Queen Mary
University of London

MTH4100 Calculus 1, Autumn 2009

Exercise sheet 5

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These questions are designed to help you understand the material covered in week n, $n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week $n+1$. You will get help on them in the exercise class on Wednesday of the same week. You should write up your solution to the starred question $(*)$ clearly and hand it in to your assigned helper during your week $n+2$ exercise class for feedback. Put your *full name and student number* on the top of your solution. It is important that you try to do all of the numbered questions. The extra question is for the more ambitious students.

(*)1. **Definition of derivative.** [(a), (b) 2008 exam questions]

(a) State the definition of the derivative of the function $f(x)$ with respect to the variable x.

(b) Differentiate from first principles $f(x) = \sqrt{x}$ by using the definition involving $h \to 0$.

(c) Does any tangent to the curve $y = \sqrt{x}$ cross the x-axis at $x = -1$? If so, find an equation for the line and the point of tangency. If not, why not?

2. **Tangent line via derivatives.** [2008 exam question]
Find equations of all lines having slope -2 that are tangent to the curve

$$y = \frac{18}{x+9} \, .$$

3. **Differentiation rules.** [(a),(b) 2008/09 exam questions]

(a) Find the first and second derivatives of $y = \dfrac{4x^5+8}{x^3}$, $x \neq 0$.

(b) Find the derivative $q'(t)$ of $q(t) = \tan \dfrac{t}{\sqrt{t+2}}$.

(c) Find the derivative $g'(t)$ of $g(t) = \cos(2 - \sin 3t)$.

Extra: Suppose that a function f satisfies the following conditions for all real values of x and y:

i. $f(x+y) = f(x)f(y)$.

ii. $f(x) = 1 + xg(x)$, where $\lim_{x \to 0} g(x) = 1$.

By differentiating from first principles, show that the derivative $f'(x)$ exists at every value of x and that $f'(x) = f(x)$.