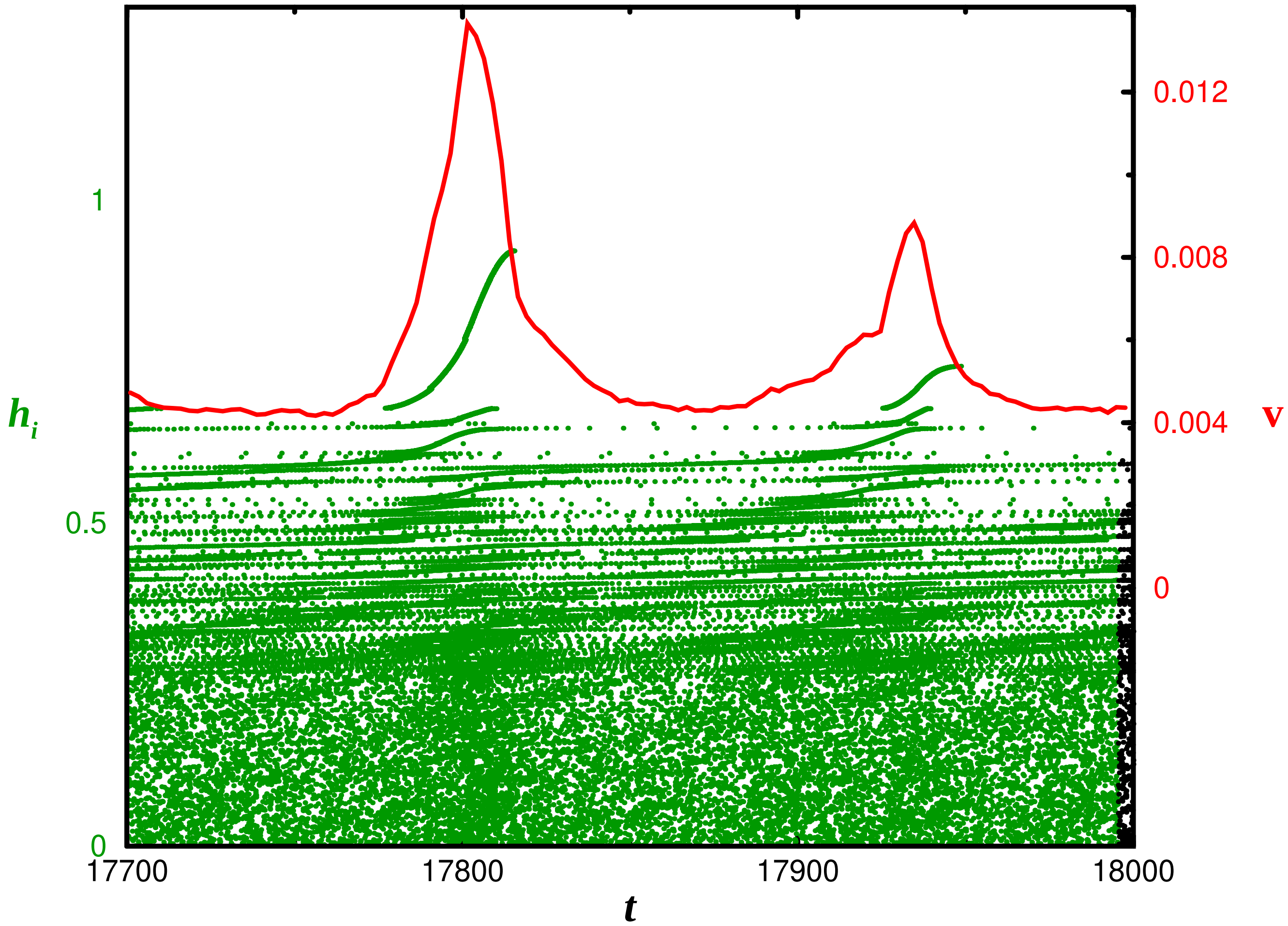


Comparison of the simulation with the analytic results:

- Noise in the upper branch due to finite system size (scales as $N^{-1/2}$).
- Fluctuations in the lower branch due to *"slip-stick" instability* (saturation for large N, macroscopic events).
- Premature switching from the upper branch to the lower branch.

The "slip-stick" instability

$\eta = 32.0$, $\tau = 0.0$, $N = 16384$, $F_{\text{ext}} = 0.2$:



Green dots depict events at which a u_i with pinning strength h_i slides over the barrier.

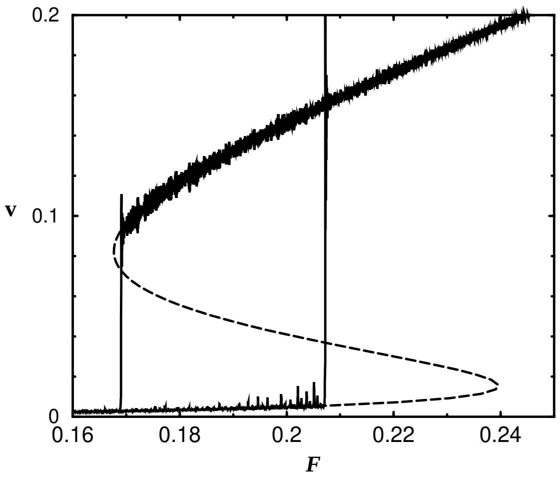
Maximal pinning force ist h_i; maximal driving force due to external force and the other u_i is $\eta v + F_{\text{ext}}$.

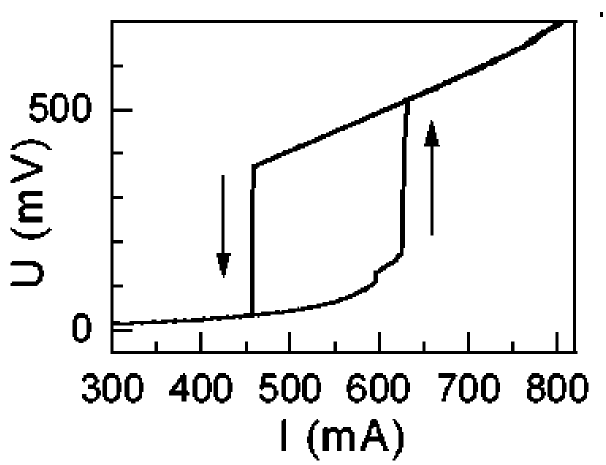
(This instability vanishes for a *smooth* pinning potential.)

Summary

- Development and analysis of a model for plastic reaction of driven systems in the presence of disorder.

- Relevant for the dynamics of *flux lines in type-II superconductors*, charge density waves, sliding friction, ...





- Confirmation of the experimentally obtained intuition: plasticity effects are relevant.

- Prediction of a possible new effect: *"slip-stick" instability.*

- *Research in Progress*: Analysis of the model in $d < \infty$ via simulations and RG.