MTH5112 Linear Algebra I 2012–2013

Coursework 4

Please hand in your solutions of the **starred** feedback exercises by **noon on Friday 26 October** using the red Linear Algebra I Collection Box in the Basement. Don't forget to put your **name** (with your surname underlined) and **student number** on your solutions, and to **staple** them.

Exercise 1. Prove the theorem on the behaviour of determinants under row operations (Theorem 3.11 in the notes) for 2×2 matrices.

Exercise 2.

(a) Let A be an invertible matrix. Show that $\det(A^{-1}) = \dfrac{1}{\det(A)}$.

(b) Let A be a square matrix. Show that if $B = S^{-1}AS$ for some invertible matrix S, then $\det(A) = \det(B)$.

Exercise* 3. Let

$$A = \begin{pmatrix} 1 & -1 & -3 \\ 1 & 1 & 4 \\ -1 & 2 & 5 \end{pmatrix}.$$

(a) Calculate $\det(A)$ and deduce that A is invertible.

(b) Determine the $(2,3)$-entry of A^{-1} by computing a quotient of two determinants.

(c) Use Cramer's rule and the results from (a) to determine y where

$$\begin{array}{rcrcrl} x & - & y & - & 3z & = 2 \\ x & + & y & + & 4z & = 3 \\ -x & + & 2y & + & 5z & = -1 \end{array}.$$

Exercise 4.

(a) Show that if A is an $n \times n$ matrix and α a scalar then $\det(\alpha A) = \alpha^n \det(A)$.

(b) Use (a) to show that $\det(A + B) \neq \det(A) + \det(B)$ in general, unless A and B are 1×1 matrices.

Exercise* 5. Determine which of the following statements (if any) are true. Justify your answers (that is, either prove the statement or give an example showing that it is false).

(a) Let A and B be square matrices of the same size. Then $\det(A + B^T) = \det(A^T + B)$.

(b) If A is a square matrix and $\mathrm{adj}(A)$ exists, then A is invertible.

(c) Let A, B, and C be square matrices of the same size such that $BA = CA$. Then $B = C$ if $\det(A) \neq 0$.

A cash prize is offered for the best solution received to the following question. Do not hand it in with your coursework, but hand it to me in person either at the Wednesday lecture (31st October) or before.

Prize question. Let A be an $n \times n$ matrix, where $n \geq 2$. Show that $\det(\mathrm{adj}(A)) = (\det(A))^{n-1}$. If A is invertible, how are $\mathrm{adj}(A^{-1})$ and $\mathrm{adj}(A)$ related? What is $\mathrm{adj}(\mathrm{adj}(A))$?