

MTH5112 Linear Algebra I 2012-2013

Coursework 2

Please hand in your solutions of the **starred** feedback exercises by **noon on Friday 12 October** using the red Linear Algebra I Collection Box in the Basement. Don't forget to put your **name** (with your surname underlined) and **student number** on your solutions, and to **staple** them. Please also indicate the day and time of the tutorial at which you will collect your marked work.

Exercise 1. Give a geometric explanation of why a homogeneous linear system consisting of two equations in three unknowns must have infinitely many solutions.

Exercise 2. Consider the matrices A and B given by

$$A = \begin{pmatrix} 2 & 4 \\ -6 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -5 \\ -3 & 2 \end{pmatrix}.$$

Find (a) $A - B$, (b) $\frac{1}{2}A + 3B$.

Exercise* 3. For the following matrices decide which products of two of them exist (including the possibility that a matrix is multiplied by itself) and calculate these products.

$$A = \begin{pmatrix} -2 & 0 \\ 7 & -1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & -2 & 1 \end{pmatrix}.$$

Exercise 4. Let A and B be square matrices of the same size. Show that

$$(A + B)^2 = A^2 + 2AB + B^2$$

if and only if A and B commute.

Exercise 5. Show that the sum and the product of two upper triangular matrices of the same size is again upper triangular.

Exercise 6. Let A and B be matrices of the same size. Show the following:

(a) $(\alpha A)^T = \alpha A^T$;

(b) $(A + B)^T = A^T + B^T$.

Exercise* 7. Let A and B be square matrices of the same size. Show the following:

(a) If $A^3 = O$ then $(I + A)$ is invertible and $(I + A)^{-1} = I - A + A^2$.

(b) If A and B are symmetric then AB is symmetric if and only if A and B commute.

Exercise 8. Show that if A is invertible then A^T is invertible and

$$(A^T)^{-1} = (A^{-1})^T.$$

A cash prize is offered for the best solution received to the following question. Do not hand it in with your coursework, but hand it to me in person at the Wednesday lecture (17th Oct) or before.

Prize question. A matrix P is called an **idempotent** if $P^2 = P$. Give examples of idempotents. Can an idempotent be invertible? If P is an idempotent, is $I - P$ invertible? What about $I - aP$, where a is a scalar?