Queen Mary
University of London

MTH4100 — Calculus 1, Autumn 2009

Exercise sheet 3 — Rainer Klages

These questions are designed to help you understand the material covered in week $n, n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week $n+1$. You will get help on them in the exercise class on Wednesday of the same week. You should write up your solution to the starred question $(*)$ clearly and hand it in to your assigned helper during your week $n+2$ exercise class for feedback. Put your *full name and student number* on the top of your solution. It is important that you try to do all of the numbered questions. The extra question is for the more ambitious students.

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1. Show by example that the following statement is wrong: *The number L is the limit of $f(x)$ as x approaches x_0 if $f(x)$ gets closer to L as x approaches x_0.* Explain why the function in your example does not have the given value of L as a limit as $x \to x_0$.

2. Compute the following limits: [2007 and 2008 exam questions]

$$\text{(a)} \quad \lim_{x\to -3^-} (x+4)\frac{|x+3|}{x+3}\,, \quad \text{(b)} \quad \lim_{u\to 3} \frac{u^3-27}{u^4-81}\,, \quad \text{(c)} \quad \lim_{x\to 0} \frac{6x+6x\cos(6x)}{\sin(6x)\cos(6x)}\,.$$

(*)3. Compute the following limits: [2008 and 2009 exam questions]

$$\text{(a)} \quad \lim_{t\to 0} \frac{\sin(4-4\cos(2t))}{1-\cos(2t)}\,, \quad \text{(b)} \quad \lim_{t\to 5} \frac{t^2+3t-40}{t^2-25}\,, \quad \text{(c)} \quad \lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2}\,.$$

4. Use the graph of the greatest integer function $y = \lfloor x \rfloor$ to determine the limits

$$\text{(a)} \quad \lim_{\theta\to 3^+} \frac{\lfloor\theta\rfloor}{\theta}\,, \quad \lim_{\theta\to 3^-} \frac{\lfloor\theta\rfloor}{\theta}\,, \qquad \text{(b)} \quad \lim_{t\to 4^+} (t-\lfloor t\rfloor)\,, \qquad \lim_{t\to 4^-} (t-\lfloor t\rfloor)\,.$$

Extra: **Roots of a quadratic equation that is almost linear.** The equation $ax^2+2x-1=0$, where a is a constant, has two roots if $a > -1$ and $a \neq 0$, one positive and one negative:

$$r_+(a) = \frac{-1+\sqrt{1+a}}{a}\,, \quad r_-(a) = \frac{-1-\sqrt{1+a}}{a}\,.$$

(a) What happens to $r_+(a)$ as $a \to 0$ and ss $a \to -1^+$?

(b) What happens to $r_-(a)$ as $a \to 0$ and as $a \to -1^+$?

(c) Support your conclusions by graphing $r_+(a)$ and $r_-(a)$ as functions of a. Describe what you see.