> *restart*;

> $(n, integer)$;

$$n, integer \tag{1}$$

> $n := 15$;

$$n := 15 \tag{2}$$

APPENDIX 4.0 17-TIMESCALE

> $deqn := \mathrm{phi}^2 \cdot diff(x(t), t, t) + x(t) + \mathrm{phi} \cdot \mathrm{epsilon} \cdot (x(t)^2 - 1) \cdot diff(x(t), t)$;

$$deqn := \phi^2 \left(\frac{d^2}{dt^2} x(t) \right) + x(t) + \phi\, \varepsilon \left(x(t)^2 - 1 \right) \left(\frac{d}{dt} x(t) \right) \tag{1.1}$$

> $x := t \to \lambda_{0,1} \cos(T) + \lambda_{1,1} \sin(3\,T)\,\varepsilon + \left(\lambda_{2,1}\cos(T) + \lambda_{2,2}\cos(3\,T) + \lambda_{2,3}\cos(5\,T)\right)\varepsilon^2 + \left(\lambda_{3,1}\sin(3\,T) + \lambda_{3,2}\sin(5\,T) + \lambda_{3,3}\sin(7\,T)\right)\varepsilon^3 + \left(\lambda_{4,1}\cos(T) + \lambda_{4,2}\cos(3\,T) + \lambda_{4,3}\cos(5\,T) + \lambda_{4,4}\cos(7\,T) + \lambda_{4,5}\cos(9\,T)\right)\varepsilon^4 + \left(\lambda_{5,1}\sin(3\,T) + \lambda_{5,2}\sin(5\,T) + \lambda_{5,3}\sin(7\,T) + \lambda_{5,4}\sin(9\,T) + \lambda_{5,5}\sin(11\,T)\right)\varepsilon^5 + \left(\lambda_{6,1}\cos(T) + \lambda_{6,2}\cos(3\,T) + \lambda_{6,3}\cos(5\,T) + \lambda_{6,4}\cos(7\,T) + \lambda_{6,5}\cos(9\,T) + \lambda_{6,6}\cos(11\,T) + \lambda_{6,7}\cos(13\,T)\right)\varepsilon^6 + \left(\lambda_{7,1}\sin(3\,T) + \lambda_{7,2}\sin(5\,T) + \lambda_{7,3}\sin(7\,T) + \lambda_{7,4}\sin(9\,T) + \lambda_{7,5}\sin(11\,T) + \lambda_{7,6}\sin(13\,T) + \lambda_{7,7}\sin(15\,T)\right)\varepsilon^7 + \left(\lambda_{8,1}\cos(T) + \lambda_{8,2}\cos(3\,T) + \lambda_{8,3}\cos(5\,T) + \lambda_{8,4}\cos(7\,T) + \lambda_{8,5}\cos(9\,T) + \lambda_{8,6}\cos(11\,T) + \lambda_{8,7}\cos(13\,T) + \lambda_{8,8}\cos(15\,T) + \lambda_{8,9}\cos(17\,T)\right)\varepsilon^8 + \left(\lambda_{9,1}\sin(3\,T) + \lambda_{9,2}\sin(5\,T) + \lambda_{9,3}\sin(7\,T) + \lambda_{9,4}\sin(9\,T) + \lambda_{9,5}\sin(11\,T) + \lambda_{9,6}\sin(13\,T) + \lambda_{9,7}\sin(15\,T) + \lambda_{9,8}\sin(17\,T) + \lambda_{9,9}\sin(19\,T)\right)\varepsilon^9 + \left(\lambda_{10,1}\cos(T) + \lambda_{10,2}\cos(3\,T) + \lambda_{10,3}\cos(5\,T) + \lambda_{10,4}\cos(7\,T) + \lambda_{10,5}\cos(9\,T) + \lambda_{10,6}\cos(11\,T) + \lambda_{10,7}\cos(13\,T) + \lambda_{10,8}\cos(15\,T) + \lambda_{10,9}\cos(17\,T) + \lambda_{10,10}\cos(19\,T) + \lambda_{10,11}\cos(21\,T)\right)\varepsilon^{10} + \left(\lambda_{11,1}\sin(3\,T) + \lambda_{11,2}\sin(5\,T) + \lambda_{11,3}\sin(7\,T) + \lambda_{11,4}\sin(9\,T) + \lambda_{11,5}\sin(11\,T) + \lambda_{11,6}\sin(13\,T) + \lambda_{11,7}\sin(15\,T) + \lambda_{11,8}\sin(17\,T) + \lambda_{11,9}\sin(19\,T) + \lambda_{11,10}\sin(21\,T) + \lambda_{11,11}\sin(23\,T)\right)\varepsilon^{11} + \left(\lambda_{12,1}\cos(T) + \lambda_{12,2}\cos(3\,T) + \lambda_{12,3}\cos(5\,T) + \lambda_{12,4}\cos(7\,T) + \lambda_{12,5}\cos(9\,T) + \lambda_{12,6}\cos(11\,T) + \lambda_{12,7}\cos(13\,T) + \lambda_{12,8}\cos(15\,T) + \lambda_{12,9}\cos(17\,T) + \lambda_{12,10}\cos(19\,T) + \lambda_{12,11}\cos(21\,T) + \lambda_{12,12}\cos(23\,T) + \lambda_{12,13}\cos(25\,T)\right)\varepsilon^{12} + \left(\lambda_{13,1}\sin(3\,T) + \lambda_{13,2}\sin(5\,T) + \lambda_{13,3}\sin(7\,T) + \lambda_{13,4}\sin(9\,T) + \lambda_{13,5}\sin(11\,T) + \lambda_{13,6}\sin(13\,T) + \lambda_{13,7}\sin(15\,T) + \lambda_{13,8}\sin(17\,T) + \lambda_{13,9}\sin(19\,T) + \lambda_{13,10}\sin(21\,T) + \lambda_{13,11}\sin(23\,T) + \lambda_{13,12}\sin(25\,T) + \lambda_{13,13}\sin(27\,T)\right)\varepsilon^{13} + \left(\lambda_{14,1}\cos(T) + \lambda_{14,2}\cos(3\,T) + \lambda_{14,3}\cos(5\,T) + \lambda_{14,4}\cos(7\,T) + \lambda_{14,5}\cos(9\,T) + \lambda_{14,6}\cos(11\,T)\right.$

$$+ \lambda_{14,7}\cos(13\,T) + \lambda_{14,8}\cos(15\,T) + \lambda_{14,9}\cos(17\,T) + \lambda_{14,10}\cos(19\,T) + \lambda_{14,11}\cos(21\,T) + \lambda_{14,12}\cos(23\,T) + \lambda_{14,13}\cos(25\,T) + \lambda_{14,14}\cos(27\,T) + \lambda_{14,15}\cos(29\,T)\Big)\,\varepsilon^{14} + \Big(\lambda_{15,1}\sin(3\,T) + \lambda_{15,2}\sin(5\,T) + \lambda_{15,3}\sin(7\,T) + \lambda_{15,4}\sin(9\,T) + \lambda_{15,5}\sin(11\,T) + \lambda_{15,6}\sin(13\,T) + \lambda_{15,7}\sin(15\,T) + \lambda_{15,8}\sin(17\,T) + \lambda_{15,9}\sin(19\,T) + \lambda_{15,10}\sin(21\,T) + \lambda_{15,11}\sin(23\,T) + \lambda_{15,12}\sin(25\,T) + \lambda_{15,13}\sin(27\,T) + \lambda_{15,14}\sin(29\,T) + \lambda_{15,15}\sin(31\,T)\Big)\,\varepsilon^{15};$$

 # first 15 x[j] solutions

$$x := t \rightarrow \lambda_{0,1}\cos(T) + \lambda_{1,1}\sin(3\,T)\,\varepsilon + \Big(\lambda_{2,1}\cos(T) + \lambda_{2,2}\cos(3\,T) + \lambda_{2,3}\cos(5\,T)\Big)\,\varepsilon^{2} + \Big(\lambda_{3,1}\sin(3\,T) + \lambda_{3,2}\sin(5\,T) + \lambda_{3,3}\sin(7\,T)\Big)\,\varepsilon^{3} + \Big(\lambda_{4,1}\cos(T) + \lambda_{4,2}\cos(3\,T) + \lambda_{4,3}\cos(5\,T) + \lambda_{4,4}\cos(7\,T) + \lambda_{4,5}\cos(9\,T)\Big)\,\varepsilon^{4} + \Big(\lambda_{5,1}\sin(3\,T) + \lambda_{5,2}\sin(5\,T) + \lambda_{5,3}\sin(7\,T) + \lambda_{5,4}\sin(9\,T) + \lambda_{5,5}\sin(11\,T)\Big)\,\varepsilon^{5} + \Big(\lambda_{6,1}\cos(T) + \lambda_{6,2}\cos(3\,T) + \lambda_{6,3}\cos(5\,T) + \lambda_{6,4}\cos(7\,T) + \lambda_{6,5}\cos(9\,T) + \lambda_{6,6}\cos(11\,T) + \lambda_{6,7}\cos(13\,T)\Big)\,\varepsilon^{6} + \Big(\lambda_{7,1}\sin(3\,T) + \lambda_{7,2}\sin(5\,T) + \lambda_{7,3}\sin(7\,T) + \lambda_{7,4}\sin(9\,T) + \lambda_{7,5}\sin(11\,T) + \lambda_{7,6}\sin(13\,T) + \lambda_{7,7}\sin(15\,T)\Big)\,\varepsilon^{7} + \Big(\lambda_{8,1}\cos(T) + \lambda_{8,2}\cos(3\,T) + \lambda_{8,3}\cos(5\,T) + \lambda_{8,4}\cos(7\,T) + \lambda_{8,5}\cos(9\,T) + \lambda_{8,6}\cos(11\,T) + \lambda_{8,7}\cos(13\,T) + \lambda_{8,8}\cos(15\,T) + \lambda_{8,9}\cos(17\,T)\Big)\,\varepsilon^{8} + \Big(\lambda_{9,1}\sin(3\,T) + \lambda_{9,2}\sin(5\,T) + \lambda_{9,3}\sin(7\,T) + \lambda_{9,4}\sin(9\,T) + \lambda_{9,5}\sin(11\,T) + \lambda_{9,6}\sin(13\,T) + \lambda_{9,7}\sin(15\,T) + \lambda_{9,8}\sin(17\,T) + \lambda_{9,9}\sin(19\,T)\Big)\,\varepsilon^{9} + \Big(\lambda_{10,1}\cos(T) + \lambda_{10,2}\cos(3\,T) + \lambda_{10,3}\cos(5\,T) + \lambda_{10,4}\cos(7\,T) + \lambda_{10,5}\cos(9\,T) + \lambda_{10,6}\cos(11\,T) + \lambda_{10,7}\cos(13\,T) + \lambda_{10,8}\cos(15\,T) + \lambda_{10,9}\cos(17\,T) + \lambda_{10,10}\cos(19\,T) + \lambda_{10,11}\cos(21\,T)\Big)\,\varepsilon^{10} + \Big(\lambda_{11,1}\sin(3\,T) + \lambda_{11,2}\sin(5\,T) + \lambda_{11,3}\sin(7\,T) + \lambda_{11,4}\sin(9\,T) + \lambda_{11,5}\sin(11\,T) + \lambda_{11,6}\sin(13\,T) + \lambda_{11,7}\sin(15\,T) + \lambda_{11,8}\sin(17\,T) + \lambda_{11,9}\sin(19\,T) + \lambda_{11,10}\sin(21\,T) + \lambda_{11,11}\sin(23\,T)\Big)\,\varepsilon^{11} + \Big(\lambda_{12,1}\cos(T) + \lambda_{12,2}\cos(3\,T) + \lambda_{12,3}\cos(5\,T) + \lambda_{12,4}\cos(7\,T) + \lambda_{12,5}\cos(9\,T) + \lambda_{12,6}\cos(11\,T) + \lambda_{12,7}\cos(13\,T) + \lambda_{12,8}\cos(15\,T) + \lambda_{12,9}\cos(17\,T) + \lambda_{12,10}\cos(19\,T) + \lambda_{12,11}\cos(21\,T) + \lambda_{12,12}\cos(23\,T) + \lambda_{12,13}\cos(25\,T)\Big)\,\varepsilon^{12} + \Big(\lambda_{13,1}\sin(3\,T) + \lambda_{13,2}\sin(5\,T) + \lambda_{13,3}\sin(7\,T) + \lambda_{13,4}\sin(9\,T) + \lambda_{13,5}\sin(11\,T) + \lambda_{13,6}\sin(13\,T) + \lambda_{13,7}\sin(15\,T) + \lambda_{13,8}\sin(17\,T) + \lambda_{13,9}\sin(19\,T) + \lambda_{13,10}\sin(21\,T) + \lambda_{13,11}\sin(23\,T) + \lambda_{13,12}\sin(25\,T) \tag{1.2}$$

$$+\lambda_{13,13}\sin(27\,T)\Big)\varepsilon^{13}+\Big(\lambda_{14,1}\cos(T)+\lambda_{14,2}\cos(3\,T)+\lambda_{14,3}\cos(5\,T)$$
$$+\lambda_{14,4}\cos(7\,T)+\lambda_{14,5}\cos(9\,T)+\lambda_{14,6}\cos(11\,T)+\lambda_{14,7}\cos(13\,T)$$
$$+\lambda_{14,8}\cos(15\,T)+\lambda_{14,9}\cos(17\,T)+\lambda_{14,10}\cos(19\,T)+\lambda_{14,11}\cos(21\,T)$$
$$+\lambda_{14,12}\cos(23\,T)+\lambda_{14,13}\cos(25\,T)+\lambda_{14,14}\cos(27\,T)+\lambda_{14,15}\cos(29\,T)\Big)\varepsilon^{14}$$
$$+\Big(\lambda_{15,1}\sin(3\,T)+\lambda_{15,2}\sin(5\,T)+\lambda_{15,3}\sin(7\,T)+\lambda_{15,4}\sin(9\,T)$$
$$+\lambda_{15,5}\sin(11\,T)+\lambda_{15,6}\sin(13\,T)+\lambda_{15,7}\sin(15\,T)+\lambda_{15,8}\sin(17\,T)$$
$$+\lambda_{15,9}\sin(19\,T)+\lambda_{15,10}\sin(21\,T)+\lambda_{15,11}\sin(23\,T)+\lambda_{15,12}\sin(25\,T)$$
$$+\lambda_{15,13}\sin(27\,T)+\lambda_{15,14}\sin(29\,T)+\lambda_{15,15}\sin(31\,T)\Big)\varepsilon^{15}$$

> $\phi := 1+\lambda_1\varepsilon^2+\lambda_2\varepsilon^4+\lambda_3\varepsilon^6+\lambda_4\varepsilon^8+\lambda_5\varepsilon^{10}+\lambda_6\varepsilon^{12}+\lambda_7\varepsilon^{14};$
 # frequency equation for the first 15 terms

$$\phi := 1+\lambda_1\varepsilon^2+\lambda_2\varepsilon^4+\lambda_3\varepsilon^6+\lambda_4\varepsilon^8+\lambda_5\varepsilon^{10}+\lambda_6\varepsilon^{12}+\lambda_7\varepsilon^{14} \qquad \textbf{(1.3)}$$

> $T :=$ phi$\cdot t$; *# T ,the stretched time variable*

$$T := \left(1+\lambda_1\varepsilon^2+\lambda_2\varepsilon^4+\lambda_3\varepsilon^6+\lambda_4\varepsilon^8+\lambda_5\varepsilon^{10}+\lambda_6\varepsilon^{12}+\lambda_7\varepsilon^{14}\right)t \qquad \textbf{(1.4)}$$

> *deqnseries* := *series*(*eval*(*subs*(*x*=*xansatz*, *deqn*)), epsilon, *n* + 1); *# deqn series in epsilon*

[Length of output exceeds limit of 1000000] **(1.5)**

> *#deqnseries:=map(collect, combine(series(deqn,* epsilon, *n* + 1), *trig*), [sin, cos]);
 # substitute x into deqn

> *deqn1* := *coeff*(*deqnseries*, epsilon, 1); *# 1st order coefficients*

$$deqn1 := -8\,\lambda_{1,1}\sin(3\,t)-\left(\lambda_{0,1}^2\cos(t)^2-1\right)\lambda_{0,1}\sin(t) \qquad \textbf{(3)}$$

> *deqn2* := *coeff*(*deqnseries*, epsilon, 2);*# 2nd order coefficients*

$$deqn2 := -4\,\lambda_{0,1}\cos(t)\,\lambda_1-24\,\lambda_{2,3}\cos(5\,t)-8\,\lambda_{2,2}\cos(3\,t)+3\left(\lambda_{0,1}^2\cos(t)^2-1\right)\lambda_{1,1}\cos(3\,t)-2\,\lambda_{0,1}^2\cos(t)\,\lambda_{1,1}\sin(3\,t)\sin(t) \qquad \textbf{(4)}$$

> *deqn3* := *coeff*(*deqnseries*, epsilon, 3) :*# 3rd order coefficients*

> *deqn4* := *coeff*(*deqnseries*, epsilon, 4)*# 4th order coefficients*

$$deqn4 := \left(\lambda_{0,1}^2\cos(t)^2-1\right)\Big(3\,\lambda_{1,1}\cos(3\,t)\,\lambda_1-9\,\lambda_{1,1}\sin(3\,t)\,\lambda_1\,t+3\,\lambda_{3,1}\cos(3\,t) \qquad \textbf{(5)}$$
$$+7\,\lambda_{3,3}\cos(7\,t)+5\,\lambda_{3,2}\cos(5\,t)\Big)-2\,\lambda_{2,1}\cos(t)\,\lambda_1-18\,\lambda_{2,2}\cos(3\,t)\,\lambda_1$$
$$-2\,\lambda_{0,1}\cos(t)\left(2\,\lambda_2+\lambda_1^2\right)+24\,\lambda_{2,2}\sin(3\,t)\,\lambda_1\,t+2\,\lambda_1\Big(-2\,\lambda_{0,1}\cos(t)\,\lambda_1$$
$$+\lambda_{0,1}\sin(t)\,\lambda_1\,t-\lambda_{2,1}\cos(t)-25\,\lambda_{2,3}\cos(5\,t)-9\,\lambda_{2,2}\cos(3\,t)\Big)-8\,\lambda_{4,2}\cos(3\,t)$$
$$-24\,\lambda_{4,3}\cos(5\,t)-48\,\lambda_{4,4}\cos(7\,t)+2\,\lambda_{0,1}\sin(t)\,\lambda_1^2\,t-50\,\lambda_{2,3}\cos(5\,t)\,\lambda_1$$
$$+3\Big(2\,\lambda_{0,1}\cos(t)\left(-\lambda_{0,1}\sin(t)\,\lambda_1\,t+\lambda_{2,1}\cos(t)+\lambda_{2,3}\cos(5\,t)+\lambda_{2,2}\cos(3\,t)\right)+$$
$$\lambda_{1,1}^2\sin(3\,t)^2+\lambda_1\left(\lambda_{0,1}^2\cos(t)^2-1\right)\Big)\lambda_{1,1}\cos(3\,t)-\Big(2\,\lambda_1\,\lambda_{0,1}\cos(t)\,\lambda_{1,1}\sin(3\,t)$$

$$+2\lambda_{1,1}\sin(3t)\left(-\lambda_{0,1}\sin(t)\lambda_1 t+\lambda_{2,1}\cos(t)+\lambda_{2,3}\cos(5t)+\lambda_{2,2}\cos(3t)\right)$$
$$+2\lambda_{0,1}\cos(t)\left(3\lambda_{1,1}\cos(3t)\lambda_1 t+\lambda_{3,1}\sin(3t)+\lambda_{3,3}\sin(7t)+\lambda_{3,2}\sin(5t)\right)\Big)$$
$$\lambda_{0,1}\sin(t)+120\lambda_{2,3}\sin(5t)\lambda_1 t-80\lambda_{4,5}\cos(9t)+2\lambda_{0,1}\cos(t)\lambda_{1,1}\sin(3t)\Big($$
$$-\lambda_{0,1}\sin(t)\lambda_1-\lambda_{0,1}\cos(t)\lambda_1 t-\lambda_{2,1}\sin(t)-5\lambda_{2,3}\sin(5t)-3\lambda_{2,2}\sin(3t)\Big)$$

> *deqn5* := *coeff*(*deqnseries*, epsilon, 5);:# *5th order coefficients*

$$deqn5 := 0 \qquad \textbf{(6)}$$

> *deqn6* := *coeff*(*deqnseries*, epsilon, 6) :# *6th order coefficients*

> *deqn7* := *coeff*(*deqnseries*, epsilon, 7) :# *7th order coefficients*

> *deqn8* := *coeff*(*deqnseries*, epsilon, 8) :# *8th order coefficients*

> *deqn9* := *coeff*(*deqnseries*, epsilon, 9) :# *9th order coefficients*

> *deqn10* := *coeff*(*deqnseries*, epsilon, 10) :# *10th order coefficients*

> *deqn11* := *coeff*(*deqnseries*, epsilon, 11) :# *11th order coefficients*

> *deqn12* := *coeff*(*deqnseries*, epsilon, 12) :# *12th order coefficients*

> *deqn13* := *coeff*(*deqnseries*, epsilon, 13) :# *13th order coefficients*

> *deqn14* := *coeff*(*deqnseries*, epsilon, 14) :# *14th order coefficients*

> *deqn15* := *coeff*(*deqnseries*, epsilon, 15) :# *15th order coefficients*

> *deqn1temp* := *collect*(*combine*(*deqn1*, *trig*), [sin, cos]);
#eqn (18) *is zero when coefficient of trig func is zero*

$$deqn1temp := \left(-\frac{1}{4}\lambda_{0,1}^3+\lambda_{0,1}\right)\sin(t)+\left(-8\lambda_{1,1}-\frac{1}{4}\lambda_{0,1}^3\right)\sin(3t) \qquad \textbf{(7)}$$

> *zero_terms* := {*coeff*(*deqn1temp*, sin(t)), *coeff*(*deqn1temp*, sin(3 t))};
#set coefficients to zero;

$$zero\_terms := \left\{-8\lambda_{1,1}-\frac{1}{4}\lambda_{0,1}^3, -\frac{1}{4}\lambda_{0,1}^3+\lambda_{0,1}\right\} \qquad \textbf{(8)}$$

> *solve*(*zero_terms*, {$\lambda_{0,1}$, $\lambda_{1,1}$}); *#solve for all constants*

$$\left\{\lambda_{0,1}=0, \lambda_{1,1}=0\right\}, \left\{\lambda_{0,1}=2, \lambda_{1,1}=-\frac{1}{4}\right\}, \left\{\lambda_{0,1}=-2, \lambda_{1,1}=\frac{1}{4}\right\} \qquad \textbf{(9)}$$

> $\lambda_{0,1} := 2;$

$$\lambda_{0,1} := 2 \qquad \textbf{(10)}$$

> $\lambda_{1,1} := -\frac{1}{4};$

$$\lambda_{1,1} := -\frac{1}{4} \qquad \textbf{(11)}$$

> *deqn2temp* := *map*(*simplify*, *collect*(*combine*(*deqn2*, *trig*), [sin, cos]));

$$deqn2temp := -\frac{1}{4}\left(1+32\lambda_1\right)\cos(t)-\frac{1}{4}\left(32\lambda_{2,2}+3\right)\cos(3t)-\frac{1}{4}\Big(5+96\lambda_{2,3}\Big)\cos(5t) \qquad \textbf{(12)}$$

> *zero_terms* := {*coeff*(*deqn2temp*, cos(t)), *coeff*(*deqn2temp*, cos(3 t)), *coeff*(*deqn2temp*, cos(5 t))};

$$zero\_terms := \left\{-\frac{5}{4}-24\lambda_{2,3}, -\frac{1}{4}-8\lambda_1, -8\lambda_{2,2}-\frac{3}{4}\right\} \qquad \textbf{(13)}$$

> $solve\left(zero\_terms, \left\{\lambda_1, \lambda_{2,2}, \lambda_{2,3}\right\}\right);$

$$\left\{\lambda_1 = -\frac{1}{32}, \lambda_{2,2} = -\frac{3}{32}, \lambda_{2,3} = -\frac{5}{96}\right\} \quad \textbf{(14)}$$

> $\lambda_1 := -\frac{1}{32};$

$$\lambda_1 := -\frac{1}{32} \quad \textbf{(15)}$$

> $\lambda_{2,2} := -\frac{3}{32};$

$$\lambda_{2,2} := -\frac{3}{32} \quad \textbf{(16)}$$

> $\lambda_{2,3} := -\frac{5}{96};$

$$\lambda_{2,3} := -\frac{5}{96} \quad \textbf{(17)}$$

> $deqn2temp := map(simplify, collect(combine(deqn2, trig), [\sin, \cos])); \# \ solution$

$$deqn2temp := 0 \quad \textbf{(18)}$$

> $deqn3temp := map(simplify, collect(combine(deqn3, trig), [\sin, \cos]));$

$$deqn3temp := -\frac{1}{32}\left(-1 + 64\lambda_{2,1}\right)\sin(t) - \frac{1}{32}\left(-9 + 256\lambda_{3,1} + 96\lambda_{2,1}\right)\sin(3t) - \frac{1}{96}\left(2304\lambda_{3,2} - 85\right)\sin(5t) - \frac{1}{12}\left(-7 + 576\lambda_{3,3}\right)\sin(7t) \quad \textbf{(19)}$$

> $zero\_terms := \{coeff(deqn3temp, \sin(t)), coeff(deqn3temp, \sin(3\,t)), coeff(deqn3temp, \sin(5\,t)), coeff(deqn3temp, \sin(7\,t))\};$

$$zero\_terms := \left\{\frac{1}{32} - 2\lambda_{2,1}, \frac{7}{12} - 48\lambda_{3,3}, -24\lambda_{3,2} + \frac{85}{96}, \frac{9}{32} - 8\lambda_{3,1} - 3\lambda_{2,1}\right\} \quad \textbf{(20)}$$

> $solve\left(zero\_terms, \left\{\lambda_{3,3}, \lambda_{3,2}, \lambda_{2,1}, \lambda_{3,1}\right\}\right);$

$$\left\{\lambda_{2,1} = \frac{1}{64}, \lambda_{3,1} = \frac{15}{512}, \lambda_{3,2} = \frac{85}{2304}, \lambda_{3,3} = \frac{7}{576}\right\} \quad \textbf{(21)}$$

> $\lambda_{2,1} := \frac{1}{64};$

$$\lambda_{2,1} := \frac{1}{64} \quad \textbf{(22)}$$

> $\lambda_{3,1} := \frac{15}{512};$

$$\lambda_{3,1} := \frac{15}{512} \quad \textbf{(23)}$$

> $\lambda_{3,2} := \frac{85}{2304};$

$$\lambda_{3,2} := \frac{85}{2304} \quad \textbf{(24)}$$

> $\lambda_{3,3} := \frac{7}{576};$

$$\lambda_{3,3} := \frac{7}{576} \tag{25}$$

> *deqn3temp* := *map*(*simplify*, *collect*(*combine*(*deqn3*, *trig*), [sin, cos]));
> # *to check we have a zero solution we substitute known constants*

$$deqn3temp := 0 \tag{26}$$

> *deqn4temp* := *map*(*simplify*, *collect*(*combine*(*deqn4*, *trig*), [sin, cos]));

$$deqn4temp := -\frac{1}{384}\left(-7+3072\,\lambda_2\right)\cos(t) - \frac{1}{1536}\left(12288\,\lambda_{4,2}-101\right)\cos(3\,t) - \frac{1}{4608}\left(110592\,\lambda_{4,3}-1865\right)\cos(5\,t) - \frac{1}{2304}\left(-1379+110592\,\lambda_{4,4}\right)\cos(7\,t) - \frac{1}{256}\left(20480\,\lambda_{4,5}-61\right)\cos(9\,t) \tag{27}$$

> *zero_terms* := {*coeff*(*deqn4temp*, cos(t)), *coeff*(*deqn4temp*, cos(3 t)), *coeff*(*deqn4temp*, cos(5 t)), *coeff*(*deqn4temp*, cos(7 t)), *coeff*(*deqn4temp*, cos(9 t))};

$$zero\_terms := \left\{\frac{7}{384} - 8\,\lambda_2, \frac{1379}{2304} - 48\,\lambda_{4,4}, -8\,\lambda_{4,2} + \frac{101}{1536}, -24\,\lambda_{4,3} + \frac{1865}{4608}, -80\,\lambda_{4,5} + \frac{61}{256}\right\} \tag{28}$$

> *solve*(*zero_terms*, $\{\lambda_2, \lambda_{4,5}, \lambda_{4,2}, \lambda_{4,4}, \lambda_{4,3}\}$);

$$\left\{\lambda_2 = \frac{7}{3072}, \lambda_{4,2} = \frac{101}{12288}, \lambda_{4,3} = \frac{1865}{110592}, \lambda_{4,4} = \frac{1379}{110592}, \lambda_{4,5} = \frac{61}{20480}\right\} \tag{29}$$

> $\lambda_2 := \frac{7}{3072}$;

$$\lambda_2 := \frac{7}{3072} \tag{30}$$

> $\lambda_{4,2} := \frac{101}{12288}$;

$$\lambda_{4,2} := \frac{101}{12288} \tag{31}$$

> $\lambda_{4,3} := \frac{1865}{110592}$;

$$\lambda_{4,3} := \frac{1865}{110592} \tag{32}$$

> $\lambda_{4,4} := \frac{1379}{110592}$;

$$\lambda_{4,4} := \frac{1379}{110592} \tag{33}$$

> $\lambda_{4,5} := \frac{61}{20480}$;

$$\lambda_{4,5} := \frac{61}{20480} \tag{34}$$

> *deqn4temp* := *map*(*simplify*, *collect*(*combine*(*deqn4*, *trig*), [sin, cos])); # *checks solution*

$$deqn4temp := 0 \tag{35}$$

> *deqn5temp* := *map*(*simplify*, *collect*(*combine*(*deqn5*, *trig*), [sin, cos]));

$$deqn5temp := -\frac{1}{24576}\left(49152\,\lambda_{4,1}+23\right)\sin(t)-\frac{1}{73728}\left(221184\,\lambda_{4,1}+493+589824\,\lambda_{5,1}\right)\sin(3\,t)-\frac{1}{55296}\left(8095+1327104\,\lambda_{5,2}\right)\sin(5\,t)-\frac{1}{276480}\left(99967+13271040\,\lambda_{5,3}\right)\sin(7\,t)-\frac{1}{30720}\left(9791+2457600\,\lambda_{5,4}\right)\sin(9\,t)-\frac{1}{61440}\left(5533+7372800\,\lambda_{5,5}\right)\sin(11\,t) \quad \textbf{(36)}$$

> $zero\_terms := \{coeff(deqn5temp, \sin(t)), coeff(deqn5temp, \sin(3\,t)), coeff(deqn5temp, \sin(5\,t)), coeff(deqn5temp, \sin(7\,t)), coeff(deqn5temp, \sin(9\,t)), coeff(deqn5temp, \sin(11\,t)), \};$

$$zero\_terms := \left\{-\frac{99967}{276480}-48\,\lambda_{5,3}, -\frac{9791}{30720}-80\,\lambda_{5,4}, -\frac{8095}{55296}-24\,\lambda_{5,2}, -\frac{5533}{61440}-120\,\lambda_{5,5}, -2\,\lambda_{4,1}-\frac{23}{24576}, -3\,\lambda_{4,1}-\frac{493}{73728}-8\,\lambda_{5,1}\right\} \quad \textbf{(37)}$$

> $solve\left(zero\_terms, \{\lambda_{5,3}, \lambda_{5,4}, \lambda_{5,2}, \lambda_{4,1}, \lambda_{5,5}, \lambda_{5,1}, \lambda_{4,1}\}\right);$

$$\left\{\lambda_{4,1}=-\frac{23}{49152}, \lambda_{5,1}=-\frac{779}{1179648}, \lambda_{5,2}=-\frac{8095}{1327104}, \lambda_{5,3}=-\frac{99967}{13271040}, \lambda_{5,4}=-\frac{9791}{2457600}, \lambda_{5,5}=-\frac{5533}{7372800}\right\} \quad \textbf{(38)}$$

> $\lambda_{4,1} := -\frac{23}{49152};$

$$\lambda_{4,1} := -\frac{23}{49152} \quad \textbf{(39)}$$

> $\lambda_{5,1} := -\frac{779}{1179648};$

$$\lambda_{5,1} := -\frac{779}{1179648} \quad \textbf{(40)}$$

> $\lambda_{5,2} := -\frac{8095}{1327104};$

$$\lambda_{5,2} := -\frac{8095}{1327104} \quad \textbf{(41)}$$

> $\lambda_{5,3} := -\frac{99967}{13271040};$

$$\lambda_{5,3} := -\frac{99967}{13271040} \quad \textbf{(42)}$$

> $\lambda_{5,4} := -\frac{9791}{2457600}$

$$\lambda_{5,4} := -\frac{9791}{2457600} \quad \textbf{(43)}$$

> $\lambda_{5,5} := -\frac{5533}{7372800};$

$$\lambda_{5,5} := -\frac{5533}{7372800} \quad \textbf{(44)}$$

> $deqn5temp := map(simplify, collect(combine(deqn5, trig), [\sin, \cos]));$# checks solutions

$$deqn5temp := 0 \tag{45}$$

> $deqn6temp := map(simplify, collect(combine(deqn6, trig), [\sin, \cos]));$

$$deqn6temp := -\frac{1}{221184}\left(-161 + 1769472\,\lambda_3\right)\cos(t) - \frac{1}{3538944}\left(28311552\,\lambda_{6,2} - 24061\right)\cos(3\,t) - \frac{1}{10616832}\left(254803968\,\lambda_{6,3} + 328835\right)\cos(5\,t) - \frac{1}{66355200}\left(10923199 + 3185049600\,\lambda_{6,4}\right)\cos(7\,t) - \frac{1}{7372800}\left(1769369 + 589824000\,\lambda_{6,5}\right)\cos(9\,t) - \frac{1}{2764800}\left(331776000\,\lambda_{6,6} + 409871\right)\cos(11\,t) - \frac{1}{22118400}\left(3715891200\,\lambda_{6,7} + 715247\right)\cos(13\,t) \tag{46}$$

> $zero\_terms := \{coeff(deqn6temp, \cos(t)), coeff(deqn6temp, \cos(3\,t)), coeff(deqn6temp, \cos(5\,t)), coeff(deqn6temp, \cos(7\,t)), coeff(deqn6temp, \cos(9\,t)), coeff(deqn6temp, \cos(11\,t)), coeff(deqn6temp, \cos(13\,t))\};$

$$zero\_terms := \left\{-\frac{10923199}{66355200} - 48\,\lambda_{6,4}, -\frac{1769369}{7372800} - 80\,\lambda_{6,5}, \frac{161}{221184} - 8\,\lambda_3, -8\,\lambda_{6,2} + \frac{24061}{3538944}, -24\,\lambda_{6,3} - \frac{328835}{10616832}, -120\,\lambda_{6,6} - \frac{409871}{2764800}, -168\,\lambda_{6,7} - \frac{715247}{22118400}\right\} \tag{47}$$

> $solve\left(zero\_terms, \left\{\lambda_{6,4}, \lambda_{6,5}, \lambda_{6,7}, \lambda_{6,3}, \lambda_{6,2}, \lambda_3, \lambda_{6,6}\right\}\right);$

$$\left\{\lambda_3 = \frac{161}{1769472}, \lambda_{6,2} = \frac{24061}{28311552}, \lambda_{6,3} = -\frac{328835}{254803968}, \lambda_{6,4} = -\frac{10923199}{3185049600}, \lambda_{6,5} = -\frac{1769369}{589824000}, \lambda_{6,6} = -\frac{409871}{331776000}, \lambda_{6,7} = -\frac{715247}{3715891200}\right\} \tag{48}$$

> $\lambda_3 := \frac{161}{1769472};$

$$\lambda_3 := \frac{161}{1769472} \tag{49}$$

> $\lambda_{6,2} := \frac{24061}{28311552};$

$$\lambda_{6,2} := \frac{24061}{28311552} \tag{50}$$

> $\lambda_{6,3} := -\frac{328835}{254803968};$

$$\lambda_{6,3} := -\frac{328835}{254803968} \tag{51}$$

> $\lambda_{6,4} := -\frac{10923199}{3185049600};$

$$\lambda_{6,4} := -\frac{10923199}{3185049600} \tag{52}$$

> $\lambda_{6,5} := -\frac{1769369}{589824000};$

(53)

$$\lambda_{6,5} := -\frac{1769369}{589824000} \tag{53}$$

> $\lambda_{6,6} := -\frac{409871}{331776000};$

$$\lambda_{6,6} := -\frac{409871}{331776000} \tag{54}$$

> $\lambda_{6,7} := -\frac{715247}{3715891200};$

$$\lambda_{6,7} := -\frac{715247}{3715891200} \tag{55}$$

> *deqn6temp* := *map*(*simplify*, *collect*(*combine*(*deqn6*, *trig*), [sin, cos]));

$$deqn6temp := 0 \tag{56}$$

> *deqn7temp* := *map*(*simplify*, *collect*(*combine*(*deqn7*, *trig*), [sin, cos]));

$$deqn7temp := -\frac{1}{84934656}\left(51619 + 169869312\,\lambda_{6,1}\right)\sin(t) - \frac{1}{424673280}\left(2656151 + 1274019840\,\lambda_{6,1} + 3397386240\,\lambda_{7,1}\right)\sin(3\,t) - \frac{1}{254803968}\left(1252495 + 6115295232\,\lambda_{7,2}\right)\sin(5\,t) - \frac{1}{7962624000}\left(-415949513 + 382205952000\,\lambda_{7,3}\right)\sin(7\,t) - \frac{1}{884736000}\left(70778880000\,\lambda_{7,4} - 117258703\right)\sin(9\,t) - \frac{1}{12386304000}\left(1486356480000\,\lambda_{7,5} - 1657839733\right)\sin(11\,t) - \frac{1}{344064000}\left(-21731177 + 57802752000\,\lambda_{7,6}\right)\sin(13\,t) - \frac{1}{12386304}\left(2774532096\,\lambda_{7,7} - 138697\right)\sin(15\,t) \tag{57}$$

> *zero_terms* := {*coeff*(*deqn7temp*, sin(t)), *coeff*(*deqn7temp*, sin(3 t)), *coeff*(*deqn7temp*, sin(5 t)), *coeff*(*deqn7temp*, sin(7 t)), *coeff*(*deqn7temp*, sin(9 t)), *coeff*(*deqn7temp*, sin(11 t)), *coeff*(*deqn7temp*, sin(13 t)), *coeff*(*deqn7temp*, sin(15 t))};

$$zero\_terms := \left\{-\frac{1252495}{254803968} - 24\,\lambda_{7,2},\ -\frac{51619}{84934656} - 2\,\lambda_{6,1},\ \frac{21731177}{344064000} - 168\,\lambda_{7,6},\ \frac{415949513}{7962624000} - 48\,\lambda_{7,3},\ -80\,\lambda_{7,4} + \frac{117258703}{884736000},\ -120\,\lambda_{7,5} + \frac{1657839733}{12386304000},\ -224\,\lambda_{7,7} + \frac{138697}{12386304},\ -\frac{2656151}{424673280} - 3\,\lambda_{6,1} - 8\,\lambda_{7,1}\right\} \tag{58}$$

> *solve*(*zero_terms*, {$\lambda_{7,7}, \lambda_{7,6}, \lambda_{7,5}, \lambda_{6,1}, \lambda_{7,2}, \lambda_{7,3}, \lambda_{7,4}, \lambda_{7,1}$}); # *solve coefficients*

$$\left\{\lambda_{6,1} = -\frac{51619}{169869312},\ \lambda_{7,1} = -\frac{4538017}{6794772480},\ \lambda_{7,2} = -\frac{1252495}{6115295232},\ \lambda_{7,3} = \frac{415949513}{382205952000},\ \lambda_{7,4} = \frac{117258703}{70778880000},\ \lambda_{7,5} = \frac{1657839733}{1486356480000},\ \lambda_{7,6} = \frac{21731177}{57802752000},\ \lambda_{7,7} = \frac{138697}{2774532096}\right\} \tag{59}$$

> $\lambda_{6,1} := -\frac{51619}{169869312};$

$$\lambda_{6,1} := -\frac{51619}{169869312} \quad (60)$$

> $\lambda_{7,1} := -\frac{4538017}{6794772480};$

$$\lambda_{7,1} := -\frac{4538017}{6794772480} \quad (61)$$

> $\lambda_{7,2} := -\frac{1252495}{6115295232};$

$$\lambda_{7,2} := -\frac{1252495}{6115295232} \quad (62)$$

> $\lambda_{7,3} := \frac{415949513}{382205952000};$

$$\lambda_{7,3} := \frac{415949513}{382205952000} \quad (63)$$

> $\lambda_{7,4} := \frac{117258703}{70778880000};$

$$\lambda_{7,4} := \frac{117258703}{70778880000} \quad (64)$$

> $\lambda_{7,5} := \frac{1657839733}{1486356480000};$

$$\lambda_{7,5} := \frac{1657839733}{1486356480000} \quad (65)$$

> $\lambda_{7,6} := \frac{21731177}{57802752000};$

$$\lambda_{7,6} := \frac{21731177}{57802752000} \quad (66)$$

> $\lambda_{7,7} := \frac{138697}{2774532096};$

$$\lambda_{7,7} := \frac{138697}{2774532096} \quad (67)$$

> *deqn7temp* := *map*(*simplify*, *collect*(*combine*(*deqn7*, *trig*), [sin, cos])); # *check solution*

$$deqn7temp := 0 \quad (68)$$

> *deqn8temp* := *map*(*simplify*, *collect*(*combine*(*deqn8*, *trig*), [sin, cos]));

$$deqn8temp := -\frac{1}{1274019840}\left(676379 + 10192158720\,\lambda_4\right)\cos(t) \quad (69)$$

$$-\frac{1}{101921587200}\left(279818087 + 815372697600\,\lambda_{8,2}\right)\cos(3\,t)$$

$$-\frac{1}{12230590464}\left(111998015 + 293534171136\,\lambda_{8,3}\right)\cos(5\,t) - \frac{1}{3822059520000}\Big($$

$$-21049213549 + 183458856960000\,\lambda_{8,4}\Big)\cos(7\,t) - \frac{1}{2972712960000}\Big(-161113663733$$

$$+ 237817036800000\,\lambda_{8,5}\big)\cos(9\,t) - \frac{1}{15606743040000}\big(1872809164800000\,\lambda_{8,6} - 1359229760383\big)\cos(11\,t) - \frac{1}{31213486080000}\big(-2076538440769 + 5243865661440000\,\lambda_{8,7}\big)\cos(13\,t) - \frac{1}{20808990720}\big(4661213921280\,\lambda_{8,8} - 526426361\big)\cos(15\,t) - \frac{1}{104044953600}\big(-392636471 + 29964946636800\,\lambda_{8,9}\big)\cos(17\,t)$$

> *zero_terms* := {*coeff*(*deqn8temp*, cos(t)), *coeff*(*deqn8temp*, cos(3 t)), *coeff*(*deqn8temp*, cos(5 t)), *coeff*(*deqn8temp*, cos(7 t)), *coeff*(*deqn8temp*, cos(9 t)), *coeff*(*deqn8temp*, cos(11 t)), *coeff*(*deqn8temp*, cos(13 t)), *coeff*(*deqn8temp*, cos(15 t)), *coeff*(*deqn8temp*, cos(17 t))}; # *equate coefficients of trig func to zero*

$$zero\_terms := \left\{-\frac{279818087}{101921587200} - 8\,\lambda_{8,2}, -\frac{111998015}{12230590464} - 24\,\lambda_{8,3}, -\frac{676379}{1274019840} - 8\,\lambda_4, \frac{392636471}{104044953600} - 288\,\lambda_{8,9}, \frac{21049213549}{3822059520000} - 48\,\lambda_{8,4}, \frac{161113663733}{2972712960000} - 80\,\lambda_{8,5}, \frac{2076538440769}{31213486080000} - 168\,\lambda_{8,7}, -120\,\lambda_{8,6} + \frac{1359229760383}{15606743040000}, -224\,\lambda_{8,8} + \frac{526426361}{20808990720}\right\} \quad \textbf{(70)}$$

> *solve*(*zero_terms*, { $\lambda_{8,9}, \lambda_{8,8}, \lambda_4, \lambda_{8,2}, \lambda_{8,3}, \lambda_{8,4}, \lambda_{8,5}, \lambda_{8,6}, \lambda_{8,7}$}); # *solve coefficients*

$$\left\{\lambda_4 = -\frac{676379}{10192158720}, \lambda_{8,2} = -\frac{279818087}{815372697600}, \lambda_{8,3} = -\frac{111998015}{293534171136}, \lambda_{8,4} = \frac{21049213549}{183458856960000}, \lambda_{8,5} = \frac{161113663733}{237817036800000}, \lambda_{8,6} = \frac{1359229760383}{1872809164800000}, \lambda_{8,7} = \frac{2076538440769}{5243865661440000}, \lambda_{8,8} = \frac{526426361}{4661213921280}, \lambda_{8,9} = \frac{392636471}{29964946636800}\right\} \quad \textbf{(71)}$$

> $\lambda_4 := -\frac{676379}{10192158720};$

$$\lambda_4 := -\frac{676379}{10192158720} \quad \textbf{(72)}$$

> $\lambda_{8,2} := -\frac{279818087}{815372697600};$

$$\lambda_{8,2} := -\frac{279818087}{815372697600} \quad \textbf{(73)}$$

> $\lambda_{8,3} := -\frac{111998015}{293534171136};$

$$\lambda_{8,3} := -\frac{111998015}{293534171136} \quad \textbf{(74)}$$

> $\lambda_{8,4} := \frac{21049213549}{183458856960000};$

$$\lambda_{8,4} := \frac{21049213549}{183458856960000} \quad \textbf{(75)}$$

> $\lambda_{8,5} := \frac{161113663733}{237817036800000};$

$$\lambda_{8,5} := \frac{161113663733}{237817036800000} \tag{76}$$

> $\lambda_{8,6} := \frac{1359229760383}{1872809164800000};$

$$\lambda_{8,6} := \frac{1359229760383}{1872809164800000} \tag{77}$$

> $\lambda_{8,7} := \frac{2076538440769}{5243865661440000};$

$$\lambda_{8,7} := \frac{2076538440769}{5243865661440000} \tag{78}$$

> $\lambda_{8,8} := \frac{526426361}{4661213921280};$

$$\lambda_{8,8} := \frac{526426361}{4661213921280} \tag{79}$$

> $\lambda_{8,9} := \frac{392636471}{29964946636800};$

$$\lambda_{8,9} := \frac{392636471}{29964946636800} \tag{80}$$

> *deqn8temp* := *map*(*simplify*, *collect*(*combine*(*deqn8*, *trig*), [sin, cos]));

$$deqn8temp := 0 \tag{81}$$

> *deqn9temp* := *map*(*simplify*, *collect*(*combine*(*deqn9*, *trig*), [sin, cos])) :

> *zero_terms* := {*coeff*(*deqn9temp*, sin(t)), *coeff*(*deqn9temp*, sin(3 t)), *coeff*(*deqn9temp*, sin(5 t)), *coeff*(*deqn9temp*, sin(7 t)), *coeff*(*deqn9temp*, sin(9 t)), *coeff*(*deqn9temp*, sin(11 t)), *coeff*(*deqn9temp*, sin(13 t)), *coeff*(*deqn9temp*, sin(15 t)), *coeff*(*deqn9temp*, sin(17 t)), *coeff*(*deqn9temp*, sin(19 t))};

$$zero\_terms := \left\{ -\frac{10150413758573}{1048773132288000} - 288\,\lambda_{9,8}, -\frac{1503060931884017}{34959104409600000} - 120\,\lambda_{9,5}, \frac{948555443}{9784472371200} - 2\,\lambda_{8,1}, \frac{3027520634887}{458647142400000} - 48\,\lambda_{9,3}, -24\,\lambda_{9,2} + \frac{215530591}{36691771392}, -80\,\lambda_{9,4} - \frac{33266232868297}{2497078886400000}, -168\,\lambda_{9,6} - \frac{218013001702691}{4369888051200000}, -224\,\lambda_{9,7} - \frac{236645090569}{7768689868800}, -360\,\lambda_{9,9} - \frac{466445839}{374561832960}, \frac{44229996751}{48922361856000} - 8\,\lambda_{9,1} - 3\,\lambda_{8,1} \right\} \tag{82}$$

> *solve*(*zero_terms*, $\{\lambda_{9,9}, \lambda_{9,6}, \lambda_{9,2}, \lambda_{8,1}, \lambda_{9,3}, \lambda_{9,4}, \lambda_{9,5}, \lambda_{9,7}, \lambda_{9,8}, \lambda_{9,1}\}$); # *solve constants*

$$\left\{ \lambda_{8,1} = \frac{948555443}{19568944742400}, \lambda_{9,1} = \frac{74231661857}{782757789696000}, \lambda_{9,2} = \frac{215530591}{880602513408}, \lambda_{9,3} = \frac{3027520634887}{22015062835200000}, \lambda_{9,4} = -\frac{33266232868297}{199766310912000000}, \lambda_{9,5} = -\frac{1503060931884017}{4195092529152000000}, \lambda_{9,6} = -\frac{218013001702691}{734141192601600000}, \lambda_{9,7} = -\frac{236645090569}{1740186530611200}, \lambda_{9,8} = -\frac{10150413758573}{302046662098944000}, \lambda_{9,9} = -\frac{466445839}{134842259865600} \right\} \tag{83}$$

> $\lambda_{8,1} := \frac{948555443}{19568944742400};$

$$\lambda_{8,1} := \frac{948555443}{19568944742400} \tag{84}$$

> $\lambda_{9,1} := \frac{74231661857}{782757789696000};$

$$\lambda_{9,1} := \frac{74231661857}{782757789696000} \tag{85}$$

> $\lambda_{9,2} := \frac{215530591}{880602513408};$

$$\lambda_{9,2} := \frac{215530591}{880602513408} \tag{86}$$

> $\lambda_{9,3} := \frac{3027520634887}{22015062835200000};$

$$\lambda_{9,3} := \frac{3027520634887}{22015062835200000} \tag{87}$$

> $\lambda_{9,4} := -\frac{33266232868297}{199766310912000000};$

$$\lambda_{9,4} := -\frac{33266232868297}{199766310912000000} \tag{88}$$

> $\lambda_{9,5} := -\frac{1503060931884017}{4195092529152000000};$

$$\lambda_{9,5} := -\frac{1503060931884017}{4195092529152000000} \tag{89}$$

> $\lambda_{9,6} := -\frac{218013001702691}{734141192601600000};$

$$\lambda_{9,6} := -\frac{218013001702691}{734141192601600000} \tag{90}$$

> $\lambda_{9,7} := -\frac{236645090569}{1740186530611200};$

$$\lambda_{9,7} := -\frac{236645090569}{1740186530611200} \tag{91}$$

> $\lambda_{9,8} := -\frac{10150413758573}{302046662098944000};$

$$\lambda_{9,8} := -\frac{10150413758573}{302046662098944000} \tag{92}$$

> $\lambda_{9,9} := -\frac{466445839}{134842259865600};$

$$\lambda_{9,9} := -\frac{466445839}{134842259865600} \tag{93}$$

> *deqn9temp* := *map*(*simplify*, *collect*(*combine*(*deqn9*, *trig*), [sin, cos]));

$$deqn9temp := 0 \tag{94}$$

> *deqn10temp* := *map*(*simplify*, *collect*(*combine*(*deqn10*, *trig*), [sin, cos])) :

> *zero_terms* := {*coeff*(*deqn10temp*, cos(t)), *coeff*(*deqn10temp*, cos(3 t)), *coeff*(*deqn10temp*, cos(5 t)), *coeff*(*deqn10temp*, cos(7 t)), *coeff*(*deqn10temp*, cos(9 t)), *coeff*(*deqn10temp*,

cos(11 t)), *coeff* (*deqn10temp*, cos(13 t)), *coeff* (*deqn10temp*, cos(15 t)), *coeff* (*deqn10temp*, cos(17 t)), *coeff* (*deqn10temp*, cos(19 t)), *coeff* (*deqn10temp*, cos(21 t)) }; # *equate coefficients of trig func to zero*

$$zero\_terms := \left\{ -\frac{164379102706183123}{110121178890240000000} - 120\,\lambda_{10,6}, -\frac{1263332565544137803}{440484715560960000000} - 168\,\lambda_{10,7}, \frac{283166833}{9172942848000} - 8\,\lambda_5, \frac{4567108919771}{19725496300339200} - 24\,\lambda_{10,3}, \frac{689487257544313}{110075314176000000} - 48\,\lambda_{10,4}, \frac{7102093905417929}{4195092529152000000} - 80\,\lambda_{10,5}, -8\,\lambda_{10,2} + \frac{177798462109}{11741366845440000}, -224\,\lambda_{10,8} - \frac{9120006426613559}{3523877724487680000}, -288\,\lambda_{10,9} - \frac{69325554291687167}{52858165867315200000}, -360\,\lambda_{10,10} - \frac{1079414079865327}{3020466620989440000}, -440\,\lambda_{10,11} - \frac{29654422883}{73383542784000} \right\} \tag{95}$$

> *solve*(*zero_terms*, { $\lambda_{10,11}, \lambda_{10,10}, \lambda_{10,2}, \lambda_{10,3}, \lambda_{10,4}, \lambda_5, \lambda_{10,5}, \lambda_{10,6}, \lambda_{10,7}, \lambda_{10,8}, \lambda_{10,9}$ }); # *solve constants*

$$\left\{ \lambda_5 = \frac{283166833}{73383542784000}, \lambda_{10,2} = \frac{177798462109}{93930934763520000}, \lambda_{10,3} = \frac{4567108919771}{473411911208140800}, \lambda_{10,4} = \frac{689487257544313}{5283615080448000000}, \lambda_{10,5} = \frac{7102093905417929}{335607402332160000000}, \lambda_{10,6} = -\frac{164379102706183123}{13214541466828800000000}, \lambda_{10,7} = -\frac{1263332565544137803}{74001432214241280000000}, \lambda_{10,8} = -\frac{9120006426613559}{789348610285240320000}, \lambda_{10,9} = -\frac{69325554291687167}{15223151769786777600000}, \lambda_{10,10} = -\frac{1079414079865327}{1087367983556198400000}, \lambda_{10,11} = -\frac{29654422883}{32288758824960000} \right\} \tag{96}$$

> $\lambda_5 := \frac{283166833}{73383542784000}$; $\lambda_{10,2} := \frac{177798462109}{93930934763520000}$; $\lambda_{10,3} := \frac{4567108919771}{473411911208140800}$; $\lambda_{10,4} := \frac{689487257544313}{5283615080448000000}$; $\lambda_{10,5} := \frac{7102093905417929}{335607402332160000000}$; $\lambda_{10,6} := -\frac{164379102706183123}{13214541466828800000000}$; $\lambda_{10,7} := -\frac{1263332565544137803}{74001432214241280000000}$; $\lambda_{10,8} := -\frac{9120006426613559}{789348610285240320000}$; $\lambda_{10,9} := -\frac{69325554291687167}{15223151769786777600000}$; $\lambda_{10,10} := -\frac{1079414079865327}{1087367983556198400000}$; $\lambda_{10,11} := -\frac{29654422883}{32288758824960000}$;

$$\lambda_5 := \frac{283166833}{73383542784000}$$

$$\lambda_{10,2} := \frac{177798462109}{93930934763520000}$$

$$\lambda_{10,3} := \frac{4567108919771}{473411911208140800}$$

$$\lambda_{10,4} := \frac{689487257544313}{5283615080448000000}$$

$$\lambda_{10,5} := \frac{7102093905417929}{335607402332160000000}$$

$$\lambda_{10,6} := -\frac{164379102706183123}{13214541466828800000000}$$

$$\lambda_{10,7} := -\frac{1263332565544137803}{740014322142412800000 0}$$

$$\lambda_{10,8} := -\frac{9120006426613559}{78934861028524032000}$$

$$\lambda_{10,9} := -\frac{69325554291687167}{1522315176978677760000}$$

$$\lambda_{10,10} := -\frac{1079414079865327}{1087367983556198400 00}$$

$$\lambda_{10,11} := -\frac{29654422883}{322887588249600000} \quad \textbf{(97)}$$

> *deqn10temp* := map(simplify, collect(combine(deqn10, trig), [sin, cos]));

$$deqn10temp := 0 \quad \textbf{(98)}$$

> *deqn11temp* := map(simplify, collect(combine(deqn11, trig), [sin, cos])) :

> *zero_terms* := {coeff(deqn11temp, sin(t)), coeff(deqn11temp, sin(3 t)), coeff(deqn11temp, sin(5 t)), coeff(deqn11temp, sin(7 t)), coeff(deqn11temp, sin(9 t)), coeff(deqn11temp, sin(11 t)), coeff(deqn11temp, sin(13 t)), coeff(deqn11temp, sin(15 t)), coeff(deqn11temp, sin(17 t)), coeff(deqn11temp, sin(19 t)), coeff(deqn11temp, sin(21 t)), coeff(deqn11temp, sin(23 t))};

$$zero\_terms := \left\{ -\frac{15570722605983324961}{352387772448768 0000000} - 80\,\lambda_{11,4}, \frac{306960354499903}{2373223773634560000} - 528\,\lambda_{11,11}, \frac{10437762921486029}{813676722388992000 0} - 440\,\lambda_{11,10}, \frac{300670130823743439 11}{17760343731417907200000} - 224\,\lambda_{11,7}, \frac{1832594120713232531083}{14800286442848256 0000000} - 168\,\lambda_{11,6}, -2\,\lambda_{10,1} + \frac{899376886003}{56358560858112000}, -24\,\lambda_{11,2} - \frac{773824093037749}{16569416892284928 00}, -48\,\lambda_{11,3} - \frac{37831797914979049}{11557907988480000000}, -120\,\lambda_{11,5} + \frac{934053336616268746 79}{49334288142827520000000}, -288\,\lambda_{11,8} + \frac{3318045553455980024 03}{266405155971268608 00000}, -360\,\lambda_{11,9} + \frac{901410814111290920 93}{16745466946765455360000}, \frac{1751270868001189}{9862748150169600000} - 8\,\lambda_{11,1} - 3\,\lambda_{10,1} \right\} \quad \textbf{(99)}$$

> *solve*(*zero_terms*, {$\lambda_{11,2}, \lambda_{11,3}, \lambda_{11,4}, \lambda_{10,1}, \lambda_{11,10}, \lambda_{11,5}, \lambda_{11,8}, \lambda_{11,6}, \lambda_{11,7}, \lambda_{11,9}, \lambda_{11,11}, \lambda_{11,1}$}); # *solve constants*

$$\left\{ \lambda_{10,1} = \frac{899376886003}{112717121716224000}, \lambda_{11,1} = \frac{3030368870850803}{157803970402713600000}, \lambda_{11,2} = -\frac{773824093037749}{39766600541483827200}, \lambda_{11,3} = -\frac{37831797914979049}{554779583447040000000}, \lambda_{11,4} = -\frac{15570722605983324961}{281910217959014400000000}, \lambda_{11,5} = \frac{93405333661626874679}{5920114577139302400000000}, \lambda_{11,6} \quad \textbf{(100)}$$

$$= \frac{1832594120713232531083}{24864481223985070080000000}, \lambda_{11,7} = \frac{30067013082374343911}{397831699583761121280000}, \lambda_{11,8} = \frac{3318045553345598002403}{767246849197253591040000}, \lambda_{11,9} = \frac{90141081411129092093}{6028368100835563929600000}, \lambda_{11,10} = \frac{10437762921486029}{35801775785115648000000}, \lambda_{11,11} = \frac{306960354499903}{12530621524790476800000} \}$$

> $\lambda_{10,1} := \frac{899376886003}{112717121716224000}; \lambda_{11,1} := \frac{3030368870850803}{157803970402713600000}; \lambda_{11,2} := -\frac{773824093037749}{397666005414838272000}; \lambda_{11,3} := -\frac{37831797914979049}{554779583447040000000}; \lambda_{11,4} := -\frac{15570722605983324961}{28191021795901440000000}; \lambda_{11,5} := \frac{93405333661626874679}{592011457713930240000000}; \lambda_{11,6} := \frac{1832594120713232531083}{24864481223985070080000000}; \lambda_{11,7} := \frac{30067013082374343911}{397831699583761121280000}; \lambda_{11,8} := \frac{3318045553345598002403}{767246849197253591040000}; \lambda_{11,9} := \frac{90141081411129092093}{6028368100835563929600000}; \lambda_{11,10} := \frac{10437762921486029}{35801775785115648000000}; \lambda_{11,11} := \frac{306960354499903}{12530621524790476800000};$

$$\lambda_{10,1} := \frac{899376886003}{112717121716224000}$$

$$\lambda_{11,1} := \frac{3030368870850803}{157803970402713600000}$$

$$\lambda_{11,2} := -\frac{773824093037749}{397666005414838272000}$$

$$\lambda_{11,3} := -\frac{37831797914979049}{554779583447040000000}$$

$$\lambda_{11,4} := -\frac{15570722605983324961}{28191021795901440000000}$$

$$\lambda_{11,5} := \frac{93405333661626874679}{592011457713930240000000}$$

$$\lambda_{11,6} := \frac{1832594120713232531083}{24864481223985070080000000}$$

$$\lambda_{11,7} := \frac{30067013082374343911}{397831699583761121280000}$$

$$\lambda_{11,8} := \frac{3318045553345598002403}{767246849197253591040000}$$

$$\lambda_{11,9} := \frac{90141081411129092093}{6028368100835563929600000}$$

$$\lambda_{11,10} := \frac{10437762921486029}{35801775785115648000000}$$

$$\lambda_{11,11} := \frac{306960354499903}{12530621524790476800000} \quad \textbf{(101)}$$

> $deqn11temp := map(simplify, collect(combine(deqn11, trig), [\sin, \cos]))$:

> $deqn12temp := map(simplify, collect(combine(deqn12, trig), [\sin, \cos]))$:

> $zero\_terms := \{coeff(deqn12temp, \cos(t)), coeff(deqn12temp, \cos(3\,t)), coeff(deqn12temp,$

cos(5 t)), *coeff* (*deqn12temp*, cos(7 t)), *coeff* (*deqn12temp*, cos(9 t)), *coeff* (*deqn12temp*, cos(11 t)), *coeff* (*deqn12temp*, cos(13 t)), *coeff* (*deqn12temp*, cos(15 t)), *coeff* (*deqn12temp*, cos(17 t)), *coeff* (*deqn12temp*, cos(19 t)), *coeff* (*deqn12temp*, cos(21 t)), *coeff* (*deqn12temp*, cos(23 t)), *coeff* (*deqn12temp*, cos(25 t)) };
 # equate coefficients of trig func to zero

$$zero\_terms := \left\{ -\frac{7277455304944278525576 7}{2368045830855720960000000} - 80\,\lambda_{12,5}, \frac{437826774352021973}{278366203790386790400\,0} - 24\,\lambda_{12,3}, \frac{5246697435916277879518531}{928368687528676845158400000} - 360\,\lambda_{12,10}, \frac{1346963739848870233767492 7}{147695018470471316275200000 0} - 288\,\lambda_{12,9}, -8\,\lambda_6 + \frac{18709525402061}{924632639078400000}, -8\,\lambda_{12,2} + \frac{2114054402463038467}{16569416892284928000000}, -48\,\lambda_{12,4} - \frac{6767517794598282239 99}{62135313346068480000000 0}, -120\,\lambda_{12,6} - \frac{8715911632372046205178 9}{414408020399751168000000 00}, -168\,\lambda_{12,7} + \frac{24322560636739368018910 57}{745934436719552102400000000}, -224\,\lambda_{12,8} + \frac{769284635444527782704 69}{89512132406346252288000 00}, -440\,\lambda_{12,11} + \frac{959193801906281485783}{451102374892457164800000}, -528\,\lambda_{12,12} + \frac{296019733193613115 69}{657857630051500032000 00}, -624\,\lambda_{12,13} + \frac{12821749966541}{313265538119761920} \right\} \quad \textbf{(102)}$$

> *solve*$\left(zero\_terms, \left\{ \lambda_{12,4}, \lambda_{12,5}, \lambda_{12,6}, \lambda_6, \lambda_{12,3}, \lambda_{12,2}, \lambda_{12,9}, \lambda_{12,7}, \lambda_{12,8}, \lambda_{12,10}, \lambda_{12,11}, \lambda_{12,12}, \lambda_{12,13} \right\} \right)$:

> $\lambda_6 := \frac{18709525402061}{7397061112627200000}$; $\lambda_{12,2} := \frac{2114054402463038467}{132555335138279424000000}$; $\lambda_{12,3} := \frac{437826774352021973}{6680788890969282969600 0}$; $\lambda_{12,4} := -\frac{676751779459828223999}{29824950406112870400000000}$; $\lambda_{12,5} := -\frac{72774553049442785255767}{1894436664684576768000000000}$; $\lambda_{12,6} := -\frac{87159116323720462051789}{4972896244797014016000000000}$; $\lambda_{12,7} := \frac{2432256063673936801891057}{125316985368884753203200000000}$; $\lambda_{12,8} := \frac{76928463544452778270469}{20050717659021560512512000 00}$; $\lambda_{12,9} := \frac{13469637398488702337674927}{4253616531949573908725760000 00}$; $\lambda_{12,10} := \frac{5246697435916277879518531}{334212727510323664257024000000}$; $\lambda_{12,11} := \frac{959193801906281485783}{198485044952681152512000000}$; $\lambda_{12,12} := \frac{29601973319361311569}{347348828667192016896000000}$; $\lambda_{12,13} := \frac{12821749966541}{195477695786731438080}$;

$$\lambda_6 := \frac{18709525402061}{7397061112627200000}$$

$$\lambda_{12,2} := \frac{2114054402463038467}{132555335138279424000000}$$

$$\lambda_{12,3} := \frac{437826774352021973}{66807888909692829696000}$$

$$\lambda_{12,4} := -\frac{676751779459828223999}{29824950406112870400000000}$$

$$\lambda_{12,5} := -\frac{7277455304944278525576 7}{189443666468457676800000000}$$

$$\lambda_{12,6} := -\frac{871591163237204620517 89}{497289624479701401600000000}$$

$$\lambda_{12,7} := \frac{2432225606367393680189105 7}{1253169853688847532032000000 00}$$

$$\lambda_{12,8} := \frac{7692846354445277827046 9}{200507176590215605125120000 0}$$

$$\lambda_{12,9} := \frac{1346963739848870233767492 7}{4253616531949573908725760000 00}$$

$$\lambda_{12,10} := \frac{5246697435916277879518531}{334212727510323664257024000000 0}$$

$$\lambda_{12,11} := \frac{959193801906281485783}{198485044952681152512000000 0}$$

$$\lambda_{12,12} := \frac{296019733193613115 69}{347348828667192016896000000}$$

$$\lambda_{12,13} := \frac{12821749966541}{19547769578673143808 0} \tag{103}$$

> $deqn12temp := map(simplify, collect(combine(deqn12, trig), [\sin, \cos]));$

$$deqn12temp := 0 \tag{104}$$

> $deqn13temp := map(simplify, collect(combine(deqn13, trig), [\sin, \cos])) :$

> $zero\_terms := \{coeff(deqn13temp, \sin(t)), coeff(deqn13temp, \sin(3\,t)), coeff(deqn13temp, \sin(5\,t)), coeff(deqn13temp, \sin(7\,t)), coeff(deqn13temp, \sin(9\,t)), coeff(deqn13temp, \sin(11\,t)), coeff(deqn13temp, \sin(13\,t)), coeff(deqn13temp, \sin(15\,t)), coeff(deqn13temp, \sin(17\,t)), coeff(deqn13temp, \sin(19\,t)), coeff(deqn13temp, \sin(21\,t)), coeff(deqn13temp, \sin(23\,t)), coeff(deqn13temp, \sin(25\,t)), coeff(deqn13temp, \sin(27\,t))\} :$

> $solve\left(zero\_terms, \left\{\lambda_{12,1}, \lambda_{13,12}, \lambda_{13,11}, \lambda_{13,5}, \lambda_{13,2}, \lambda_{13,3}, \lambda_{13,4}, \lambda_{13,6}, \lambda_{13,7}, \lambda_{13,8}, \lambda_{13,9}, \lambda_{13,10}, \lambda_{13,13}, \lambda_{13,1}\right\}\right) :$

> $\lambda_{12,1} := -\frac{1043558042177218117}{31813280433187061760000 0}; \lambda_{13,1} := -\frac{30272184448705772281 97}{445385926064618864640000000}; \lambda_{13,2} := -\frac{273333340767208840031}{2805931334207098847232000 0}; \lambda_{13,3} := \frac{47340136081522495436341}{25052958341134811136000000000}; \lambda_{13,4} := \frac{27316618549621483126137203}{159132679833504448512000000000 00}; \lambda_{13,5} := \frac{545178681407652325036038834 7}{300760764885323407687680000000000 0}; \lambda_{13,6} := \frac{161978662112639894861499574 7}{631597606259179156144128000000000 0}; \lambda_{13,7} := -\frac{153820003662979889788438581 1}{111161178701615531481366528000000 0}; \lambda_{13,8} := -\frac{432122164210127016965076231953}{2358205005312843774997561344000000 00}; \lambda_{13,9} := -\frac{23550165395927585934733962662 3}{1852875361317234394640941056000000 00}; \lambda_{13,10} :=$

$- \frac{6116955247262910119431748 9}{11004010892176643095265280000000}; \lambda_{13, 11} :=$
$- \frac{15437380498809076600986314 9}{10013649911880745216691404800000 0}; \lambda_{13, 12} :=$
$- \frac{447497082273611260487}{1803323966814887450278625280}; \lambda_{13, 13} := - \frac{17689067479651037}{1003934832682472570880000};$

$$\lambda_{12, 1} := - \frac{1043558042177218117}{318132804331870617600000}$$

$$\lambda_{13, 1} := - \frac{302721844487057722819 7}{445385926064618864640000000}$$

$$\lambda_{13, 2} := - \frac{27333334076720884003 1}{280593133420709884723200 00}$$

$$\lambda_{13, 3} := \frac{473401360815224954363 41}{250529583411348111360000000 00}$$

$$\lambda_{13, 4} := \frac{273166185496214831261372 03}{15913267983350444851200000000 00}$$

$$\lambda_{13, 5} := \frac{545178681407652325036038834 7}{30076076488532340768768000000000 0}$$

$$\lambda_{13, 6} := \frac{161978662112639894861499574 7}{63159760625917915614412800000000 0}$$

$$\lambda_{13, 7} := - \frac{153820003662979889788438581 1}{11116117870161553148136652800000 0}$$

$$\lambda_{13, 8} := - \frac{4321221642101270169650762319 53}{23582050053128437749975613440000000}$$

$$\lambda_{13, 9} := - \frac{2355016539592758593473396266 23}{18528753613172343946409410560000000}$$

$$\lambda_{13, 10} := - \frac{6116955247262910119431748 9}{11004010892176643095265280000000}$$

$$\lambda_{13, 11} := - \frac{15437380498809076600986314 9}{10013649911880745216691404800000 0}$$

$$\lambda_{13, 12} := - \frac{447497082273611260487}{1803323966814887450278625280}$$

$$\lambda_{13, 13} := - \frac{17689067479651037}{1003934832682472570880000} \qquad \textbf{(105)}$$

> *deqn13temp* := *map*(*simplify*, *collect*(*combine*(*deqn13*, *trig*), [sin, cos]));

$$deqn13temp := 0 \qquad \textbf{(106)}$$

> *deqn14temp* := *map*(*simplify*, *collect*(*combine*(*deqn14*, *trig*), [sin, cos])) :

> *zero_terms* := {*coeff*(*deqn14temp*, cos(t)), *coeff*(*deqn14temp*, cos(3 t)), *coeff*(*deqn14temp*, cos(5 t)), *coeff*(*deqn14temp*, cos(7 t)), *coeff*(*deqn14temp*, cos(9 t)), *coeff*(*deqn14temp*, cos(11 t)), *coeff*(*deqn14temp*, cos(13 t)), *coeff*(*deqn14temp*, cos(15 t)), *coeff*(*deqn14temp*, cos(17 t)), *coeff*(*deqn14temp*, cos(19 t)), *coeff*(*deqn14temp*, cos(21 t)), *coeff*(*deqn14temp*, cos(23 t)), *coeff*(*deqn14temp*, cos(25 t)), *coeff*(*deqn14temp*, cos(27 t)), *coeff*(*deqn14temp*, cos(29 t))} :
 # equate coefficients of trig func to zero

> *solve*(*zero_terms*, $\{\lambda_7, \lambda_{14, 15}, \lambda_{14, 2}, \lambda_{14, 3}, \lambda_{14, 4}, \lambda_{14, 5}, \lambda_{14, 6}, \lambda_{14, 7}, \lambda_{14, 8}, \lambda_{14, 9}, \lambda_{14, 10}, \lambda_{14, 11},$

$\lambda_{14,12}, \lambda_{14,13}, \lambda_{14,14}\}) :$

$$> \lambda_7 := -\frac{4926458600140905907}{1043873264213950464000000}; \lambda_{14,2} := -\frac{6852305401828116022817 83}{374124177894279846297600000000};$$

$$\lambda_{14,3} := -\frac{107661122917891135844527 7}{188558585658717042533990400000}; \lambda_{14,4} := -\frac{16546335822603558710578393 7}{42088970013106482708480000000000}; \lambda_{14,5} := \frac{117581448257344743326159356 79}{267342902120287473500160000000000 0}; \lambda_{14,6} := \frac{392367462575471034485573687394 7}{378958563755507493686476800000000000}; \lambda_{14,7} := \frac{392337695781091849469029848588 7}{547121426422013944009850880000000000}; \lambda_{14,8} := -\frac{108165520931093116791384716022 59}{616277574721756506532696031232000000 00}; \lambda_{14,9} := -\frac{105461968574931184712801821635849 37}{130738885494544058885864800911360000000 00}; \lambda_{14,10} := -\frac{844296283159120305375587173097152 9}{102723410031427474838893772144640000000 00}; \lambda_{14,11} := -\frac{390849275466668413561290180875641}{79308107302095502116195926016000000 00}; \lambda_{14,12} := -\frac{278331630055718236695148240348 43}{144340755289813813851476585349120000 00}; \lambda_{14,13} := -\frac{316130036683604181358606259 3}{649845824681412841582405405900800 0}; \lambda_{14,14} := -\frac{729547964918096196101783}{101297827765527628379848704000 00}; \lambda_{14,15} := -\frac{378242952723832503289}{7969234701833467267645440000 00};$$

$$\lambda_7 := -\frac{4926458600140905907}{1043873264213950464000000}$$

$$\lambda_{14,2} := -\frac{685230540182811602281783}{374124177894279846297600000000}$$

$$\lambda_{14,3} := -\frac{1076611229178911358445277}{188558585658717042533990400000}$$

$$\lambda_{14,4} := -\frac{165463358226035587105783937}{42088970013106482708480000000000}$$

$$\lambda_{14,5} := \frac{11758144825734474332615935679}{2673429021202874735001600000000000}$$

$$\lambda_{14,6} := \frac{3923674625754710344855736873947}{378958563755507493686476800000000000}$$

$$\lambda_{14,7} := \frac{3923376957810918494690298485887}{547121426422013944009850880000000000}$$

$$\lambda_{14,8} := -\frac{108165520931093116791384716022 59}{616277574721756506532696031232000000 00}$$

$$\lambda_{14,\,9} := -\frac{10546196857493118471280182163584937}{1307388854945440588858648009113600000000}$$

$$\lambda_{14,\,10} := -\frac{844296283159120305375587173097152 9}{10272341003142747483889377214464000000000}$$

$$\lambda_{14,\,11} := -\frac{39084927546668413561290180875641}{79308107302095502116195926016000000000}$$

$$\lambda_{14,\,12} := -\frac{278331630055718236695148240348 43}{14434075528981381385147658534912000000}$$

$$\lambda_{14,\,13} := -\frac{31613003668360418135860625 93}{6498458246814128415824054059008000}$$

$$\lambda_{14,\,14} := -\frac{7295479649180961961017 83}{10129782776552762837984870400000}$$

$$\lambda_{14,\,15} := -\frac{3782429527238325032 89}{79692347018334672676454400000} \tag{107}$$

> *deqn14temp* := *map*(*simplify*, *collect*(*combine*(*deqn14*, *trig*), [sin, cos]));

$$deqn14temp := 0 \tag{108}$$

> *deqn15temp* := *map*(*simplify*, *collect*(*combine*(*deqn15*, *trig*), [sin, cos])) :

> *zero_terms* := {*coeff*(*deqn15temp*, sin(t)), *coeff*(*deqn15temp*, sin(3 t)), *coeff*(*deqn15temp*, sin(5 t)), *coeff*(*deqn15temp*, sin(7 t)), *coeff*(*deqn15temp*, sin(9 t)), *coeff*(*deqn15temp*, sin(11 t)), *coeff*(*deqn15temp*, sin(13 t)), *coeff*(*deqn15temp*, sin(15 t)), *coeff*(*deqn15temp*, sin(17 t)), *coeff*(*deqn15temp*, sin(19 t)), *coeff*(*deqn15temp*, sin(21 t)), *coeff*(*deqn15temp*, sin(23 t)), *coeff*(*deqn15temp*, sin(25 t)), *coeff*(*deqn15temp*, sin(27 t)), *coeff*(*deqn15temp*, sin(29 t)), *coeff*(*deqn15temp*, sin(31 t)), } :

> $solve\left(zero\_terms, \left\{ \lambda_{15,\,14}, \lambda_{15,\,15}, \lambda_{15,\,7}, \lambda_{15,\,6}, \lambda_{15,\,13}, \lambda_{15,\,4}, \lambda_{15,\,12}, \lambda_{15,\,11}, \lambda_{15,\,8}, \lambda_{15,\,9}, \lambda_{14,\,1}, \lambda_{15,\,2}, \lambda_{15,\,3}, \lambda_{15,\,5}, \lambda_{15,\,10}, \lambda_{15,\,1} \right\}\right)$:

> $\lambda_{14,\,1} := -\frac{29681065808320718421871}{448949013473135815557120000000}; \lambda_{15,\,1} := -\frac{18375904026802852139690491 1}{6285286188623901417799680000000000}; \lambda_{15,\,2} := \frac{3425302960574532296343183 13}{1583892119533223157285519360000000}; \lambda_{15,\,3} := \frac{122128531264366736034145931233}{35354734811009445475123200000000000000}; \lambda_{15,\,4} := \frac{15391971578599979302273417277 89}{224568037781041477740134400000000000000}; \lambda_{15,\,5} := -\frac{1484517119251730903407005522133825 1}{381990232265551553635968614400000000000000}; \lambda_{15,\,6} := -\frac{1033196162578724751406224779714873251 7}{1941274360373532995577992498380800000000000}; \lambda_{15,\,7} := -\frac{759180343285926231558495684474110429}{34166428742574180722172667971502080000000000}; \lambda_{15,\,8} := \frac{15522364347745086196297961763799451921 3}{724816381181752262463234456252579840000000000}; \lambda_{15,\,9} := \frac{3104870896321785342960819677866318890593}{74034816077850409665887519460084940800000000000}; \lambda_{15,\,10}$

$$:= \frac{20076340767335292929629690843097076887}{57158939094766270285184727798251520000000000}; \lambda_{15,\,11}$$
$$:= \frac{96606919656260678761162407126018756317}{520146345762373059595181022964088832000000000}; \lambda_{15,\,12}$$
$$:= \frac{3079126084842417876792809090174911}{4683568827643878631852712241408245760000}; \lambda_{15,\,13}$$
$$:= \frac{1110408111401712892870241629651}{7300737042717107232592455794688000000}; \lambda_{15,\,14}$$
$$:= \frac{4788695875262519698796723767}{2297434733722166611654968606720000000}; \lambda_{15,\,15}$$
$$:= \frac{29414496968053043925761}{229513959412803857308188672000000};$$

$$\lambda_{14,\,1} := -\frac{29681065808320718421871}{448949013473135815557120000000}$$
$$\lambda_{15,\,1} := -\frac{18375904026802852139690491 1}{62852861886239014177996800000000}$$
$$\lambda_{15,\,2} := \frac{342530296057453229634318313}{158389211953322315728551936000000}$$
$$\lambda_{15,\,3} := \frac{122128531264366736034145931233}{35354734811009445475123200000000000}$$
$$\lambda_{15,\,4} := \frac{1539197157859997930227341727789}{22456803778104147774013440000000000000}$$
$$\lambda_{15,\,5} := -\frac{14845171192517309034070055221338251}{38199023226555155363596861440000000000000}$$
$$\lambda_{15,\,6} := -\frac{10331961625787247514062247797148732517}{19412743603735329955779924983808000000000000}$$
$$\lambda_{15,\,7} := -\frac{759180343285926231558495684474110429}{3416642874257418072217266797150208000000000}$$
$$\lambda_{15,\,8} := \frac{155223643477450861962979617637994519213}{7248163811817522624632344562525798400000000000}$$
$$\lambda_{15,\,9} := \frac{3104870896321785342960819677866318890593}{74034816077850409665887519460084940800000000000}$$
$$\lambda_{15,\,10} := \frac{20076340767335292929629690843097076887}{57158939094766270285184727798251520000000000}$$
$$\lambda_{15,\,11} := \frac{96606919656260678761162407126018756317}{520146345762373059595181022964088832000000000}$$
$$\lambda_{15,\,12} := \frac{3079126084842417876792809090174911}{4683568827643878631852712241408245760000}$$
$$\lambda_{15,\,13} := \frac{1110408111401712892870241629651}{7300737042717107232592455794688000000}$$
$$\lambda_{15,\,14} := \frac{4788695875262519698796723767}{2297434733722166611654968606720000000}$$
$$\lambda_{15,\,15} := \frac{29414496968053043925761}{229513959412803857308188672000000} \qquad \textbf{(109)}$$

> $Deqn := diff(X(t), t, t) + X(t) + \text{epsilon}\cdot\left(X(t)^2 - 1\right)\cdot diff(X(t), t);$

$$Deqn := \frac{d^2}{dt^2} X(t) + X(t) + \varepsilon \left(X(t)^2 - 1\right) \left(\frac{d}{dt} X(t)\right) \quad \textbf{(110)}$$

> *foo* := *dsolve*([*subs*(epsilon = 5 / 4, *Deqn*), *X*(0) = 2.0, D(*X*)(0) = 0], *range* = 0. ..100, *numeric*);

$$foo := \textbf{proc}(x\_rkf45) \; ... \; \textbf{end proc} \quad \textbf{(111)}$$

> *xapprox* := *subs*(epsilon = $\frac{5}{4}$, *x*(*t*)) :

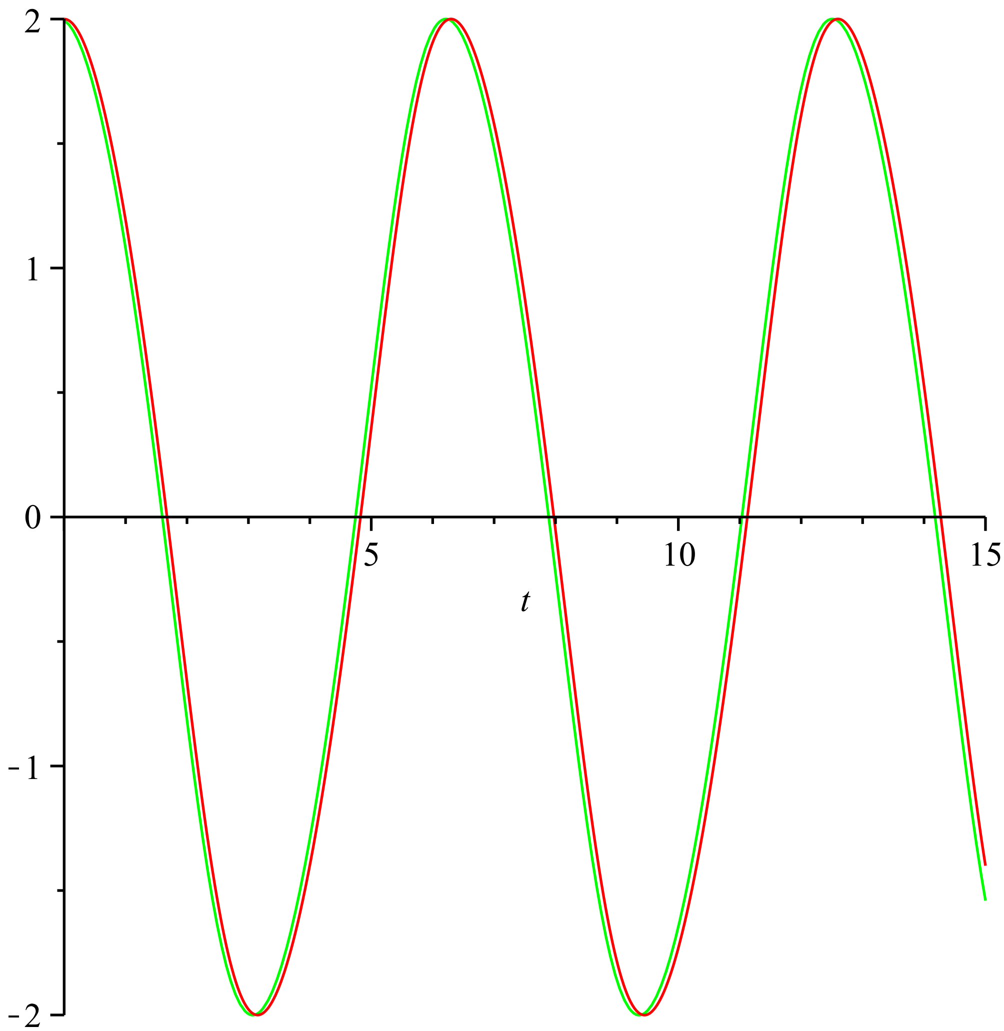
$$xapprox := x(t) \quad \textbf{(112)}$$

> *with*(*plots*) :

> *pnum* := *odeplot*(*foo*, *t* = 0 ..15, *refine* = 1, *color* = *red*) :

> *pexp* := *plot*(*xapprox*, *t* = 0 ..15, *color* = *green*) :

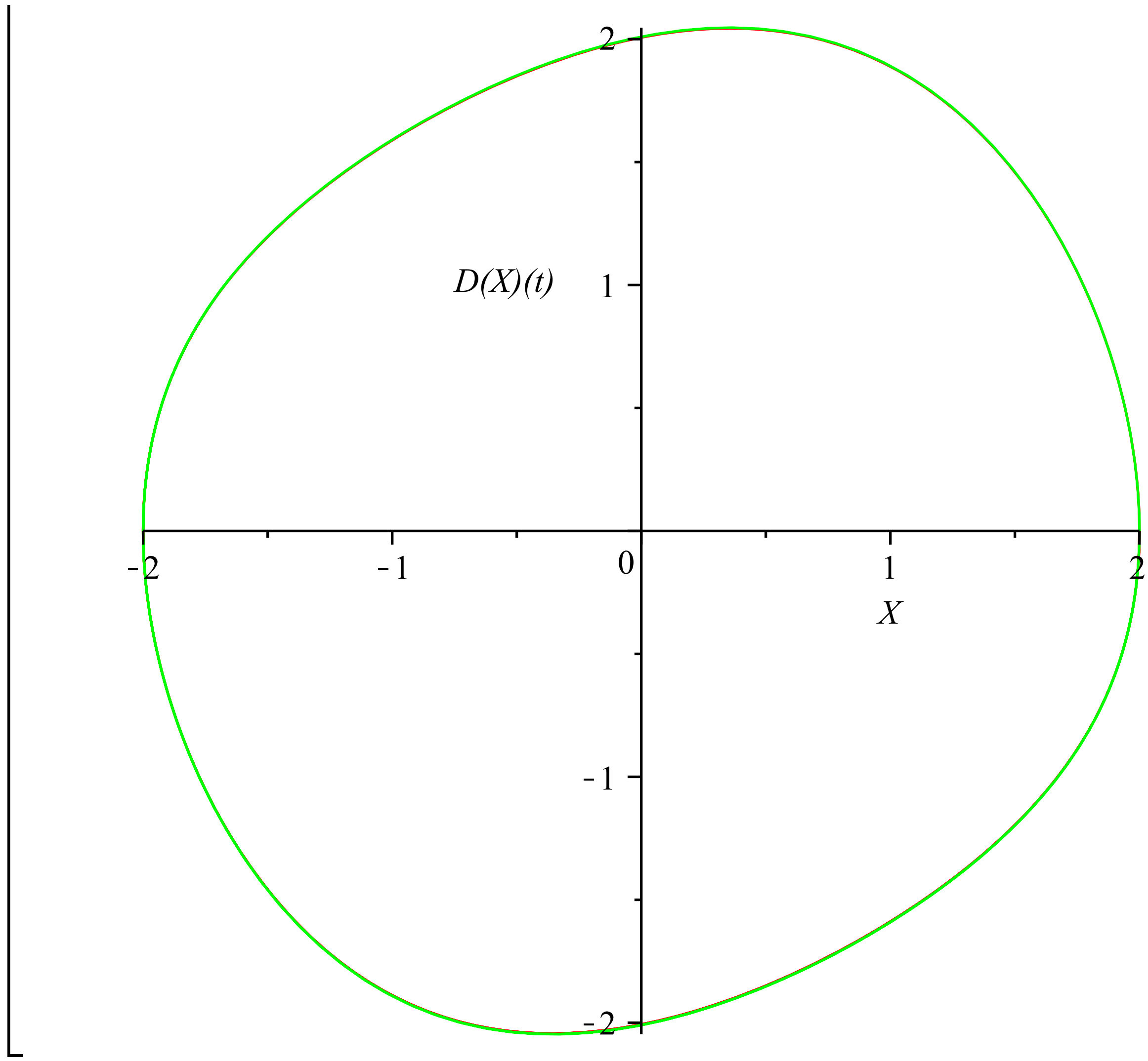
> *display*(*pexp*, *pnum*);



> *pnum2* := *odeplot*(*foo*, [*X*(*t*), D(*X*)(*t*)], *t* = 0 ..10, *refine* = 1, *color* = *red*) :

> *pexp2* := *plot*([*xapprox*, *diff*(*xapprox*, *t*), *t* = 0 ..10], *color* = *green*) :

> *display*(*pnum2*, *pexp2*);


D(X)(t)
2
1
-1
-2
-2
-1
0
1
2
X