MTH5112 Linear Algebra I 2012–2013

Coursework 3

Please hand in your solutions of the **starred** feedback exercises by **noon on Friday 19th October** using the red Linear Algebra I Collection Box in the Basement. Don't forget to put your **name** (with your surname underlined) and **student number** on your solutions, and to **staple** them.

Exercise* 1. A square matrix A is said to be **left-invertible** if there is a matrix C of the same size as A such that $CA = I$. Use the Invertible Matrix Theorem to show that a left-invertible matrix is invertible.

Exercise* 2.

(a) Use Gauss-Jordan inversion to determine the inverse of the matrix

$$A = \begin{pmatrix} 1 & -1 & 3 \\ -2 & 3 & -6 \\ 1 & -1 & 4 \end{pmatrix}.$$

(b) Write the system

$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & 3x_3 & = & 4 \\ -2x_2 & + & 3x_2 & - & 6x_3 & = & -6 \\ x_1 & - & x_2 & + & 4x_3 & = & 5 \end{array}$$

in matrix form. Using the result from (a), determine the solution set of the system without performing any elementary row operations.

Exercise 3.

(a) Using the Gauss–Jordan algorithm, bring the following matrix to reduced row echelon form and record the elementary row operations used.

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 3 & 0 \\ -2 & -8 & 1 \end{pmatrix}.$$

(b) Using the results from (a), explain why A is invertible. Write A^{-1} and A as a product of elementary matrices.

Exercise* 4. Verify the Cofactor Expansion Theorem in a particular case by calculating the determinant of the matrix

$$A = \begin{pmatrix} -2 & 1 & 3 \\ 5 & -1 & -4 \\ 7 & 0 & -1 \end{pmatrix}$$

by expanding down the third column and across the second row.

Exercise 5. Compute the following determinants. Remember to indicate which method you are using, that is, which row operations you carry out or whether you are using a cofactor expansion.

(a) $\begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 4 & 1 \\ 1 & 1 & 6 & 6 \\ 2 & 4 & 1 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & 0 & 0 & 0 & 5 \\ -4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{vmatrix}$