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B.Sc. EXAMINATION BY COURSE UNITS

MAS115 Calculus I (late summer paper)

Monday 11 August 2008, 14:30 – 16:30

The duration of this examination is 2 hours.

You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Candidates must not remove the question paper from the examination room.

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

1. (a) [5 marks] Find the limit
$$\lim_{t \to 16} \frac{t^2 + 3t - 304}{t^2 - 256} \ .$$

(b) [5 marks] Find the limit
$$\lim_{t \to 0} \frac{\sin(4 - 4\cos(2t))}{1 - \cos(2t)} \ .$$

(c) [5 marks] Define $g(5)$ in a way that extends
$$g(x) = \frac{4x^2 - 100}{4x - 20}$$

to be continuous at $x = 5$.

(d) [5 marks] Find an equation for the line tangent to
$$y = 5 - 3x^2$$

at $(2, -7)$.

(e) [5 marks] Find the derivative $q'(t)$ of
$$q = \tan \frac{t}{\sqrt{t + 2}} \ .$$

(f) [5 marks] Find the area of the region enclosed by the functions
$$y = x^6 \qquad \text{and} \qquad y = 8x^3 \ .$$

(g) [5 marks] Find the derivative $y'(x)$ of
$$y = \ln(8 \ln x) \ .$$

(h) [5 marks] Using a substitution, evaluate the integral
$$\int_0^{\pi/2} \frac{2\sin(2t)}{5 - \cos(2t)} \, dt \ .$$

(i) [5 marks] Evaluate the integral
$$\int_{-2}^0 \frac{11 \, dt}{\sqrt{12 - 4t - t^2}}$$

using the method of completing the square and an appropriate substitution.

(j) [5 marks] Evaluate the integral
$$\int_0^\pi 6\sqrt{\frac{1 - \cos x}{2}} \, dx$$

after simplifying the integrand with the help of a half-angle formula.

[Next question overleaf]

2. [20 marks] Consider the curve $y = f(x)$ for the function

$$f(x) = e^{2x} - 2e^x .$$

    (a) Identify the domain of f and any symmetries the curve may have.

    (b) Find $f'(x)$ and $f''(x)$.

    (c) Find the critical points of f, and identify the function's behaviour at each one.

    (d) Find where the curve is increasing and where it is decreasing.

    (e) Find the points of inflection, if any occur, and determine the concavity of the curve.

    (f) Identify any asymptotes.

    (g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve.

    (h) Is the area enclosed by the curve and the x-axis finite? If so, what is its value?

3. [10 marks] For $a > 0$ fixed, consider the two curves given by

$$x^2 + y^2 = ax \qquad \text{and} \qquad x^2 + y^2 = y .$$

    (a) Compute the point(s) of intersection.

    (b) For which values of a do the curves intersect orthogonally?

4. [10 marks]

    (a) What are the hypotheses and conclusions of the Intermediate Value Theorem?

    (b) Using this Theorem, explain why the equation

$$e^x + x = 0$$

has at least one solution.

5. [10 marks]

    (a) State the definition of the derivative of the function $f(x)$ with respect to the variable x.

    (b) Differentiate from first principles $f(x) = x^{1/2}$.

[End of examination paper]