

B.Sc. EXAMINATION BY COURSE UNITS

MAS115 Calculus I (first sit paper)

Thursday 1 May 2008, 14:30 – 16:30

*The duration of this examination is 2 hours.*

*You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.*

Candidates must not remove the question paper from the examination room.

**YOU ARE NOT PERMITTED TO START READING THIS QUESTION  
PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR**

1. (a) [5 marks] Find the limits

$$\lim_{x \rightarrow -3^-} (x+4) \frac{|x+3|}{x+3} \quad \text{and} \quad \lim_{x \rightarrow -3^+} (x+4) \frac{|x+3|}{x+3} .$$

- (b) [5 marks] Find equations of all lines having slope  $-2$  that are tangent to the curve

$$y = \frac{18}{x+9} .$$

- (c) [5 marks] If

$$x^3 + y^3 = 56 ,$$

find the values of  $dy/dx$  and  $d^2y/dx^2$  at the point  $(-2, 4)$ .

- (d) [5 marks] Find the absolute maximum and minimum values of the function

$$f(x) = \frac{5}{6}x - 9$$

on the interval  $[-4, 7]$ .

- (e) [5 marks] Find the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos(6x)}{36x^2} .$$

- (f) [5 marks] Find

$$\frac{d}{dx} \int_{\sqrt[3]{x}}^{\pi/6} \cos(t^3) dt .$$

- (g) [5 marks] Find the area of the region enclosed by the curves

$$y^2 - x = 1 \quad \text{and} \quad x - y = 1 .$$

- (h) [5 marks] Find the derivative  $y'(t)$  of

$$y = 7^{\sin(6t)} .$$

- (i) [5 marks] Evaluate the integral

$$\int \frac{16 dx}{1 - \cos(8x)} .$$

- (j) [5 marks] Evaluate the integral

$$\int_{1/3}^7 8x \ln(3x) dx .$$

[Next question overleaf]

2. [20 marks] Consider the curve  $y = f(x)$  for the function

$$f(x) = \frac{x}{1+x^2}.$$

- (a) Identify the domain of  $f$  and any symmetries the curve may have.  
 (b) Find  $f'(x)$  and  $f''(x)$ .  
 (c) Find the critical points of  $f$ , and identify the function's behaviour at each one.  
 (d) Find where the curve is increasing and where it is decreasing.  
 (e) Find the points of inflection, if any occur, and determine the concavity of the curve.  
 (f) Identify any asymptotes.  
 (g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve.  
 (h) Is the area enclosed by the curve and the positive  $x$ -axis finite? If so, what is its value?
3. [10 marks] Consider the family of curves given by

$$f_a(x) = 2x^3 + ax^2 + 1, \quad a \in \mathbb{R}.$$

- (a) For fixed  $a$ , compute the critical point(s) of each curve.  
 (b) When varying  $a$ , the set of all critical points lie on a new curve. Compute the equation of that curve.
4. [10 marks]

- (a) What are the hypotheses and conclusions of the Mean Value Theorem For Definite Integrals?  
 (b) Using this Theorem, deduce the following statement:  
 If

$$\int_a^b f(x) dx = 0$$

for a function  $f$  which is continuous on the interval  $[a, b]$  with  $a < b$ , then  $f(x) = 0$  at least once in  $[a, b]$ .

5. [10 marks]

- (a) State the definition of the derivative of the function  $f(x)$  with respect to the variable  $x$ .  
 (b) Given that

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1,$$

differentiate from first principles  $f(x) = e^x$ .