

B.Sc. EXAMINATION BY COURSE UNITS

MAS115 Calculus I (first sit paper)

Friday 04 May 2007, 14:30 – 16:30

The duration of this examination is 2 hours.

You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Candidates must not remove the question paper from the examination room.

**YOU ARE NOT PERMITTED TO START READING THIS QUESTION
PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR**

1. Marks are only awarded for the final answer, so indicate this answer clearly.

(a) [5 marks] Evaluate in terms of radicals

$$\cos \frac{\pi}{12} .$$

(b) [5 marks] Find the limit

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2} .$$

(c) [5 marks] For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x ?

(d) [5 marks] Find the limit

$$\lim_{\theta \rightarrow 0} \tan \left(1 - \frac{\sin \theta}{\theta} \right) .$$

(e) [5 marks] Find the derivative $f'(t)$ of

$$f(t) = 4 \sin \sqrt{1 + \sqrt{t}} .$$

(f) [5 marks] Find the area enclosed by the two curves

$$y = x^2 - 2 \quad \text{and} \quad y = 2 .$$

(g) [5 marks] Evaluate the integral

$$\int \frac{dx}{\sqrt{x}(\sqrt{x} + 1)} .$$

(h) [5 marks] Evaluate the integral

$$\int_0^{\pi} \sqrt{1 - \cos 2x} \, dx .$$

(i) [5 marks] Evaluate the integral

$$\int_1^e x^3 \ln x \, dx .$$

(j) [5 marks] Evaluate the integral

$$\int \frac{dx}{x^2 + 2x} .$$

[Next question overleaf]

2. [20 marks] Consider the curve $y = f(x)$ for the function

$$f(x) = \frac{12}{3 + x^2}.$$

- Identify the domain of f and any symmetries the curve may have.
 - Find $f'(x)$ and $f''(x)$.
 - Find the critical points of f , and identify the function's behaviour at each one.
 - Find where the curve is increasing and where it is decreasing.
 - Find the points of inflection, if any occur, and determine the concavity of the curve.
 - Identify any asymptotes.
 - Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve.
 - Is the area enclosed by the curve and the x -axis finite? If so, what is its value?
3. [10 marks] Consider the curve given by

$$x^2 + 7xy + y^2 = 9.$$

- Find y' by implicit differentiation.
 - Are there any straight lines through the origin that are normal to this curve? If so, find the points of orthogonal intersection and the equation for each line.
4. [10 marks]
- What are the hypotheses and conclusions of the Mean Value Theorem?
 - Using the Mean Value Theorem, deduce the following statement:
If $f'(x) = 0$ at each point x of an open interval (a, b) , then $f(x) = C$ for all $x \in (a, b)$, where C is a constant.

5. [10 marks]

- State the definition of the derivative of the function $f(x)$ with respect to the variable x .
- Given

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0 \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1,$$

differentiate from first principles $f(x) = \cos x$.