MTH4100 — Calculus 1, Autumn 2008

Exercise sheet 3

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- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problem in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after the exercise class took place.

(*)1. **Two wrong statements about limits.** Show by example that the following statements are wrong.

(a) The number L is the limit of $f(x)$ as x approaches x_0 if $f(x)$ gets closer to L as x approaches x_0.

(b) The number L is the limit of $f(x)$ as x approaches x_0 if, given any $\epsilon > 0$, there exists a value of x for which $|f(x) - L| < \epsilon$.

Explain why the functions in your examples do not have the given value of L as a limit as $x \to x_0$.

2. Compute the following limits: [2007 and 2008 exam questions]

$$\text{(a)} \quad \lim_{x \to -3^-} (x+4)\frac{|x+3|}{x+3}\,, \quad \text{(b)} \quad \lim_{u \to 3} \frac{u^3 - 27}{u^4 - 81}\,, \quad \text{(c)} \quad \lim_{x \to 0} \frac{6x + 6x\cos(6x)}{\sin(6x)\cos(6x)}\,.$$

3. Use the graph of the greatest integer function $y = \lfloor x \rfloor$ to determine the limits

$$\text{(a)} \quad \lim_{\theta \to 3^+} \frac{\lfloor \theta \rfloor}{\theta}\,, \quad \lim_{\theta \to 3^-} \frac{\lfloor \theta \rfloor}{\theta}\,, \qquad \text{(b)} \quad \lim_{t \to 4^+} (t - \lfloor t \rfloor)\,, \qquad \lim_{t \to 4^-} (t - \lfloor t \rfloor)\,.$$

Extra: **Roots of a quadratic equation that is almost linear.** The equation $ax^2 + 2x - 1 = 0$, where a is a constant, has two roots if $a > -1$ and $a \neq 0$, one positive and one negative:

$$r_+(a) = \frac{-1 + \sqrt{1+a}}{a}\,, \quad r_-(a) = \frac{-1 - \sqrt{1+a}}{a}\,.$$

(a) What happens to $r_+(a)$ as $a \to 0$? As $a \to -1^+$?

(b) What happens to $r_-(a)$ as $a \to 0$? As $a \to -1^+$?

(c) Support your conclusions by graphing $r_+(a)$ and $r_-(a)$ as functions of a. Describe what you see.