

# Coursework 1

Please hand in your solution of the **starred** exercise by **noon on Friday 5 October** using the red Linear Algebra I Collection Box in the basement.

Don't forget to put your **name** and **student number** on your solutions, and to **staple** them. Please also put the **day** and **time** of your tutorial so that we can return your marked work to you.

**Exercise 1.** Which of the following matrices are in row echelon form? Which are in reduced row echelon form?

$$\begin{array}{llll} \text{(a)} \begin{pmatrix} 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} & \text{(b)} \begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 2 & 3 \end{pmatrix} & \text{(c)} \begin{pmatrix} 1 & 8 & 0 & 5 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} & \text{(d)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \\ \text{(e)} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \text{(f)} \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \text{(g)} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} & \end{array}$$

**Exercise 2.** In each of the following, the augmented matrix is in reduced row echelon form. In each case, identify the leading variables and the free variables, and determine the solution set of the corresponding linear system.

(a)  $\left(\begin{array}{cccc|c} 1 & 3 & 0 & 2 & 4 \\ 0 & 0 & 1 & 1 & 3 \end{array}\right)$     (b)  $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)$     (c)  $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -2 \end{array}\right)$     (d)  $\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$

**Exercise 3.** Consider the linear system

$$\begin{array}{rclcl} x_1 & - & x_2 & + & 3x_3 & = & 2 \\ 2x_1 & - & x_2 & & & = & 5 \\ -x_1 & + & 2x_2 & - & 6x_3 & = & 0 \end{array}$$

- Write down the augmented matrix of the system.
- Use Gaussian elimination to bring the augmented matrix to row echelon form and, as in the lectures, indicate which elementary row operations are used at each step.
- Identify the leading and the free variables, and write down the solution set of the system.

### Exercise\* 4.

- (a) Give an example of a matrix with 4 rows and 3 columns in row echelon form, but not in reduced row echelon form. Explain why your chosen example has the required properties.
- (b) Consider the linear system

$$\begin{array}{rclcl} x_1 & + & x_2 & + & x_3 & - & 3x_4 & = & -2 \\ 2x_1 & + & 3x_2 & & & - & 4x_4 & = & 1 \\ -3x_1 & - & 4x_2 & - & x_3 & + & 6x_4 & = & -1 \end{array}$$

- (i) Write down the augmented matrix of the system.
- (ii) Use Gauss-Jordan reduction to bring the augmented matrix to reduced row echelon form and, as in the lectures, indicate which elementary row operations are used at each step.
- (iii) Identify the leading and the free variables, and write down the solution set of the system.

**Exercise 5.** Determine all quadratic polynomials  $p(x) = ax^2 + bx + c$  that satisfy  $p(-1) = 1$  and  $p(2) = 10$ . [Hint: Write down a system of linear equations for the coefficients and solve it using Gaussian elimination]