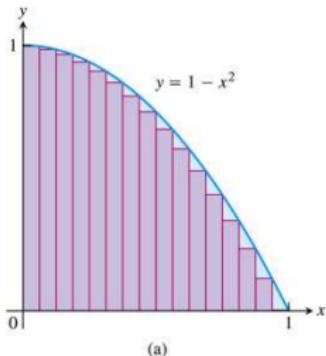


**TABLE 4.2** Antiderivative formulas

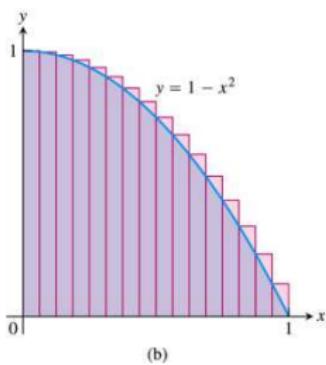
<b>Function</b>	<b>General antiderivative</b>
1. $x^n$	$\frac{x^{n+1}}{n+1} + C, \quad n \neq -1, n \text{ rational}$
2. $\sin kx$	$-\frac{\cos kx}{k} + C, \quad k \text{ a constant, } k \neq 0$
3. $\cos kx$	$\frac{\sin kx}{k} + C, \quad k \text{ a constant, } k \neq 0$
4. $\sec^2 x$	$\tan x + C$
5. $\csc^2 x$	$-\cot x + C$
6. $\sec x \tan x$	$\sec x + C$
7. $\csc x \cot x$	$-\csc x + C$

**TABLE 4.3** Antiderivative linearity rules

	<b>Function</b>	<b>General antiderivative</b>
1. <i>Constant Multiple Rule:</i>	$kf(x)$	$kF(x) + C, \quad k \text{ a constant}$
2. <i>Negative Rule:</i>	$-f(x)$	$-F(x) + C,$
3. <i>Sum or Difference Rule:</i>	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$



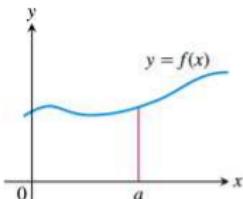
(a)



(b)

**TABLE 5.1** Finite approximations for the area of  $R$ 

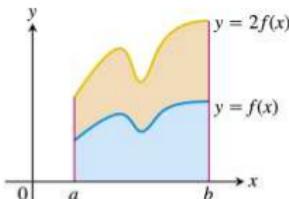
Number of subintervals	Lower sum	Midpoint rule	Upper sum
2	.375	.6875	.875
4	.53125	.671875	.78125
16	.634765625	.6669921875	.697265625
50	.6566	.6667	.6766
100	.66165	.666675	.67165
1000	.6661665	.66666675	.6671665



(a) Zero Width Interval:

$$\int_a^a f(x) dx = 0.$$

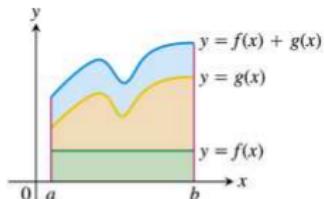
(The area over a point is 0.)



(b) Constant Multiple:

$$\int_a^b kf(x) dx = k \int_a^b f(x) dx.$$

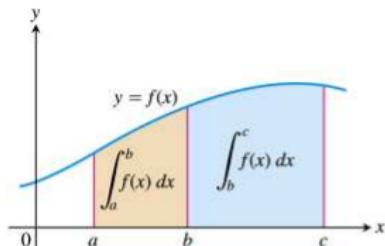
(Shown for  $k = 2$ .)



(c) Sum:

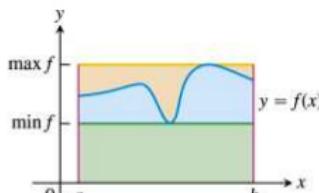
$$\int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

(Areas add)



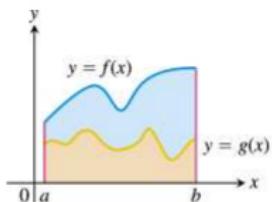
(d) Additivity for definite integrals:

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$



(e) Max-Min Inequality:

$$\begin{aligned} \min f \cdot (b - a) &\leq \int_a^b f(x) dx \\ &\leq \max f \cdot (b - a) \end{aligned}$$



(f) Domination:

$$\begin{aligned} f(x) &\geq g(x) \text{ on } [a, b] \\ \Rightarrow \int_a^b f(x) dx &\geq \int_a^b g(x) dx \end{aligned}$$