Queen Mary
University of London

MTH4100 Calculus 1, Autumn 2009

Exercise sheet 2

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These questions are designed to help you understand the material covered in week n, $n \in \mathbb{N}$ lectures. Exercise sheets will typically be handed out in the Tuesday lecture of week $n+1$. You will get help on them in the exercise class on Wednesday of the same week. You should write up your solution to the starred question $(*)$ clearly and hand it in to your assigned helper during your week $n+2$ exercise class for feedback. Put your *full name and student number* on the top of your solution. It is important that you try to do all of the numbered questions. The extra question is for the more ambitious students.

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1. Recall the identity

$$\cos^2\theta + \sin^2\theta = 1$$

and the *addition formulas*

$$\cos(A+B) = \cos A \cos B - \sin A \sin B\,,\ \sin(A+B) = \sin A \cos B + \cos A \sin B\,,$$

which are valid for all angles θ, A, B.

(a) Derive the two *double-angle formulas*

$$\cos 2\theta = \cos^2\theta - \sin^2\theta\,,\ \sin 2\theta = 2\sin\theta\cos\theta$$

by using (some of) the above three formulas.

(b) Derive the two *half-angle formulas*

$$\cos^2\theta = (1+\cos 2\theta)/2\,,\ \sin^2\theta = (1-\cos 2\theta)/2$$

by using (some of) the first three formulas and one of the formulas derived in 1.(a).

(c) Use the above formulas to evaluate in terms of radicals $\sin\frac{7\pi}{12}$.

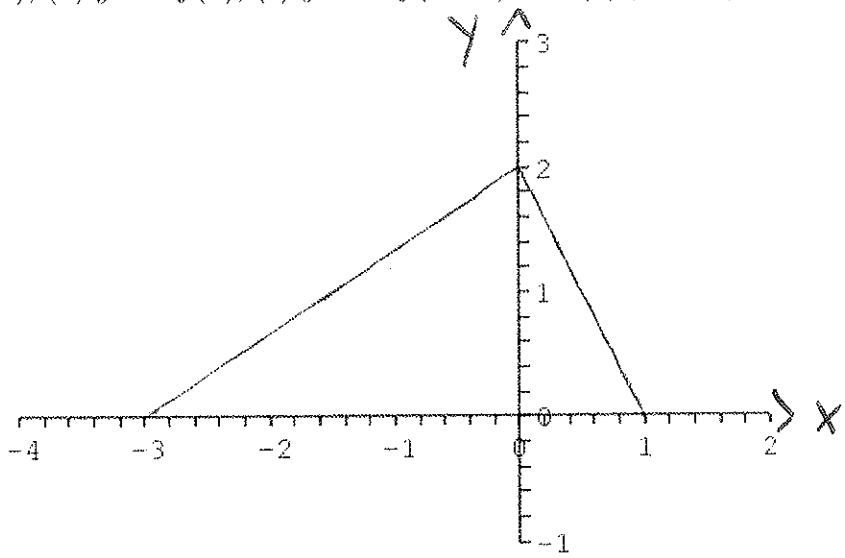
(d) Evaluate in terms of radicals $\cos\frac{\pi}{12}$. [2007 exam question]

2. Find a formula for $f \circ g$ and $g \circ f$ and find the domain and range of each:

$$(a) \qquad f(x) = 2 - x^2\,, \qquad g(x) = \sqrt{x+2}$$
$$(b) \qquad f(x) = \sqrt{x}\,, \qquad g(x) = \sqrt{1-x}$$

[please turn over]

(*)3. The graph of f is shown below. Draw the graph of each of the following functions: (a) $y = f(-x)$, (b) $y = -f(x)$, (c) $y = -2f(x+1)+1$, (d) $y = 3f(x-2)-2$.



4. Define what is meant by even and odd functions. Then determine whether the function

$$f(x) = 3x^5 - 4x^2$$

is even, odd, or neither. [2009 exam question]

Extra: Graph the equations (a) $|x| + |y| = 1 + x$ and (b) $y + |y| = x + |x|$.