MTH5112 Linear Algebra I 2012–2013

Coursework 5

Please hand in your solutions of the **starred** feedback exercises by **noon on Friday 2 November** using the red Linear Algebra I Collection Box in the Basement. Don't forget to put your **name** (with your surname underlined) and **student number** on your solutions, and to **staple** them.

Exercise* 1. Explain what is wrong with the following solution to Exercise 7(a) on Coursework 2:

If $(I + A)$ is invertible then

$$(I + A)(I + A)^{-1} = (I + A)(I - A + A^2) = I - A + A^2 + A - A^2 + A^3 = I + A^3 = I$$
$$\Rightarrow I = I.$$

Exercise 2. Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in a vector space. Prove that if $\mathbf{u} + \mathbf{w} = \mathbf{v} + \mathbf{w}$ then $\mathbf{u} = \mathbf{v}$. As in the lectures, clearly indicate which vector space axiom you are using at each step.

Exercise 3. In each case, determine whether H is a subspace of $\mathbb{R}^3$ and give reasons for your answer:

(a) $H_1 = \left\{ (r, s, t)^T \;\middle|\; r, s, t \in \mathbb{R} \text{ and } 3r + s - 2t = 0 \right\}$,

(b) $H_2 = \left\{ (r + 1, 0, r)^T \;\middle|\; r \in \mathbb{R} \right\}$,

(c) $H_3 = \left\{ (r, s, t)^T \;\middle|\; r, s, t \in \mathbb{R} \text{ and } r^2 + s^2 + t^2 \leq 1 \right\}$.

Exercise 4. Let $C^1[a, b]$ denote the subset of those functions in $C[a, b]$ that have a continuous derivative on $[a, b]$. Explain why $C^1[a, b]$ is a subspace of $C[a, b]$.

Exercise 5. Determine the nullspace of the following matrix

$$A = \begin{pmatrix} 1 & 3 & -2 & 1 \\ -3 & -9 & 6 & -2 \\ 2 & 6 & -4 & 1 \end{pmatrix} .$$

Exercise 6. Let U and V be subspaces of a vector space W. Define their **intersection** $U \cap V$ and their **sum** $U + V$ by

$$U \cap V = \{ \mathbf{w} \in W \mid \mathbf{w} \text{ belongs to both U and V} \} ,$$
$$U + V = \{ \mathbf{w} \in W \mid \mathbf{w} = \mathbf{u} + \mathbf{v} \text{ where } \mathbf{u} \in U \text{ and } \mathbf{v} \in V \} .$$

Show that $U \cap V$ and $U + V$ are subspaces of W.

Exercise* 7. Which of the following statements (if any) are true? Justify your answers.

(a) $H_1 = \left\{ (r, s, t, u)^T \;\middle|\; r, s, t, u \in \mathbb{R} \text{ and } r + s - 3t + 5u = 0 \right\}$ is a subspace of $\mathbb{R}^4$.

(b) $H_2 = \{ A \in \mathbb{R}^{n\times n} \mid A \text{ is symmetric} \}$ is a subspace of $\mathbb{R}^{n\times n}$.

(c) $H_3 = \{ \mathbf{f} \in C[0, 1] \mid \mathbf{f}(1) = 1 \}$ is a subspace of $C[0, 1]$.

(d) $S_1 = \left\{(1, 0, 2)^T, (3, 0, 4)^T, (5, 0, 6)^T\right\}$ spans $\mathbb{R}^3$.

(e) $S_2 = \{M_1, M_2\}$ is a spanning set for $V = \{ A \in \mathbb{R}^{2\times 2} \mid A \text{ is diagonal} \}$, where

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \quad M_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} .$$