

B.Sc. EXAMINATION BY COURSE UNITS

MAS115 Calculus I (late summer paper)

13th August 2007, 14:30 – 16:30

The duration of this examination is 2 hours.

You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Candidates must not remove the question paper from the examination room.

**YOU ARE NOT PERMITTED TO START READING THIS QUESTION
PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR**

1. (a) [5 marks] Find the limit

$$\lim_{u \rightarrow 3} \frac{u^3 - 27}{u^4 - 81} .$$

- (b) [5 marks] Find any horizontal, vertical, or oblique asymptotes of

$$f(x) = \frac{2x^2}{x - 7} .$$

- (c) [5 marks] Find

$$\frac{d^{999}}{dx^{999}}(\sin x) .$$

- (d) [5 marks] Find all values of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the function $f(x) = 2x^2 + 5x - 3$ and the interval $[-2, 1]$.

- (e) [5 marks] Evaluate

$$\lim_{x \rightarrow \infty} (2x - \sqrt{4x^2 + 3x}) .$$

- (f) [5 marks] Find

$$\frac{d}{dt} \int_0^{t^2} \sqrt{u} \, du .$$

- (g) [5 marks] Find the total area of the region between the x -axis and the graph

$$y = 3x^3 - 9x^2 + 6x, \quad 0 \leq x \leq 2 .$$

- (h) [5 marks] Evaluate the integral

$$\int x^3 \sqrt{x^2 - 4} \, dx .$$

- (i) [5 marks] Evaluate the integral

$$\int x \sin\left(\frac{x}{8}\right) \, dx .$$

- (j) [5 marks] Evaluate the integral

$$\int_0^6 \frac{x^3 \, dx}{x^2 + 12x + 36} .$$

[Next question overleaf]

2. [12 marks] Suppose that f has a negative derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true of the function

$$h(x) = \int_0^x f(t) dt ?$$

Give reasons for your answers.

- (a) h is a twice-differentiable function of x .
 - (b) h and dh/dx are both continuous.
 - (c) The graph of h has a horizontal tangent at $x = 1$.
 - (d) h has a local minimum at $x = 1$.
 - (e) The graph of h has an inflection point at $x = 1$.
 - (f) The graph of dh/dx crosses the x -axis at $x = 1$.
3. [10 marks]
- (a) State the definition of the derivative of the function $f(x)$ with respect to the variable x .
 - (b) Compute from first principles $f'(0)$ for

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x \neq 0, \\ 1 & \text{for } x = 0. \end{cases}$$

4. [20 marks] Consider the curve $y = f(x)$ for the function

$$f(x) = (x^2 - 3)^2 .$$

- (a) Identify the domain of f and any symmetries the curve may have.
 - (b) Find $f'(x)$ and $f''(x)$.
 - (c) Find the critical points of f , and identify the function's behaviour at each one.
 - (d) Find where the curve is increasing and where it is decreasing.
 - (e) Find the points of inflection, if any occur, and determine the concavity of the curve.
 - (f) Identify any asymptotes.
 - (g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve.
 - (h) Compute the area enclosed by the curve and the x -axis.
5. [8 marks] For which values of the constant p does the integral

$$\int_1^2 \frac{(\ln x)^p}{x} dx$$

converge? What is its value?