School of Mathematical Sciences
Mile End, London E1 4NS

MTH5112 Linear Algebra I
TEST

Date: 10 November 2011 *Time:* 3.00–3.40pm

Arts 2 Lecture Theatre	Surnames A to M
Mason Lecture Theatre	Surnames N to Z

Complete the following information:

Name	
Student Number (9 digit code)	

The duration of the test is **40 minutes**. Answer **all** questions **in the spaces provided**. Write the final answer clearly. Calculators are **not** allowed.

Total Marks	

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(You can use this page for draft.)

Nothing on this page will be marked!

1. Let A be an $m \times n$ real matrix. Give a definition of the *null space*, $N(A)$, of A.

   Let
   $$A = \begin{pmatrix} 1 & 2 & -3 & 1 \\ -1 & -1 & 4 & -1 \\ -2 & -4 & 7 & -1 \end{pmatrix}.$$
   Determine the null space $N(A)$.

2. Let
$$A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}.$$

Give a reason to show that A is invertible.

Further, find the $(1,3)$-entry of the inverse A^{-1}.

3. Determine, **with a reason**, whether the following vectors are linearly independent in the vector space $\mathbb{R}^4$:

$$\begin{pmatrix} 2 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 3 \end{pmatrix}.$$

Do they form a basis of $\mathbb{R}^4$?

4. Let V be a real vector space and let $x, y, z \in V$. Give a definition of the linear span
$$\mathrm{Span}\,(x, y, z)$$
of the vectors x, y and z.

Let x, y, z be linearly independent. Show that each vector v in Span (x, y, z) is a **unique** linear combination of x, y and z.

Let $\mathbb{R}^{4\times 4}$ be the real vector space of 4×4 real matrices. Determine, **with a reason**, if the subset
$$W = \{A \in \mathbb{R}^{4\times 4} : \det A = 0\}$$
is a subspace of $\mathbb{R}^{4\times 4}$.

END