

1. Riemann sums.

(\*) a. Evaluate

$$\lim_{n \rightarrow \infty} \frac{1^5 + 2^5 + 3^5 + \dots + n^5}{n^6}$$

by showing that the limit is

$$\int_0^1 x^5 dx$$

and evaluating the integral.

b. Evaluate

$$\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^4}.$$

2. L'Hôpital or not L'Hôpital?

Find the following limits:

a.  $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2}$

[2007 exam question]

b.  $\lim_{x \rightarrow 0} \frac{1 - \cos(6x)}{36x^2}$

[2008 exam question]

c.  $\lim_{x \rightarrow \infty} \frac{\sqrt{x+5}}{\sqrt{x}+5}$

3. Finite sums.

Which formula is not equivalent to the other two?

a.  $\sum_{j=2}^4 \frac{(-1)^{j-1}}{j-1}$

b.  $\sum_{k=0}^2 \frac{(-1)^k}{k+1}$

c.  $\sum_{l=-1}^1 \frac{(-1)^l}{l+2}$

Extra: Let  $f(x)$ ,  $g(x)$  be two continuously differentiable functions satisfying the relationships  $f'(x) = g(x)$  and  $f''(x) = -f(x)$ . Let  $h(x) = f^2(x) + g^2(x)$ . If  $h(0) = 5$ , find  $h(10)$ .