Queen Mary
University of London

MTH4100 — Calculus 1, Autumn 2009

Exercise sheet 6 — Rainer Klages

---

1. **Parametric equations**
   Identify the particle's path for the given parametric equation by finding a Cartesian equation for it. Graph this equation, indicate the portion of the graph traced by the particle and the direction of motion.

$$(a)\ x = \cos 2t\ ,\ y = \sin 2t\ ,\ 0 \leq t \leq \pi \quad (b)\ x = -\sqrt{t}\ ,\ y = t\ ,\ t \geq 0$$

2. **Implicit differentiation.** [2008 exam question]
   If
$$x^3 + y^3 = 56\ ,$$
   find the values of dy/dx and d^2y/dx^2 at the point $(-2, 4)$.

3. **Linearisation of trigonometric functions.**
   Find the linearisation of $f(x) = \cos x$ at $x = \pi/2$.

(*)4. **Critical points.** [2008 exam question]
   Consider the family of curves given by
$$f_a(x) = 2x^3 + ax^2 + 1\ , \qquad a, x \in \mathbb{R}\ .$$
   (a) For fixed a, compute the critical point(s) of each curve.
   (b) When varying a, the set of all a-dependent critical points lie on a new curve. Compute the equation of that curve.

5. **The Extreme Value Theorem.**
   Give an example of a function that violates both the assumption of continuity in the Extreme Value Theorem and its conclusions.

Extra: We know how to find the extreme values of a continuous function $f(x)$ by investigating its values at critical points and endpoints. But what if there *are* no critical points or endpoints? What happens then? Do such functions really exist? Give reasons for your answers.