Queen Mary
University of London

MTH4100 Calculus 1, Autumn 2009

Exercise sheet 4

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These questions are designed to help you understand the material covered in week n, $n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week $n+1$. You will get help on them in the exercise class on Wednesday of the same week. You should write up your solution to the starred question $(*)$ clearly and hand it in to your assigned helper during your week $n+2$ exercise class for feedback. Put your *full name and student number* on the top of your solution. It is important that you try to do all of the numbered questions. The extra question is for the more ambitious students.

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1. Find any horizontal, vertical, or oblique asymptotes of [2007 exam question]

$$f(x) = \frac{2x^2}{x-7} \quad .$$

2. (a) Define $g(5)$ in a way that extends [2008 exam questions]

$$g(x) = \frac{4x^2 - 100}{4x - 20}$$

to be continuous at $x = 5$.

(b) For what value of a is [2007 exam questions]

$$f(x) = \begin{cases} x^2 - 1\,, & x < 3 \\ 2ax\,, & x \geq 3 \end{cases}$$

continuous at every x? Justify your answer.

(*)(c) Can $f(x) = x(x^2-1)/|x^2-1|$ be extended to be continuous at $x = 1$ or $x = -1$? Give reasons for your answers.

4. (a) What are the assumptions and conclusions of the Intermediate Value Theorem? [2007 exam question]

(b) Give an example of a function that violates both the assumptions and the conclusions of the Intermediate Value Theorem.

Extra Using this theorem, explain why the equation $\cos x = x$ has at least one solution. *Hint:* Use $f(x) = \cos x - x$ and the theorem to prove that there is an x_0 such that $f(x_0) = 0$. [2007 exam question]