Queen Mary University of London

MTH4100 — Calculus 1, Autumn 2008

Exercise sheet 6 — **Rainer Klages**

- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problem in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after the exercise class took place.

(*)1. **Critical points.** [2008 exam question]

Consider the family of curves given by

$$f_a(x) = 2x^3 + ax^2 + 1 \,, \qquad a, x \in \mathbb{R} \,.$$

(a) For fixed a, compute the critical point(s) of each curve.

(b) When varying a, the set of all a-dependent critical points lie on a new curve. Compute the equation of that curve.

2. **Linearisation of trigonometric functions.**

Find the linearisation of $f(x) = \cos x$ at $x = \pi/2$.

3. **Absolute extrema.** [2008 exam question]

Find the absolute maximum and minimum values of the function $f(x) = \frac{5}{6}x - 9$ on the interval $[-4, 7]$.

4. **The mean value theorem.** [2007 exam question]

Explain why the function $f(x) = 2x^2 + 5x - 3$ defined on $[a, b]$ with $a = -2, b = 1$ satisifies all assumptions of the mean value theorem. Then find all values of $c \in (a, b)$ that satisfy the equation in the conclusion of this theorem,

$$\frac{f(b) - f(a)}{b - a} = f'(c) \,.$$

Extra: We know how to find the extreme values of a continuous function $f(x)$ by investigating its values at critical points and endpoints. But what if there *are* no critical points or endpoints? What happens then? Do such functions really exist? Give reasons for your answers.