MTH5112 Linear Algebra I 2012–2013

Coursework 6

Please hand in your solution of the **starred** feedback exercise by **noon on Friday 16 November** using the red Linear Algebra I Collection Box in the Basement. Don't forget to put your **name** (with your surname underlined) and **student number** on your solutions, and to **staple** them.

Exercise 1. Determine which of the following collections of vectors in $\mathbb{R}^3$ are linearly independent:

(a) $(1,1,1)^T$, $(3,4,3)^T$, $(2,1,3)^T$, $(1,1,3)^T$;

(b) $(2,-1,5)^T$, $(1,3,2)^T$, $(3,2,7)^T$;

(c) $(3,3,-6)^T$, $(-2,-1,4)^T$, $(1,4,-1)^T$;

(d) $(1,2,3)^T$, $(4,5,0)^T$.

Exercise 2. Show the following:

(a) If $\{\mathbf{v}_1,\dots,\mathbf{v}_n\}$ is a spanning set for a vector space V and $\mathbf{v}$ is any vector in V, then $\mathbf{v},\mathbf{v}_1,\dots,\mathbf{v}_n$ are linearly dependent.

(b) If $\mathbf{v}_1,\dots,\mathbf{v}_n$ are linearly independent vectors in a vector space V, then $\mathbf{v}_2,\dots,\mathbf{v}_n$ cannot span V.

Exercise 3. Let $\mathbf{x}_1,\dots,\mathbf{x}_k$ be linearly independent vectors in $\mathbb{R}^n$, and let $A\in\mathbb{R}^{n\times n}$ be invertible. Define $\mathbf{y}_i = A\mathbf{x}_i$ for $i=1,\dots,k$. Show that $\mathbf{y}_1,\dots,\mathbf{y}_k$ are linearly independent.

Exercise 4. For each of the collection of vectors in Exercise 1, decide whether they form a basis for $\mathbb{R}^3$. Justify your answer.

Exercise 5. Let H be the subspace of P_3 consisting of all polynomials $\mathbf{p}\in P_3$ with $p(1)=0$. Find a basis for H and determine its dimension.

Exercise* 6. Which of the following statements (if any) are true? Justify your answers.

(a) $(6,5,4)^T,(3,2,1)^T,(0,-1,-2)^T,(-3,-4,-5)^T$ are linearly independent vectors in $\mathbb{R}^3$.

(b) $(1,1,2)^T,(-2,1,-4)^T,(2,3,8)^T$ are linearly independent vectors in $\mathbb{R}^3$.

(c) $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ are linearly independent vectors in P_2, where

$$\mathbf{p}_1(t)=1\,,\quad \mathbf{p}_2(t)=1+t\,,\quad \mathbf{p}_3(t)=1+t+t^2\,.$$

(d) $\begin{pmatrix}1&0\\0&-2\end{pmatrix}$, $\begin{pmatrix}0&3\\3&0\end{pmatrix}$, $\begin{pmatrix}1&6\\6&-2\end{pmatrix}$ are linearly independent vectors in $\mathbb{R}^{2\times 2}$.

(e) If $\mathbf{a}_1,\dots,\mathbf{a}_n$ are the columns of an invertible $n\times n$ matrix A, then $\{\mathbf{a}_1,\dots,\mathbf{a}_n\}$ is a basis for $\mathbb{R}^n$.