

Integration tables

TABLE 8.1 Basic integration formulas

1. $\int du = u + C$
2. $\int k du = ku + C$ (any number k)
3. $\int (du + dv) = \int du + \int dv$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C$ ($n \neq -1$)
5. $\int \frac{du}{u} = \ln |u| + C$
6. $\int \sin u du = -\cos u + C$
7. $\int \cos u du = \sin u + C$
8. $\int \sec^2 u du = \tan u + C$
9. $\int \csc^2 u du = -\cot u + C$
10. $\int \sec u \tan u du = \sec u + C$
11. $\int \csc u \cot u du = -\csc u + C$
12. $\int \tan u du = -\ln |\cos u| + C$
 $= \ln |\sec u| + C$
13. $\int \cot u du = \ln |\sin u| + C$
 $= -\ln |\csc u| + C$
14. $\int e^u du = e^u + C$
15. $\int a^u du = \frac{a^u}{\ln a} + C$ ($a > 0, a \neq 1$)
16. $\int \sinh u du = \cosh u + C$
17. $\int \cosh u du = \sinh u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \left(\frac{u}{a} \right) + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$
21. $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1} \left(\frac{u}{a} \right) + C$ ($a > 0$)
22. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1} \left(\frac{u}{a} \right) + C$ ($u > a > 0$)

Integration tricks

Procedures for Matching Integrals to Basic Formulas

PROCEDURE

Making a simplifying substitution

Completing the square

Using a trigonometric identity

Eliminating a square root

Reducing an improper fraction

Separating a fraction

Multiplying by a form of 1

EXAMPLE

$$\frac{2x - 9}{\sqrt{x^2 - 9x + 1}} dx = \frac{du}{\sqrt{u}}$$

$$\sqrt{8x - x^2} = \sqrt{16 - (x - 4)^2}$$

$$\begin{aligned} (\sec x + \tan x)^2 &= \sec^2 x + 2 \sec x \tan x + \tan^2 x \\ &= \sec^2 x + 2 \sec x \tan x \\ &\quad + (\sec^2 x - 1) \\ &= 2 \sec^2 x + 2 \sec x \tan x - 1 \end{aligned}$$

$$\sqrt{1 + \cos 4x} = \sqrt{2 \cos^2 2x} = \sqrt{2} |\cos 2x|$$

$$\frac{3x^2 - 7x}{3x + 2} = x - 3 + \frac{6}{3x + 2}$$

$$\frac{3x + 2}{\sqrt{1 - x^2}} = \frac{3x}{\sqrt{1 - x^2}} + \frac{2}{\sqrt{1 - x^2}}$$

$$\begin{aligned} \sec x &= \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x} \\ &= \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \end{aligned}$$