Queen Mary
University of London

MTH6109 Combinatorics

Assignment 1 For handing in on 6 October 2011

Your solution to one question of your own choice should be handed in to the RED box on the SECOND floor by 3.30pm on Thursday.

Include with your solution a brief *self-assessment*, explaining how well you have understood the week's work, any areas of particular difficulty, why you have chosen to hand in this particular question, how well you think you have answered it, and (important:) what help you had with it.

I will hand back marked work (and discuss it with you, where necessary) at the tutorials the following week.

The learning objectives for this week are: to understand the basic counting principles (multiplication and addition principles), and to be able to use them accurately to count sequences and subsets of various kinds; also to understand the binomial theorem, its proof and some combinatorial applications.

If these questions are too easy, then there are some more challenging questions on last year's exercises which you can find on the module web-site.

1 Let $X = \{1,2,3,4,5,6,7\}$. Answers should be given as actual numbers, not formulae.

(i) How many ordered sequences of length five using the elements of X are there if repetitions are allowed?

(ii) How many ordered sequences of length five using distinct elements of X are there?

(iii) How many of the ordered sequences in (ii) begin with two odd numbers and end with three even numbers?

(iv) How many of the ordered sequences in (ii) have the property that no two consecutive numbers are odd and no two consecutive numbers are even?

(v) How many of the ordered sequences in (ii) contain exactly two of the numbers 1,2,3?

2 Let $X = \{A, B, C, D, E, F, G, H, I\}$.

(i) How many subsets of size five does X have?

(ii) How many of the subsets in (i) contain exactly two vowels?

(iii) How many of the subsets in (i) contain at least two vowels?

(iv) How many 11 letter sequences can be made using each of the letters in MISSISSIPPI exactly once?

3 Let $A = \{1, 2, 3, ..., n\}$. Let $\mathcal{X}$ be the set of all subsets of A which have an even number of elements and $\mathcal{Y}$ be the set of all subsets of A which have an odd number of elements. Define a bijection $f : \mathcal{X} \to \mathcal{Y}$ and hence deduce that $|\mathcal{X}| = |\mathcal{Y}|$.