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The Statistical Mechanics of Vesicles

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I discuss two-dimensional lattice models of closed fluctuating membranes, or vesicles [1]. The underlying mathematical model is that of self-avoiding polygons and their enumeration by perimeter and area. Physically, this model is of interest as its phase diagram contains a point at which tricritical scaling is observed [2].

By adding the constraint of partial directedness, one gets combinatorial models related to the partitioning of integers. These models are solvable in the sense that an explicit expression for the generating function can be given [3,4]. The derivation of this generating function involves a nonlinear functional equation, and the solution is given in terms of alternating q-series.

The asymptotic analysis of these generating functions can be done with the help of a new contour integral representation. This leads to an explicit calculation of the scaling behaviour around the critical point in terms of the Airy function [5]. In particular, one gets a uniform asymptotic expansion of the involved q-series as q approaches 1 from below.

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