University of London

MTH4100
Exercise sheet 5

Calculus 1, Autumn 2008
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- Make sure you attend the excercise class that you have been assigned to!
- The instructor will present the starred problem in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after the exercise class took place.


## ${ }^{(*)} 1$. Definition of derivative.

[(a), (b) 2008 exam questions]
(a) State the definition of the derivative of the function $f(x)$ with respect to the variable $x$.
(b) Differentiate from first principles $f(x)=\sqrt{x}$ by using the definition involving $h \rightarrow 0$.
(c) Does any tangent to the curve $y=\sqrt{x}$ cross the $x$-axis at $x=-1$ ? If so, find an equation for the line and the point of tangency. If not, why not?
2. Tangent line via derivatives.
[2008 exam question]
Find equations of all lines having slope -2 that are tangent to the curve

$$
y=\frac{18}{x+9} .
$$

3. Implicit differentiation.
[2008 exam question]
If

$$
x^{3}+y^{3}=56
$$

find the values of $d y / d x$ and $d^{2} y / d x^{2}$ at the point $(-2,4)$.

## 4. Differentiation rules

[2008 exam question]
Find the derivative $q^{\prime}(t)$ of

$$
q=\tan \frac{t}{\sqrt{t+2}}
$$

Extra: Suppose that a function $f$ satisfies the following conditions for all real values of $x$ and $y$ :
i. $f(x+y)=f(x) f(y)$.
ii. $f(x)=1+x g(x)$, where $\lim _{x \rightarrow 0} g(x)=1$.

Show that the derivative $f^{\prime}(x)$ exists at every value of $x$ and that $f^{\prime}(x)=f(x)$.

