Teaching mathematics: satnav or map?

R. A. Bailey

r.a.bailey@qmul.ac.uk

London Mathematical Society
20 November 2009

What do I know about it, anyway?

- Teaching Mathematics (including Statistics) undergraduates since 1972, in particular, “non-mature” home students at the University of London since 1991.
- Returning to Year 1 lecturing in Spring 2009 after a gap from Christmas 2005.
- Member of the Higher Education Section of the Joint Mathematical Council, 1993–1996.
- Member of the Royal Society’s *ad hoc* group on Mathematics Education, 1996–1997.
- Consulted by *Reform*—“the right-wing think-tank” during their 2009 review of A levels.

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  (I do not necessarily agree with all of the causes supported by *Reform*, but I do think that university mathematicians should be engaged in this discussion.)

Reform: May–June 2009

I looked at

- Mathematics A-level examination papers:
  - Cambridge, 1951, 1970, 1990
  - OCR, 2000, 2008
- Syllabuses:
  - Cambridge, 1952, 1960, 1970, 1980, 1990
  - OCR, 2000
- AQA Specification for A-level Mathematics 2010
- Examiners' reports:
  - Cambridge, 1954–56, 1971, 1982, 1990
  - OCR, 2008
- Mark scheme: OCR, 2008
- *A comparability study in advanced level Mathematics—A study based on the Summer 1994 and 1989 examinations*
  by Mick Quinlan, organized by the University of London Examinations and Assessment Council

These items were chosen by *Reform*, not by me.

OCR Core Mathematics 1, June 2008, Question 8

The curve $y = x^3 - kx^2 + x - 3$ has two stationary points.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Given that there is a stationary point when $x = 1$, find the value of k.

(iii) Determine whether this stationary point is a minimum or maximum point.

(iv) Find the x-coordinate of the other stationary point.

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I found all the Pure Mathematics questions boring.
They had no proofs, no excitement, no feel for the subject.

(Part of) an A-level question about mathematics in biology

OCR Core Mathematics 3, June 2008, Question 7

It is claimed that the number of plants of a certain species in a particular locality is doubling every 9 years. The number of plants is now 42. The number of plants is treated as a continuous variable and is denoted by N. The number of years from now is denoted by t.

(i) Two equivalent expressions giving N in terms of t are

$$N = A \times 2^{kt} \qquad \text{and} \qquad N = Ae^{mt}.$$

Determine the values of each of the constants A, k and m.

(ii) Find the value of t for which $N = 100$, giving your answer correct to 3 significant figures.

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Do they not know that most UK plants have an annual life-cycle? The supposed application is bogus.

An A-level question about mathematics in physics

OCR Mechanics 1, June 2008, Question 5

A railway wagon A of mass 2400 kg and moving with speed 5 m s^{-1} collides with railway wagon B which has mass 3600 kg and is moving towards A with speed 3 m s^{-1}. Immediately after the collision the speeds of A and B are equal.

(i) Given that the two wagons are moving in the same direction after the collision, find their common speed. State which wagon has changed its direction of motion.

(ii) Given instead that A and B are moving with equal speeds in opposite directions after the collision, calculate (a) the speed of the wagons, (b) the change in the momentum of A as a result of the collision.

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Part (i) makes physical sense, because the wagons might couple together.

Part (ii) does not. There is no physical reason why this should happen. This is a bogus application, solving equations for the sake of it, rather than to solve a genuine problem.

Reform's report 'A new level': 16 June 2009

From roadmap to sat-nav

The most important change in exams over the period 1951–2008 is that sitting an A-level paper now is more like using a sat-nav system than reading a map. If you read a map to get from A to B, you remember the route and learn about other things on the way. If you use a sat-nav you do neither of those things. The questions in the 2008 paper are heavily structured in this way and the result is that students will retain very little knowledge and develop very little understanding.

Reform's report 'A new level': 16 June 2009

Questions have evolved over the decades. In 1951 there were almost exclusively single questions; by 1970 most questions were split into numbered sub-parts, with part-marks shown. In 1990 this process went a lot further. Each question was very short, so that candidates had to keep changing topics. Also, the ratio of words to formulae became much smaller. But by this stage not all papers had been balkanized like this and there was little change from 1990 to 2000.

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However the questions were further subdivided and directed by 2008 where there was a change from two substantial three-hour papers to a modular system with several 90-minute papers. The questions became orders: do this, do that. They include hints and instructions about which method to use. The questions on the Core (Pure Mathematics) papers are mind-numbingly boring, apart from those that are mind-numbingly stupid.

Reform's report 'A new level': 16 June 2009

Compartmentalised not holistic

Another important change is that now there is a wide choice of topics to study for A-level Mathematics. Given that Mathematics is a cumulative subject, with each topic depending on several previous steps, any topic that is studied by some A-level students, but not all, must be re-taught to all of them in their first year at university. The amount of choice in this selection of modules presents a headache for universities. Any topic which is not a compulsory part of A-level has to be taught to all first-year undergraduates, because later topics depend on them.

Reform's report 'A new level': 16 June 2009

The 2000 syllabus contained the following laudable statement:
"It should be noted that individual questions may involve more than one section of the following list and that topics may be tested in the context of solving problems and in the application of mathematics."
It is questionable whether that is still true. We expect our students to take a holistic approach to the subject, but it seems from these A-level questions that they expect it to be compartmentalised.

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[From 1952 to 2000] the obvious changes are the removal of geometry, both practical and theoretical, and the gradual decreasing importance of proof.

There are questions which seem to have been set by people who do not understand the subject. ... There are some statistics questions whose basis will encourage bad practice in statistics in later life, whether as a mathematician, a scientist or a policy maker.

Critical thinking and reasoning

The AQA specification for 2010 does not bode well. Under "Key Skills" it says that "the study of mathematics does not easily lend itself to developing the key skill of problem solving". On the contrary, problem solving is one half of Mathematics, the other being logical argument and proof. When you have solved a number of similar problems then you develop a mathematical theory about them. This problem-solving ability is one of the main skills that has been lost from school Mathematics.

From discovery to contract

The syllabus changes suggest a new approach to marking. The 1952 syllabus is written in normal text, in paragraphs; by 2008, we have a numbered list, with very precise details as well as "curriculum objectives". From 1952 to 1980, and maybe even until 1990, the syllabus suggests that the examiners intend to find out if the candidates know any Mathematics; in 2000, the syllabus seems more like a contract: you do exactly this, this and this and then we will reward you. On the surface, that approach looks fairer, but it cannot promote genuine learning.

Reform's report 'A new level': 16 June 2009

Examiners not trusted

. . . One cannot avoid the suspicion that the main reason for the change to the "sat-nav" type of question is to enable consistent marking from people who may not be trusted to actually understand the Mathematics.

The press: 17 June 2009

A-levels 'too much like sat-nav'—BBC online

A levels 'create unthinking students'
Pupils 'sit sat-nav exams and then forget'—*The Times*

A-levels being turned into 'sat nav' qualifications—*The Daily Telegraph*

'SATNAV' A-LEVEL STUDENTS CAN 'REVISE AND FORGET'—*The Daily Express*

Satnav-style A-level exams are creating a generation of school-leavers 'who can't think for themselves'—*Mail* Online

'Satnav students' can't think for themselves, says A-level study—*The Guardian*

So what is new?

A. D. Gardiner has been saying all these things for several years.

The ‘sat-nav’ analogy seems to have caught people’s imagination.

An A-level student: June 2009

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> "When they use such a predictable system, the way to win is to follow the system."

The Daily Mail: August 2009

In the *Daily Mail* on 18 August 2009, Allyson Pearson wrote:

> **Spoon-fed A-levels that fail everyone**
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> Few things are certain in August . . .
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I challenge ANY cynics to sit the exams without any preparation and see how they fare. . . . Me and my friends have worked VERY hard over these past 2 years and its very upsetting that instead of celebrating huge successes from a generation which is involved in so much crime, a negative spin is put on it. Us young people can never win and this isn't fair. **–Kate, Manchester, 20/8/2009 11:17**

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I quote Kate of Manchester....“Me and my friends.....”
I rest my case...! **–Raymond, Poole, 20/8/2009 16:30**

The LMS and the IMA: Media release, 20 August 2009

Maths A-level numbers bounce back

The London Mathematical Society and the Institute of Mathematics and its Applications warmly congratulated all who have successfully completed these examinations.

Jenny's Blog, 13 November 2009

What we teach is what we test

I acknowledge that most of us feel constrained by the system and that the structure in which we work encourages us to "teach to the test".

Jenny's Blog, 13 November 2009

The Edexcel examiner was describing how the maths GCSE papers would change and was (I assume) trying to allay fears by saying that there would be very little change and that teachers should not worry. In the description on how the board would meet the challenges of the three assessment objectives, in particular the new A03 (Interpret and analyse problems and generate strategies to solve them), the examiner used words to the effect:

In the past we have written questions with several parts so that the student is led through the question step by step. So a question might have been [divided] into parts a, b and c. In the new exam sections [assessing] A03, questions will not be split into parts. They will be written as a single question. What you will need to do is to teach your students how to split it back up into parts a, b and c again before they start. Then they can carry on just as before.

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- I work out what the question is asking me to do.
- If I cannot do it, I try special cases.

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  But this request comes almost entirely from those students who have not bothered to collect their own marked coursework.

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(The set of those who passed in May is roughly the same as the set of those who had collected most of their marked coursework.)

A question from the August resit examination

A simple model for the sex of newborn rabbits is that there is some probability p such that each newborn rabbit has probability p of being male, independent of all other rabbits. If this model is correct, then the number of males in litters of three rabbits should have distribution $\text{Bin}(3,p)$.

A rabbit breeder notes the number of males in each of 80 litters of three rabbits. Here are the data.

number of males	0	1	2	3	Total
frequency	20	31	22	7	80

(i) Use these data to estimate p.

(ii) The null hypothesisis is that there is some p such that the distribution is $\text{Bin}(3,p)$. Calculate a test statistic suitable for testing this null hypothesis.

(iii) Test the hypothesis at the 5% significance level and report your conclusion.

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Answers:
everyone who answered this question appeared to think that it said
"test the hypothesis that $p = 0.5$."

Another question from the August resit examination

Let $Z_1, Z_2, \ldots, Z_{10}$ be mutually independent standard normal random variables. Put

$$\bar{Z} = \frac{Z_1 + Z_2 + \cdots + Z_{10}}{10} \qquad \text{and} \qquad S^2 = \frac{1}{9}\sum_{i=1}^{10}\left(Z_i - \bar{Z}\right)^2.$$

State the distribution of each of the following random variables.

(i) $6Z_1 + 3$

(ii) $\bar{Z}$

(iii) $9S^2$

(iv) $\displaystyle\sum_{i=1}^{10} Z_i^2$

(v) $\dfrac{\bar{Z}}{\sqrt{S^2/10}}$

(vi) $\dfrac{7\left(Z_1^2 + Z_2^2 + Z_3^2\right)}{3\left(Z_4^2 + Z_5^2 + Z_6^2 + Z_7^2 + Z_8^2 + Z_9^2 + Z_{10}^2\right)}$

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"Poisson" and "Binomial" were common answers,
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although a few could do part (i) $(6Z_1 + 3)$.

"Poisson" and "Binomial" were common answers,
even though these are discrete random variables.

Why?
I had never set a question like this before (apart from part (i)),
and the question refers to more than one section of the lecture notes.

How can we teach our students how to learn?

Learning Tips

The purpose of coursework questions is for me to guide you into brain activities that will help you to understand and learn the subject.

The purpose of examination questions is for you to demonstrate to me that you do understand the subject.

These two purposes are quite different, so you should not expect coursework questions to be like examination questions.

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Learning Tips

Watching your friend do a work-out at the gym will not make you any fitter.

Listening to Ashkenazy playing *The Moonlight Sonata* will not make you a better pianist.

So why do you think that looking at my model solutions will improve your mathematical ability?

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So

- How should we teach Mathematics?
- How should Mathematics be examined?
- Should university Mathematics departments get together to set an examination to replace Mathematics A-level?