## The Physics of Foraging: Bumblebee Flights under Predation Risk

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## Outline

(1) The physics of foraging: Can biologically relevant search strategies be identified by mathematical modeling?


- the albatross story and the Lévy flight hypothesis
- further biological data, their analysis and interpretation
(2) Bumblebees foraging under predation risk:
- the experiment
- the analysis
- the modeling



## Part 1:

## The Physics of Foraging

## Lévy flight search patterns of wandering albatrosses

famous paper by Viswanathan et al., Nature 381, 413 (1996):
for albatrosses foraging in the South Atlantic the flight times were recorded

the distribution of flight times was fitted with a Lévy flight model (power law)


## Lévy flights in a nutshell

Lévy flights have well-defined mathematical properties:

- a Markovian stochastic process
- with probability distribution function of flight lengths exhibiting power law tails, $\rho(\ell) \simeq \ell^{-1-\alpha}, 0<\alpha<2$;
- it has infinite variance, $\left\langle\ell^{2}\right\rangle=\infty$,
- satisfies a generalized central limit theorem (Gnedenko, Kolmogorov, 1949) and
- is scale invariant
for an outline see, e.g., Shlesinger at al., Nature 363, 31 (1993)
(remark: $\exists$ the more physical model of Lévy walks)


## Optimizing the success of random searches

another paper by Viswanathan et al., Nature 401, 911 (1999):

- question posed about "best statistical strategy to adapt in order to search efficiently for randomly located objects"
- random walk model leads to Lévy flight hypothesis:

Lèvy flights provide an optimal search strategy for sparsely, randomly distributed, revisitable targets


Brownian motion (left) vs. Lévy flights (right)

- Lévy flights also obtained for bumblebee and deer data


## Revisiting Lévy flight search patterns

Edwards et al., Nature 449, 1044 (2007):

- Viswanathan et al. results revisited by correcting old data (Buchanan, Nature 453, 714, 2008):

- Lévy flight behavior clearly ruled out: On the basis of new, more precise data some other (gamma distributed) stochastic process revealed
- refined data analysis yields no evidence for Lévy flights in bumblebee and deer data either


## Lévy or not Lévy?

## Lévy paradigm: Look for power law tails in pdf's!

- Sims et al., Nature 451, 1098 (2008): scaling laws of marine predator search behaviour; > $10^{6}$ data points!

- prey distributions also display Lévy-like patterns...


## Lévy flights induced by the environment?

- Humphries et al., Nature 465, 1066 (2010): environmental context explains Lévy and Brownian movement patterns of marine predators; $>10^{7}$ data points!


blue: exponential; red: truncated power law
- note: $\exists$ day-night cycle, cf. oscillations; suggests to fit with two different pdf's (not done)


## Optimal searches: adaptive or emergent?

## strictly speaking two different Lévy flight hypotheses:

(1) Lévy flights represent an (evolutionary) adaptive optimal search strategy
Viswanathan et al. (1999) the 'conventional' Lévy flight hypothesis

(2) Lévy flights emerge from the interaction with a scale-free food source distribution
Viswanathan et al. (1996) more recent reasoning


## Biological cell migration: further trouble

suggested by Reynolds (Physica A, 2009) that biological cells also perform Lévy dynamics
single biological cell crawling on a substrate; trajectory recorded with a video camera (Dieterich et al., PNAS, 2008)


## Position distribution function for cell migration

- two types: wildtype and deficient one
- $P(x, t) \rightarrow$ Gaussian $(t \rightarrow \infty)$ and kurtosis $\kappa(t):=\frac{\left\langle x^{4}(t)\right\rangle}{\left\langle x^{2}(t)\right\rangle^{2}} \rightarrow 3(t \rightarrow \infty)$ for Brownian motion (green lines, in 1d)
- other solid lines: fits from our model
- also extracted: mean square displacement, velocity autocorrelation fct.



$\Rightarrow$ crossover from peaked to broad non-Gaussian distributions


## An alternative to Lévy flight search strategies

Bénichou et al., Rev. Mod. Phys. 83, 81 (2011):

- for non-revisitable targets intermittent search strategies minimize the search time

- popular account of this work in Shlesinger, Nature 443, 281 (2006): "How to hunt a submarine?"
- cf. also protein binding on DNA


## In search of a mathematical foraging theory

## Summary of Part 1:

- two different Lévy flight hypothesis: adaptive and emergent
- scale-free Lévy flight paradigm
- problems with the data analysis
- different dynamics on different time scales and intermittent search strategies



## Part 2:

## Bumblebee Flights under Predation Risk

## Motivation

find food (nectar, pollen) in complex landscapes


## try to avoid predators

## The bumblebee experiment

Ings, Chittka, Current Biology 18, 1520 (2008): bumblebee foraging in a cube of $\simeq 75 \mathrm{~cm}$ side length

- artificial yellow flowers: $4 \times 4$ grid on one wall
- two cameras track the position (50fps) of a single bumblebee (Bombus terrestris)

- advantages: systematic variation of the environment; easier than tracking bumblebees on large scales
- disadvantage: no typical free flight of bumblebees; no test of the Lévy hypothesis (but questioning of the Lévy paradigm!)


## Variation of the environmental conditions


safe and dangerous flowers

- two types of artificial spiders: white (easily visible) and yellow (cryptic)
- three experimental stages:
(1) spider-free foraging
(2) foraging under predation risk
(3) memory test 1 day later
\#bumblebees=30, \#data per bumblebee for each stage $\approx 7000$


## Bumblebee experiment: two main questions

(1) What type of motion do the bumblebees perform in terms of stochastic dynamics?

(2) Are there changes of the dynamics under variation of the environmental conditions?

## Velocity distributions: analysis



left: experimental data yielding pdf of $v_{y}$-velocities of a single bumblebee in the spider-free stage (black crosses) with max. likelihood fits of mixture of 2 Gaussians; exponential; power law; single Gaussian
right: quantile-quantile plot of a Gaussian mixture against the experimental data (black) plus surrogate data

## Velocity distributions: interpretation

- best fit to the data by a mixture of two Gaussians with different variances (verified by information criteria with resp. weights)
- biological explanation: models spatially different flight modes near the flower vs. far away, cf. intermittent dynamics
- no contradiction to Lévy hypothesis; but Lévy paradigm 'suggests': all relevant information captured by pdf's
$\Rightarrow \begin{aligned} & \text { big surprise: no difference in pdf's between different } \\ & \text { stages under variation of environmental conditions! }\end{aligned}$


## Velocity autocorrelation function $\perp$ to the wall

$$
V_{x}^{A C}(\tau)=\frac{\left\langle\left(v_{x}(t)-\mu\right)\left(v_{x}(t+\tau)-\mu\right)\right\rangle}{\sigma^{2}} \text { with average over all bees }
$$



- plot: spider-free stage, predation thread, memory test
- $\exists$ anti-correlations for $\tau \simeq 0.5$ : bees return to flowers
- only small quantitative changes under predation thread, cf. shift of minimum in $V_{x}^{A C}(\tau)$ and changes in pdf of flight times (inset): more flights with long durations


## Velocity autocorrelation function || to the wall



- plot: spider-free stage, predation thread, memory test
- $\exists$ profound qualitative change of correlations from positive for spider-free to negative in case of spiders
- resampling of data (inset) confirms existence of positive correlations
$\Rightarrow$ all changes are in the velocity correlations, not the pdf's!


## Mathematical modeling of bumblebee flights

- trivial modeling of data via overdamped Langevin eq.:

$$
\mathbf{v}=\chi_{\mathbf{f} \mathbf{z}}(\mathbf{r}) \xi_{1}(t)+\left(1-\chi_{\mathbf{f z}}(\mathbf{r})\right) \xi_{2}(t)
$$

with chararacteristic function of feeding zone $\chi_{\mathrm{fz}}$ and Gaussian noise $\xi_{i}, i=1,2$ correlated according to velocity correlation of data: non-Brownian motion

- advanced modeling via Langevin eq. in comoving frame:

$$
\begin{aligned}
& \dot{\beta}=-\gamma \beta+\xi_{v}(t) \\
& \dot{v}=g(v)+\xi(t)
\end{aligned}
$$


with nonlinear drift term $g(v)$, noise $\xi_{v}(t), \xi(t)$ and noise correlations to be determined from data (work in progress): generalized correlated random walk model

## Clever bumblebees!

## Summary of Part 2:

- mixture of two Gaussian velocity distributions reflects spatial adjustment of bumblebee dynamics to flower carpet
- all changes to predation thread are contained in the velocity autocorrelation functions that exhibit highly non-trivial temporal behaviour
- no problem with the Lévy hypothesis but with the Lévy paradigm, which suggests that all relevant foraging information is contained in pdf's
- bumblebees exhibit strongly non-Brownian motion: modeling by generalized Langevin dynamics


## Conclusion

suggestion: replace the fundamental question
What is the mathematically most efficient search strategy?
by
How can we statistically quantify changes in foraging dynamics due to interactions with the environment?
(Is nature necessarily ‘simple’?)
reference:
F.Lenz, T.Ings, A.V.Chechkin, L.Chittka, R.Klages,

Spatio-temporal dynamics of bumblebees foraging under predation risk, arXiv:1108.1278 (2011)
nb: pdf-file of the talk available on homepage RK

