Queen Mary
University of London

MTH4100 Calculus 1, Autumn 2008

Exercise sheet 4

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- **Make sure you attend the excercise class that you have been assigned to!**
- The instructor will present the starred problem in class.
- You should then work on the other problems on your own.
- The instructor and helper will be available for questions.
- Solutions will be available online after the exercise class took place.

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1. **Continuity.**

(*) (a) Can $f(x) = x(x^2-1)/|x^2-1|$ be extended to be continuous at $x = 1$ or $x = -1$? Give reasons for your answers.

(b) For what value of a is [2007 exam questions]

$$f(x) = \begin{cases} x^2 - 1\,, & x < 3 \\ 2ax\,, & x \geq 3 \end{cases}$$

continuous at every x? Justify your answer.

2. **Asymptotes.** [2007 exam question]
Find any horizontal, vertical, or oblique asymptotes of

$$f(x) = \frac{2x^2}{x-7} \quad .$$

3. **The Intermediate Value Theorem.** [2007 exam question]

(a) What are the assumptions and conclusions of the intermediate value theorem?

(b) Using this theorem, explain why the equation $\cos x = x$ has at least one solution. *Hint:* Use $f(x) = \cos x - x$ and the theorem to prove that there is an x_0 such that $f(x_0) = 0$.

Extra **A function continuous at only one point.** Let

$$f(x) = \begin{cases} x\,, & \text{if } x \text{ is rational} \\ 0\,, & \text{if } x \text{ is irrational.} \end{cases}$$

Show that f is continuous at $x = 0$.