Fluctuation relations: a pedagogical overview

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The fluctuation relations have received considerable attention since their emergence and development in the 1990s. As an aid to newcomers to the field, we present a summary of the main results and suggest ways to interpret this material. Starting with a consideration of the under-determined time evolution of a simple open system, formulated using continuous Markovian stochastic dynamics, an expression for the entropy generated with time is proposed in terms of the probability of observing a trajectory associated with a prescribed driving protocol, and the probability of its time-reverse. This forms the basis for a more complete theoretical description of non-equilibrium thermodynamic processes. Having established a connection between entropy production and an inequivalence in probability for forward and time-reversed events, we proceed in the manner of Sekimoto and Seifert, in particular, to derive results in stochastic thermodynamics: a description of the evolution of a system between equilibrium states that ties in with well-established thermodynamic expectations. We derive fluctuation relations, state conditions for their validity, and illustrate their operation in some simple cases, thereby providing some introductory insight into the various celebrated symmetry relations that have emerged in this field.

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