MTH4100 — Calculus 1, Autumn 2009

Exercise sheet 9 — Rainer Klages

(*)1. **Fundamental Theorem of Calculus.** [2007 exam question]

Suppose that f has a negative derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true for the function

$$h(x) = \int_0^x f(t)dt \ ?$$

Give reasons for your answers.

(a) h is a twice-differentiable function of x.

(b) h and dh/dx are both continuous.

(c) The graph of h has a horizontal tangent at $x = 1$.

(d) h has a local maximum at $x = 1$.

(e) h has a local minimum at $x = 1$.

(f) The graph of h has an inflection point at $x = 1$.

(g) The graph of dh/dx crosses the x-axis at $x = 1$.

2. **The substitution rule.**

Sometimes it helps to reduce an integral step by step, using a trial substitution to simplify the integral a bit and then another one to simplify it some more. Practice this on

$$\int \sqrt{1 + \sin^2(x-1)}\,\sin(x-1)\cos(x-1)dx\ .$$

(a) $u = x - 1$, followed by $v = \sin u$, then by $w = 1 + v^2$

(b) $u = \sin(x-1)$, followed by $v = 1 + u^2$

(c) $u = 1 + \sin^2(x-1)$

3. **Integration, differentiation, and the chain rule** [2008 exam question]

Find

$$\frac{d}{dx}\int_{\sqrt[3]{x}}^{\pi/6} \cos(t^3)\,dt\ .$$

(*)4. **Area between curves** [2007 exam question]

Find the area enclosed by the two curves $y = x^2 - 2$ and $y = 2$.

Extra: Prove that

$$\int_0^x \left(\int_0^u f(t)dt\right) du = \int_0^x f(u)(x-u)du\ .$$

Hint: Express the integral on the right hand side as the difference of two integrals. Then show that both sides of the equation have the same derivative with respect to x.