Irregular diffusion in the bouncing ball billiard

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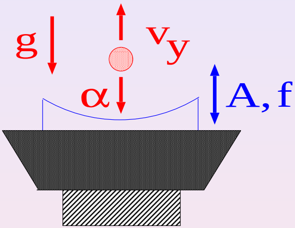
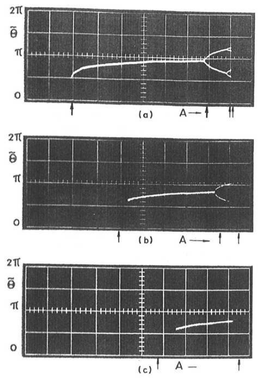
Outline

1. **Motivation:** the bouncing ball billiard
2. Frequency locking, diffusion and correlated random walks
3. Spiral modes and diffusion
4. **Outlook:** experiments on vibro-transporters

The bouncing ball: experiments



Pieranski (1983ff)
Tufillaro (1986ff)
“Jugend forscht” (2004?)



Pieranski, J.Phys. (1985)

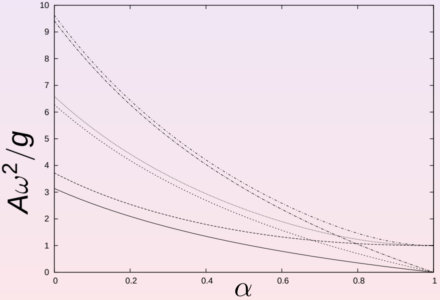
Luck, Mehta (1993): “chattering”

bifurcations into chaotic motion?

Linz (2003?)

The bouncing ball: 'theory'

linear stability analysis of the exact (implicit) equations of motion yields frequency locking regions ('tongues'):



Hongler et al. (1989)
Luck, Mehta (1993)

high bounce approximation: for displacement amplitude $A \ll y_{max}$ ball's max. height eom's become

$$\theta_{k+1} = \theta_k + v_k$$
$$v_{k+1} = \alpha v_k + \gamma \cos \theta_{k+1}$$

dissipative standard map

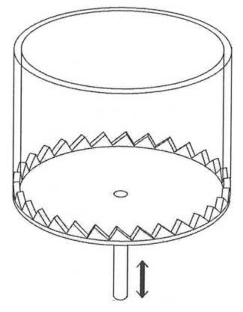
with θ_k: phase of the table; v_k: ball velocity at the kth collision and $\gamma = 2\omega^2(1+\alpha)A/g$

Tufillaro (1986ff)

cp. with driven pendulum and Fermi acceleration

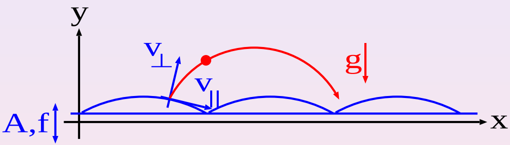
The bouncing ball billiard

study gas of granular particles on vibrating surface coated with periodic scatterers:



Farkas et al. (1999)
Urbach et al. (2002)

motivated our one dimensional **bouncing ball billiard**:



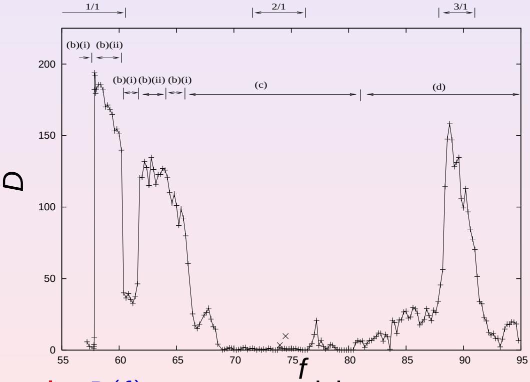
at collision: two friction coefficients α perpendicular and β tangential to the surface

Q: $\exists$ frequency locking in diffusion?

Frequency locking and diffusion

parameters: scatterer radius $R = 25$mm, amplitude $A = 0.1mm$, restitution $\alpha = 0.5$, $\beta = 0.99$

diffusion coefficient $D(f)$ from MD computer simulations:

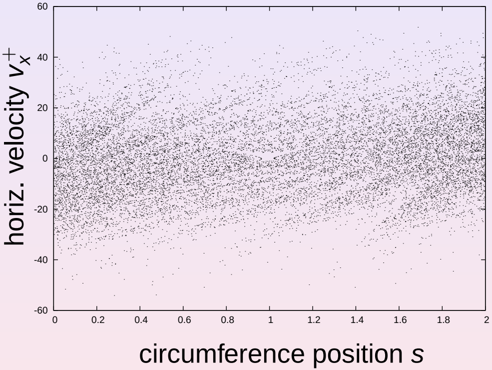


- highly irregular $D(f)$, no monotonicity
- frequency locking $\leftrightarrow$ largest maxima of $D(f)$

Numerical analysis of the dynamics: resonance

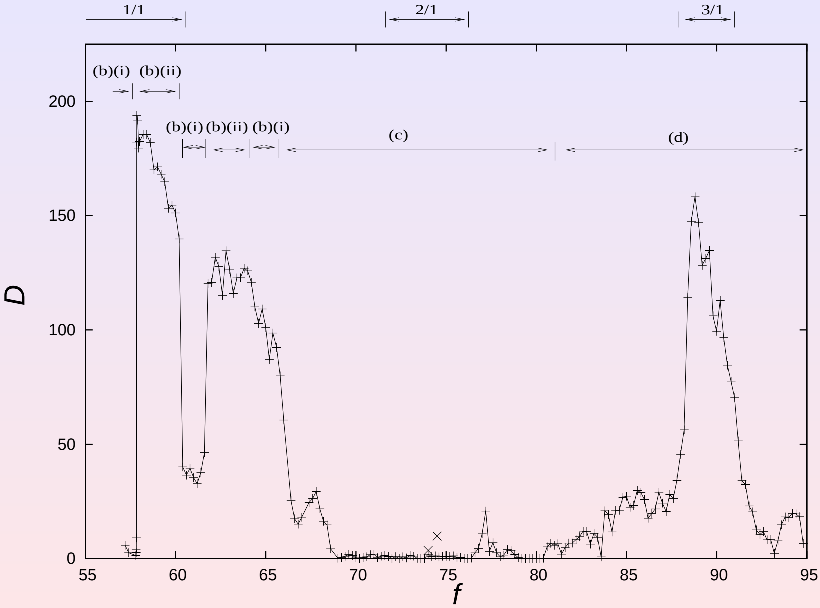
$\exists$ two types of attractors; projections at collisions:

attractor 1:



- $\exists$ 1/1-resonance *vertically*, irregular motion *horizontally*
- traces of harmonic oscillator *separatrix*
- fan-shaped structure by *chaotic* scatterers

$\Rightarrow$ defines **regime (b)(ii)**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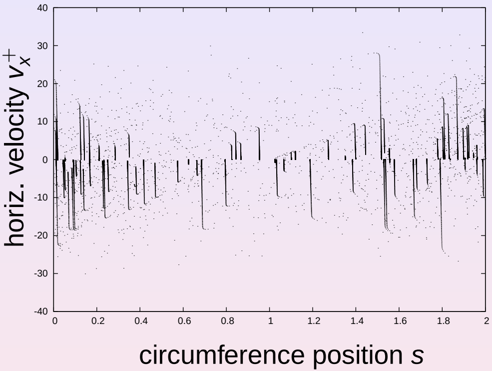
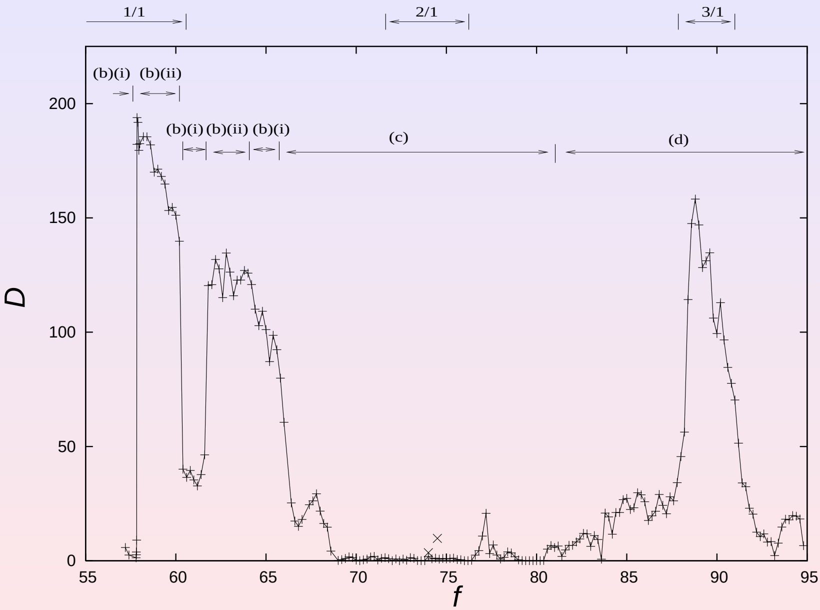
Numerical analysis of the dynamics: creeps

attractor 2:



- non-resonant irregular motion in x and y
- long creeps: sequences of correlated tiny jumps along the surface: **regime (c)**

both types of dynamics can be linked to each other ergodically **(d)** or exist on different attractors non-ergodically **(b)(i)**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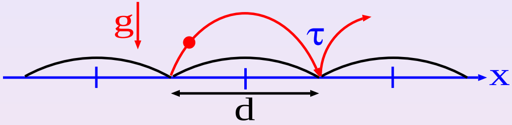
Simple random walk approximation

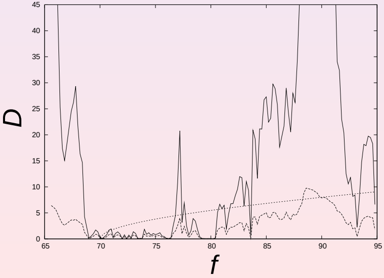
diffusion as a random walk on the line:



$$D_{\mathrm{rw}}(f) = \frac{d^2}{2\tau(f)}$$

distance d between wedges and escape time τ out of wedge

$D_{\mathrm{rw}}(f)$ for τ numerically:



$\tau \simeq d / < v_x > \simeq d/\sqrt{2E_x}$ links $D_{\mathrm{rw}}(f)$ to kinetic energy $E_x(f)$

dotted line: energy balance

$E = E_x + E_y + E_{pot}$ with

$E_{pot} \simeq g\overline{y} \simeq gA$, $E \simeq A^2\omega^2/2$ and

$E_y \simeq 19E_x$ leads to

$$D_{\mathrm{stoch}}(f) \simeq \frac{d}{2}\sqrt{2E_x} \simeq \frac{d}{2}\sqrt{\frac{A^2\omega^2}{20} - \frac{gA}{10}}$$

Correlated random walk approximation

diffusion via Taylor-Green-Kubo formula:

$$\boxed{D(f) = \frac{d^2}{2\tau} + \frac{1}{\tau}\sum_{k=1}^{\infty} < h(x_0) \cdot h(x_k) >}$$

with lattice vectors $h(x_k) = \pm d$ and equilibrium ensemble average $< \ldots >$ (R.K., Korabel, 2002)

truncate series and express it by conditional probabilities

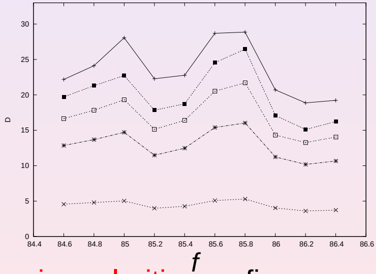
$$D_n(f) = d^2/2\tau + \frac{1}{\tau} \sum_{s_1 \ldots s_n} p(s_1 s_2 \ldots) h \cdot h(s_1 s_2 \ldots)$$

examples: 1st order approximation by forward- and backward scattering: $D_1 = D_0 + 2D_0(p_f - p_b) = D_0 + 2D_0(1 - 2p_b)$

2nd order approximation: $D_2 = D_1 + 2D_0(p_{ff} - p_{fb} + p_{bf} - p_{bb})$

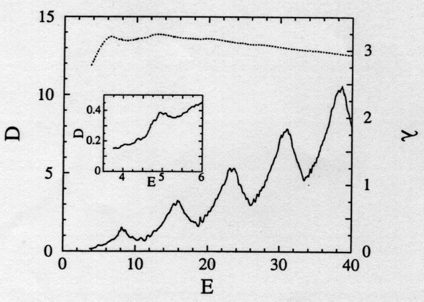
Understanding correlations in deterministic diffusion

compute probabilities numerically and check convergence of higher-order terms to $D(f)$:



$\Rightarrow$ irregularities on fine scales are *real* and due to dynamical correlations

Hamiltonian billiard without vibrations and friction:



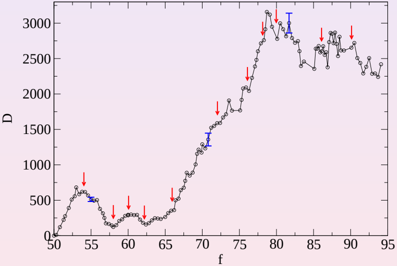
Harayama, Gaspard (2001)

fractal diffusion coefficient in energy E, cp. to yesterday's talk

Irregular diffusion for other parameters

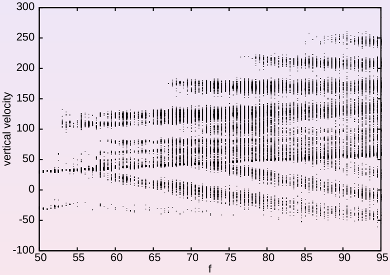
2nd set of parameters closer to experiments: $R = 15\text{mm}$, $A = 0.1\text{mm}$, $\alpha = 0.7$, $\beta = 0.99$

$D(f)$ from simulations:



• highly irregular diffusion coefficient, but very different from previous one

projections of velocities v_y^+ around $y = 0$:

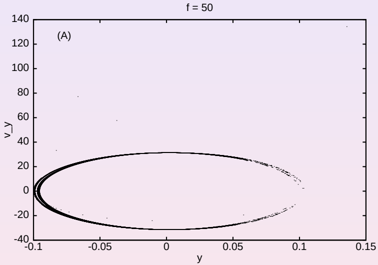


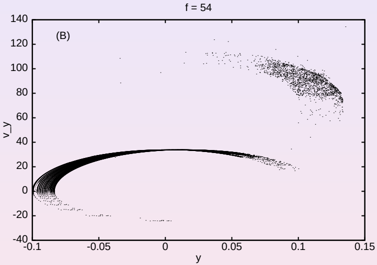
• local extrema ↔ frequency locking?

• cp. ‘bifurcations’ ↔ local extrema!

Spiral modes and diffusion 1

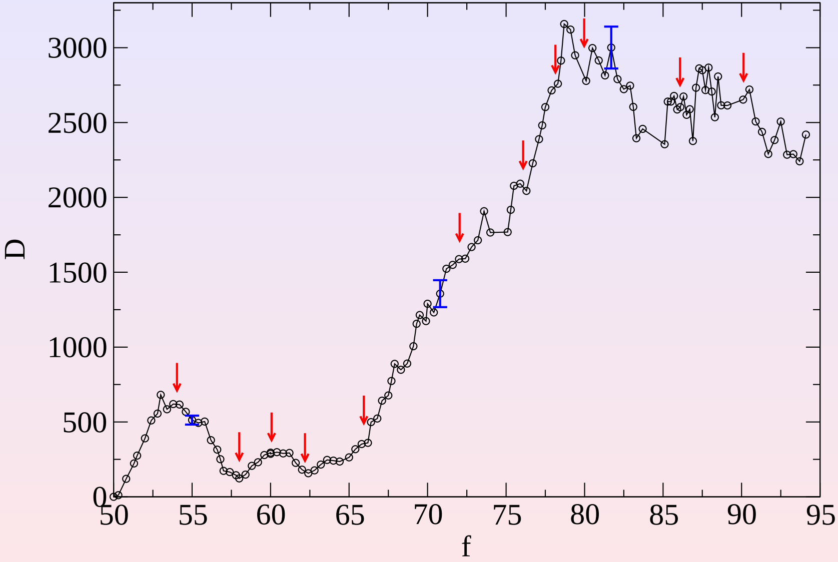
projections of orbits onto the (y, v_y^+)-plane:





(A) **onset of diffusion:** particles oscillate harmonically with the surface

(B) **onset of $1/1$-resonance**: enhancement of diffusion; coexistence with creeping orbits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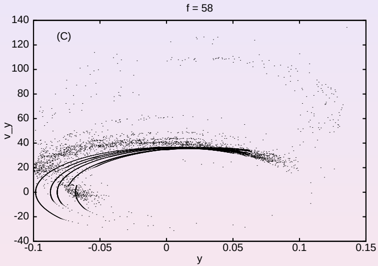
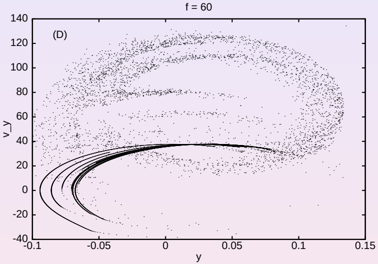
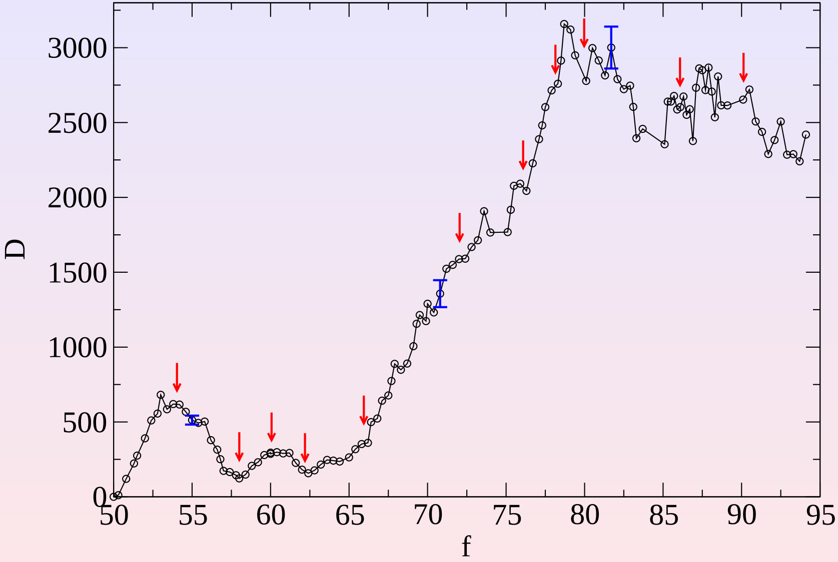
Spiral modes and diffusion 2



(C) **destruction of** $1/1$**-resonance:** existence of a local minimum in the diffusion coefficient



(D) **new type of resonance:** a virtual harmonic oscillator mode (VHO) is forming; explains the second peak in $D(f)$; unstable around $f \simeq 62$



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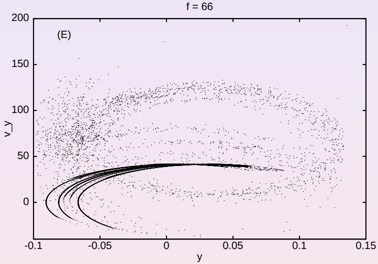
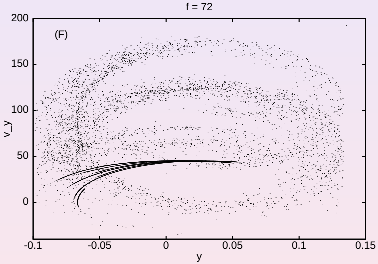
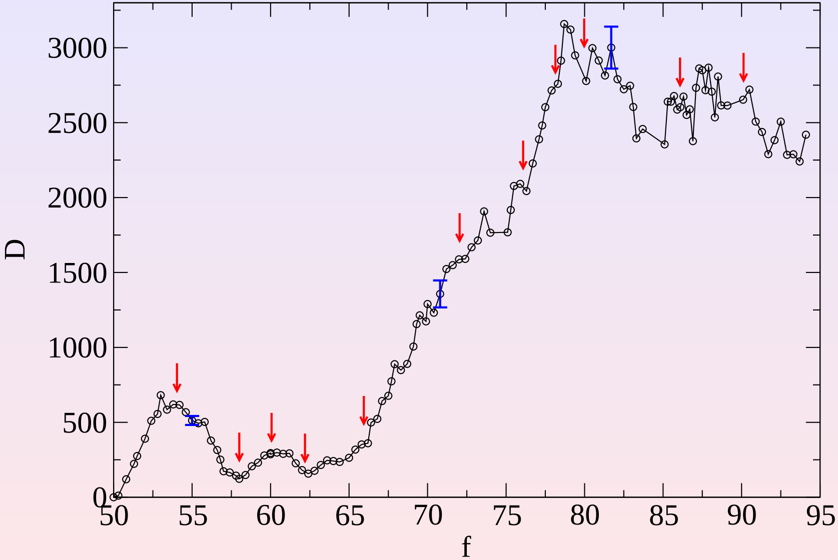
Spiral modes and diffusion 3



(E) the VHO spirals out: further enhancement of diffusion



(F) two-loop spiral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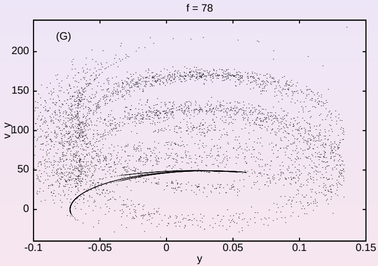
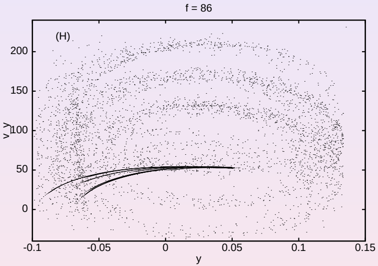
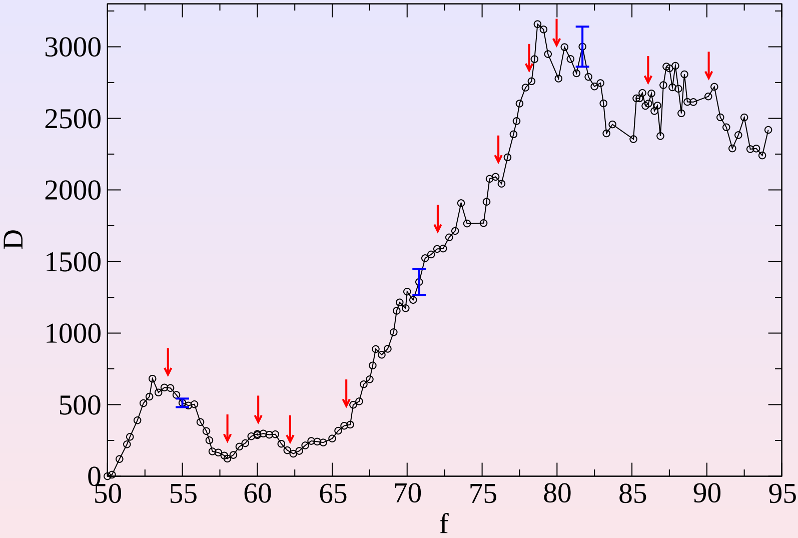
Spiral modes and diffusion 4



(G) onset of a third loop around $f \simeq 76$: explains third local maximum



(H) onset of a fourth loop: related to fourth local maximum



note: diffusion coefficient is also irregular with respect to other control parameters α, β, R

Spiral modes quantitatively

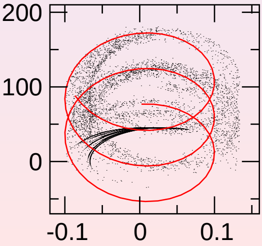
frequency locking condition: $k := T_p/T_f = 2v_y^+ f/g$ with T_p particle time of flight and T_f period of vibration

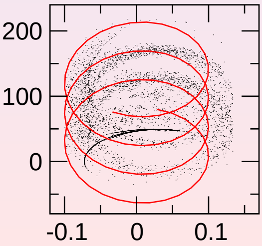
numerical finding: $D(f)$ has local maxima with complete VHO loops at half-integer k

spiral equation: assume flat surface and no correlations between collisions; from eom's (Luck, Mehta, 1993):

$y \ = -A\sin(2\pi f t_1)\,,\ v_y = \alpha g/2(t_1 - t_0) - A2\pi f(1+\alpha)\cos(2\pi f t_1)$

with particle launched at time t_0 and first collision at t_1, cp. with simulations for $f = 72, 78$:

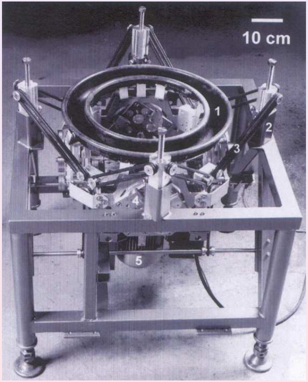




Outlook: Vibro-transporters

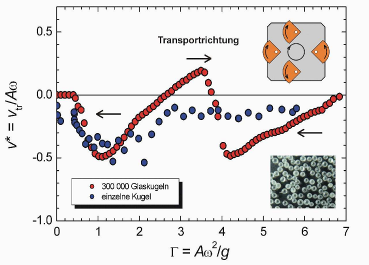
for agricultural material, also corrugated (Persson et al. (1992))

current generated by symmetry breaking;:



Grochowski et al. (2003)

current reversals under asymmetric vibrations:



reproduced in simulations with frequency locking (Elhor, Linz, 2003); cf. also Hongler et al. (1989); Han, Lee (2001)

Summary

- bouncing ball billiard models diffusion of a granular particle on a vibrating corrugated floor
- computer simulations show a highly irregular frequency-dependent diffusion coefficient; main impact by frequency locking and spiral modes
- highly correlated nonlinear dynamics yields further irregularities on fine scales, understood by correlated random walk approximations
- fractal transport coefficients in experiments?

References:

L. Matyas, R. Klages, Physica D **187**, 165 (2004)
R.Klages, I.F.Barna, L.Matyas, Physics Letters A **333**, 79 (2004)