Queen Mary
University of London

MTH6109 Combinatorics

Assignment 7 For handing in on 24 November 2011

---

1 An opinion poll reports that the percentage of voters who would be satisfied with each of the three candidates A, B, C for President is 65%, 57%, 58% respectively. Further, 28% would accept A or B, 30% A or C, 27% B or C, and 12% would be content with any of the three. What do you conclude?

2 I have 25 sweets to distribute to a class of 10 children.

(a) In how many ways can I distribute the sweets?

(b) In how many ways can I distribute the sweets if I give Alice at least four sweets?

(c) In how many ways can I distribute the sweets if I give both Alice and Bob at least four sweets?

(d) Use the Principle of Inclusion and Exclusion to count the number of ways I can distribute the sweets if no child is to have more than three sweets.

3 A student has to choose eight courses from a list containing five pure maths courses, five applied maths courses, four statistics courses and four computing courses. Use the principle of inclusion and exclusion to find how many ways there are to choose the eight courses such that at least one course of each of the four types is chosen.

4 Ten dice are rolled. Use the principle of inclusion and exclusion to calculate the probability that each of the numbers 1,2,3,4,5,6 occur at least once.

5 Use the principle of inclusion and exclusion to determine the number of permutations f of $\{1, 2, 3, \ldots, 2n\}$ with the property that $f(i) \neq i$ for all $i = 1, 3, 5, \ldots, 2n - 1$.

6 Let $A_1, A_2, \ldots, A_n$ be sets each of cardinality m, with the property that any two of the sets intersect in exactly one point, but no point lies in three of the sets. Show that the union of all the sets has cardinality $n(2m - n + 1)/2$.