Queen Mary
University of London

MTH4100 — Calculus 1, Autumn 2008

Exercise sheet 9

Rainer Klages

(*)1. **Fundamental theorem of calculus.** [2007 exam question]

Suppose that f has a negative derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true for the function

$$h(x) = \int_0^x f(t)dt \ ?$$

(a) h is a twice-differentiable function of x.

(b) h and dh/dx are both continuous.

(c) The graph of h has a horizontal tangent at $x = 1$.

(d) h has a local maximum at $x = 1$.

(e) h has a local minimum at $x = 1$.

(f) The graph of h has an inflection point at $x = 1$.

(g) The graph of dh/dx crosses the x-axis at $x = 1$.

2. **The substitution rule.**

Sometimes it helps to reduce an integral step by step, using a trial substitution to simplify the integral a bit and then another one to simplify it some more. Practice this on

$$\int \sqrt{1 + \sin^2(x-1)}\,\sin(x-1)\cos(x-1)dx \ .$$

(a) $u = x - 1$, followed by $v = \sin u$, then by $w = 1 + v^2$

(b) $u = \sin(x - 1)$, followed by $v = 1 + u^2$

(c) $u = 1 + \sin^2(x - 1)$

3. **Integration, differentiation, and the chain rule** [2008 exam question]

Find

$$\frac{d}{dx}\int_{\sqrt[3]{x}}^{\pi/6} \cos(t^3)\,dt \ .$$

4. **Area between curves** [2007 exam question]

Find the area enclosed by the two curves $y = x^2 - 2$ and $y = 2$.

Extra: Let f be continuous for all $x \in [a, b]$ and let F be any antiderivative of f on $[a, b]$. Show that

$$F(b) - F(a) = \int_a^b f(x)dx \ .$$

Hint: Use $F(x) = G(x) + C$, where $G(x) = \int_a^x f(t)dt$ is a specific antiderivative of f, and C is some constant.