Design and analysis of experiments testing for diversity in ecology

R. A. Bailey

Queen Mary University of London

r.a.bailey@qmul.ac.uk

Joint work with Julia Reiss, University of Roehampton

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Abstract

It is now widely believed that biological diversity is good for the environment. One way that ecologists test this is to place random collections of species in mini-environments and then measure some outcome. I have been working with a group of fresh-water ecologists to improve this in two ways. The first is that our subsets of species are carefully chosen, not random. The second is that we fit a nested family of plausible models. Our results suggest that the underlying model is not diversity at all.

Biodiversity experiments

This seems to be the received wisdom.

Treatments:	random sets of species
Measured response Y:	some eco-desirable outcome
Conclusion:	the greater the number of different species, the better the outcome.

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The experiment was carried out in 4 blocks of 41 jars.

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I suggested the model ‘Type’ with 6 parameters $\alpha_A, \ldots, \alpha_F$:

monoculture A	α_A
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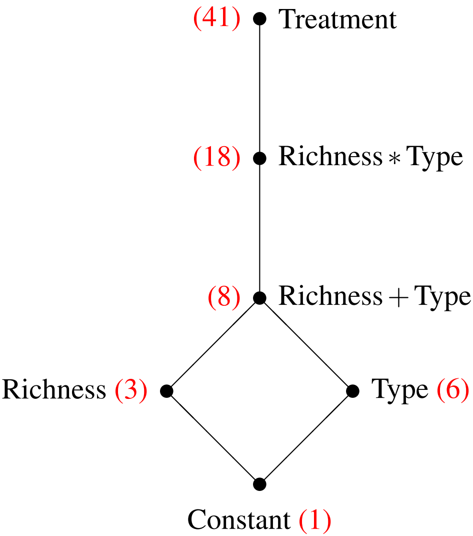
In other words, if there are x_i shrimps of type i then

$$\mathbb{E}(Y) = \sum_{i=1}^{6} a_i x_i \qquad \text{where } 12a_i = \alpha_i.$$

($\sum x_i = 12$ always, so no need for intercept.)

Family of expectation models



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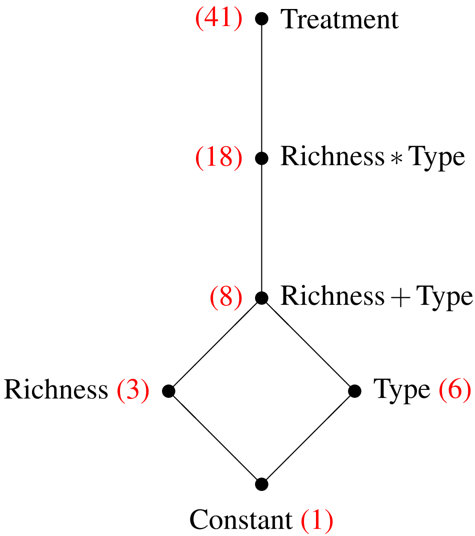

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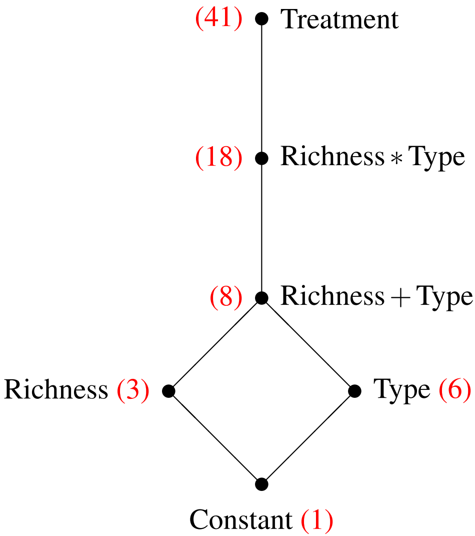

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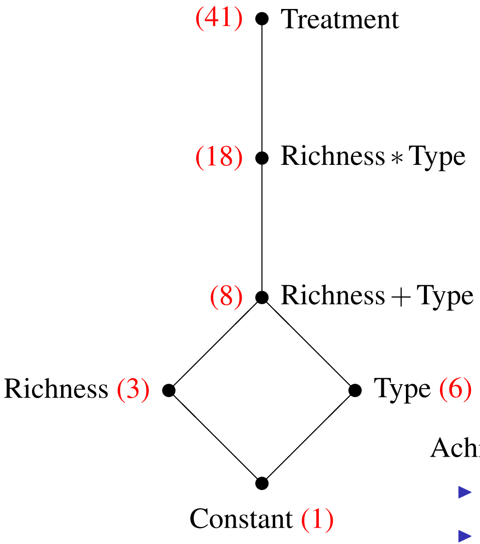

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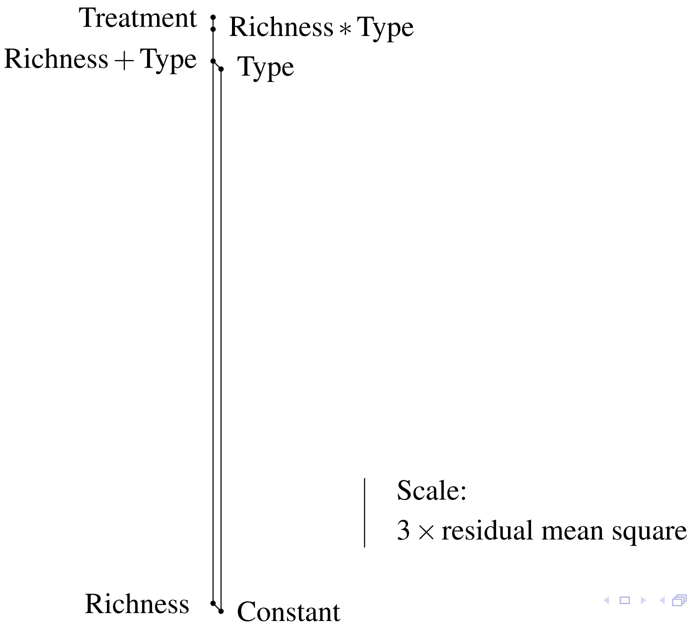
Achievement: an ecology journal published

- diagram of family of models
- statement that each row of an ANOVA table is for a difference between models.

What the data showed: mean squares



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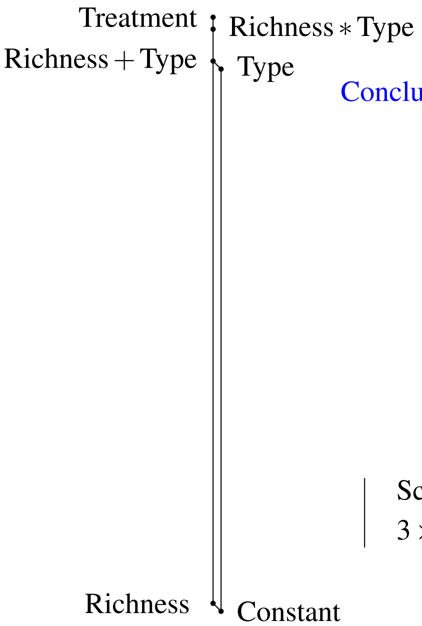

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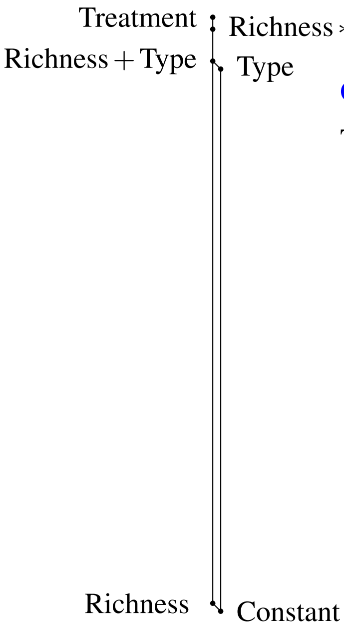

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Richness $*$ Type

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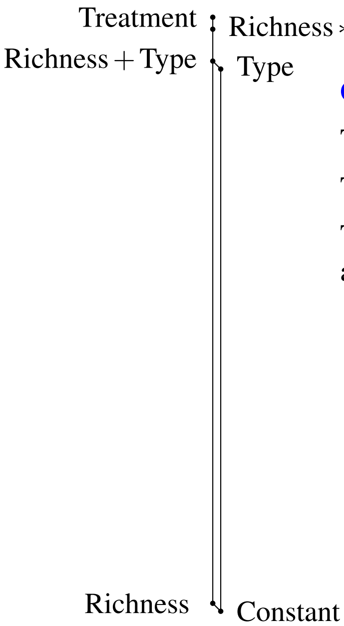
Richness

Constant



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Two experiments, with two responses each, all led to similar conclusions.

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A new experiment on a different ecosystem (7 types)

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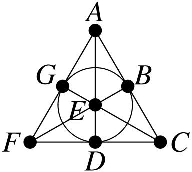

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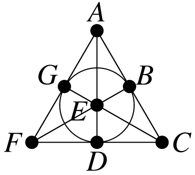

Another achievement:
an ecology journal published
a picture of the Fano plane.

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  - If so, we should not include any polycultures.

How should we choose which subsets to include?

Suppose that there are t types in all.
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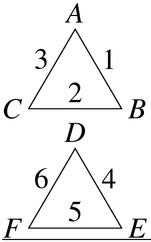
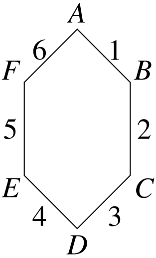
- adopt some other strategy?

6 subsets of size 2, from 6 types

design		
		
		

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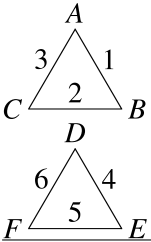		
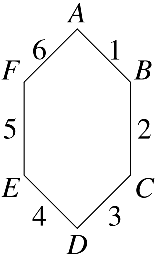		

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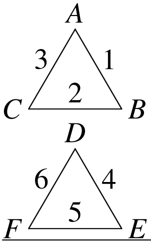
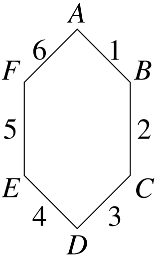
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$	
		

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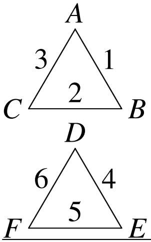
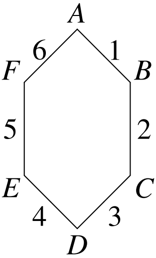
 	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \dfrac{4\sigma^2}{3}$	
 		

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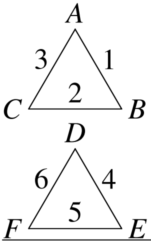
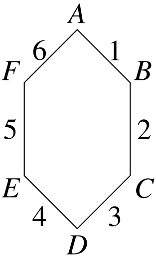
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \dfrac{4\sigma^2}{3}$ $\alpha_A - \alpha_D$ is not estimable	
		

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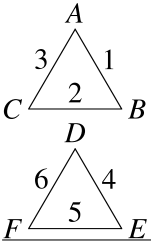
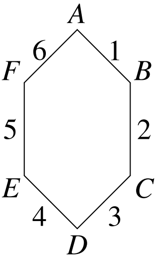
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{4\sigma^2}{3}$ $\alpha_A - \alpha_D$ is not estimable	
		

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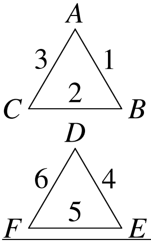
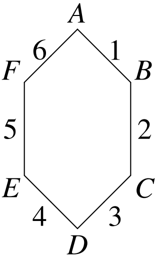
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \dfrac{4\sigma^2}{3}$ $\alpha_A - \alpha_D$ is not estimable	$\mathbb{E}(Y_1) = \dfrac{\alpha_A + \alpha_B}{2}$
		

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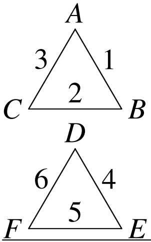
<tr><td>
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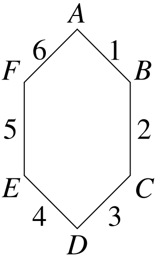
<tr><td>
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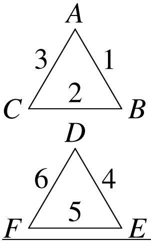
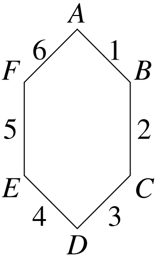
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{4\sigma^2}{3}$ $\alpha_A - \alpha_D$ is not estimable	$\mathbb{E}(Y_1) = \frac{\alpha_A + \alpha_B}{2}$ $\hat{\alpha}_A = Y_1 + Y_3 - Y_2$ $\mathrm{Var}(\hat{\alpha}_A) = 3\sigma^2$
		

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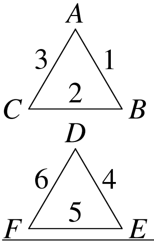
<tr><td>
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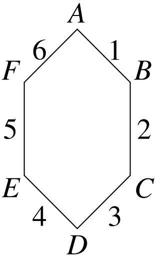
<tr><td>
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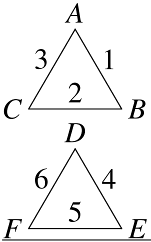
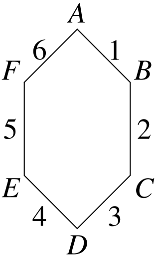
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{4\sigma^2}{3}$ $\alpha_A - \alpha_D$ is not estimable	$\mathbb{E}(Y_1) = \frac{\alpha_A + \alpha_B}{2}$ $\hat{\alpha}_A = Y_1 + Y_3 - Y_2$ $\mathrm{Var}(\hat{\alpha}_A) = 3\sigma^2$ $\sum_i \mathrm{Var}(\hat{\alpha}_i)$ is minimized
	$\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_D) = 3\sigma^2$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_C) = \frac{8\sigma^2}{3}$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{5\sigma^2}{3}$	

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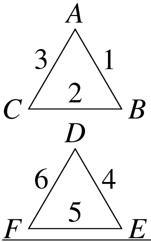
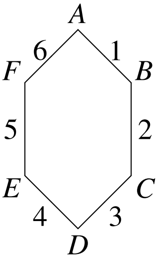
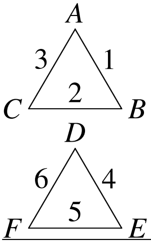
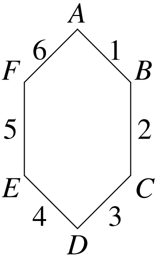
<tr><td></td><td>$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$

$\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{4\sigma^2}{3}$

$\alpha_A - \alpha_D$ is not estimable</td><td>$\mathbb{E}(Y_1) = \frac{\alpha_A + \alpha_B}{2}$
$\hat{\alpha}_A = Y_1 + Y_3 - Y_2$
$\mathrm{Var}(\hat{\alpha}_A) = 3\sigma^2$
$\sum_i \mathrm{Var}(\hat{\alpha}_i)$ is minimized</td></tr>
<tr><td></td><td>$\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_D) = 3\sigma^2$
$\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_C) = \frac{8\sigma^2}{3}$
$\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{5\sigma^2}{3}$
$\sum_{i \neq j} \mathrm{Var}(\hat{\alpha}_i - \hat{\alpha}_j)$ is minimized</td><td></td></tr>
</table>

6 subsets of size 2, from 6 types

design	usual model 12 responses	duoculture 6 responses
	$\mathbb{E}(Y_{1A}) = \alpha_A + \beta_B$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{4\sigma^2}{3}$ $\alpha_A - \alpha_D$ is not estimable	$\mathbb{E}(Y_1) = \frac{\alpha_A + \alpha_B}{2}$ $\hat{\alpha}_A = Y_1 + Y_3 - Y_2$ $\mathrm{Var}(\hat{\alpha}_A) = 3\sigma^2$ $\sum_i \mathrm{Var}(\hat{\alpha}_i)$ is minimized
	$\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_D) = 3\sigma^2$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_C) = \frac{8\sigma^2}{3}$ $\mathrm{Var}(\hat{\alpha}_A - \hat{\alpha}_B) = \frac{5\sigma^2}{3}$ $\sum_{i \neq j} \mathrm{Var}(\hat{\alpha}_i - \hat{\alpha}_j)$ is minimized	α_A is not estimable

A non-intuitive result

Consider incomplete-block designs for t treatments in b blocks of size k.

Usual model	expected response on any unit with treatment i in block B is $\alpha_i + \beta_B$
Polyculture model	expected response on any unit whose treatment is an equal mixture of the species in subset B is $\sum_{i \in B} \alpha_i / k$

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If there is no balanced incomplete-block design for t treatments in b blocks of size k then a design which is best for one situation may be worst for the other.

References

1. Julia Reiss, R. A. Bailey, Fernanda Cássio, Guy Woodward and Cláudia Pascoal: Assessing the contribution of micro-organisms and macrofauna to biodiversity-ecosystem functioning relationships in freshwater microcosms. *Advances in Ecological Research*, **43**, 2010, 151–176.
2. Julia Reiss, R. A. Bailey, Daniel M. Perkins, Angela Pluchinotta and Guy Woodward: Testing effects of consumer richness, evenness and body size on ecosystem functioning. *Journal of Animal Ecology*, 2011, in press.