Anomalous Diffusion, Anomalous Time Series, and the models that describe them.

I will first talk briefly about why we need to use stochastic (or partly stochastic) models in physics and the geosciences, particularly in time series analysis. The motivation is not only classic problems but also the new importance of topics like extremes and large deviations. I will review how our intuition tends to have been formed by the simplest textbook stochastic time series model: white, Gaussian, iid, stationary noise, for which, following Fisher and others, a highly developed theory of statistical inference exists. Such noise has a short tailed amplitude distribution, it is delta-correlated in time, and has constant finite moments, so its range does not grow.

I will then describe how, in stark contrast, Mandelbrot's classic work in the 1960s and early 1970s focused particularly on 3 effects seen in time series drawn from the natural and economic sciences, each of which represented a strong departure from one of the above properties. These were the   ”Noah effect" (heavy tails in amplitude); the ``Joseph effect" (long range serial dependence in time) ; and “volatility clustering" (correlations between the absolute value of the time series). Mandelbrot's work has been highly influential, with one notable subsequent model being Bak et al’s self-organised criticality, introduced as a mechanism to explain, and unify, the Noah and Joseph effects through self-similar “avalanches”.

I will then discuss three resulting issues which have been of personal interest to me and my co-workers [1,2,3]. We were initially motivated by our interest in extreme fluctuations in space physics and other areas, and the need to compare paradigms like SOC to experimental data. One issue arises from the fact that a measured fluctuating quantity need not always be stationary and noise-like. Instead natural fluctuations may have been integrated or multiplied by the system's physics to create the observed variable(s). Aggregated fluctuations already have rather different properties to noise, some of which can be traps for the unwary. For example, the first passage time of even an “ordinary” Brownian random walk is already a heavy tailed random variable with infinite expectation value. A second problem arises from the fact that some popular diagnostics are constructed to measure self-similarity, while others in fact measure long-range dependence, so some confusion can arise when interpreting their outputs [2].

Finally I will discuss the models which modify the Brownian random walk, including those for “anomalous diffusion”. I will build on, and where necessary correct, the classification we presented in [1]. Three particularly important classes of such models are: additive and undamped, including fractional stable models, the fractional CTRW, and generalised shot noises; additive and mean reverting (damped) models like the Ornstein-Uhlenbeck process; and multiplicative processes.   I will try and dispel the misconception that such models are ``just statistics", as many embody a close correspondence with a physical scenario, which can be used as a guide when trying to choose the most suitable one to use.

[1] **Watkins, N. W.,** Credgington, D., Sanchez, R., Rosenberg, S., and Chapman, S. C. Kinetic equation of linear fractional stable motion and applications to modeling the scaling of intermittent bursts. *PRE* (2009).

[2] C. L. E. Franzke, T. Graves, **N. W. Watkins**, R. B. Gramacy and C. Hughes. Robustness of estimators of long-range dependence and self-similarity under non-Gaussianity. *Phil. Trans. Roy. Soc. A (2012).*

[3] **N. W. Watkins**, B. Hnat and S. C. Chapman. On selfsimilar and multifractal models for the scaling of extreme bursty fluctuations in space plasmas. *AGU Monograph on Complexity and Extreme Events in Geoscience (To appear, 2012)*.