Neighbour balance in a strip-block design for an experiment on irrigation

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(Joint work with Silvio Zocchi, ESALQ)

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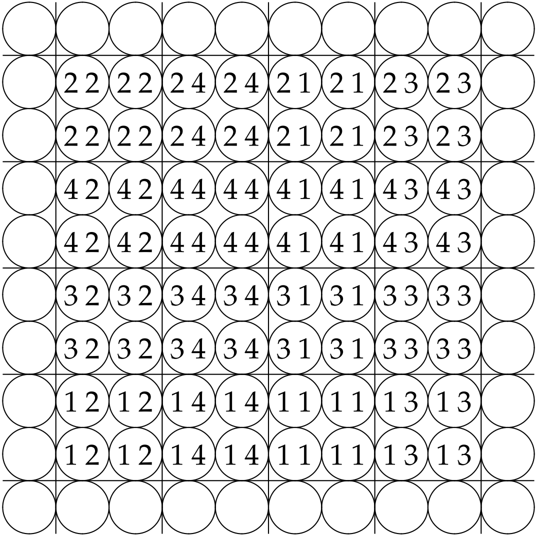
There are 4 quantities of nitrogen (which affects salinity of the water), one applied to each whole column.

This is called a strip-block design or criss-cross design.

One block of the design



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In order not to lose degrees of freedom,
it may be better not to model these neighbour effects explicitly but to use a neighbour-balanced design.

Constructing a neighbour-balanced design

- Start with a special column whose differences are all different modulo 4: $0 - 3 = 1$; $3 - 1 = 2$; $1 - 2 = 3$.

0
3
1
2

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- Develop this column modulo 4.

0	1	2	3	0	1	2	3
3	0	1	2	3	0	1	2
1	2	3	0	1	2	3	0
2	3	0	1	2	3	0	1

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- Now every level has every other level on the immediate North the same number of times (ditto South).

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2	3	0	1	2	3	0	1

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factors	N, L (water), R (rows), C (columns), B_1	B_2
levels modulo 4	0, 1, 2, 3	0, 2

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Put $L \equiv R + B_1$ and $N \equiv C + B_1 + B_2$. (This is the design key.)

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		C			
		0	3	1	2
R	0	1 3	1 2	1 0	1 1
	3	0 3	0 2	0 0	0 1
	1	2 3	2 2	2 0	2 1
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The 3 df for L are aliased with 3 of the df for $R.B$:
are these for ‘rows within blocks’ or
for ‘the interaction of rows and blocks’?

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$R,\ 2R,\ 3R$	3 df	contrasts between row-positions
$B_1,\ 2B_1,\ 3B_1,$ $B_2,$ $B_1+B_2,\ 2B_1+B_2,\ 3B_1+B_2$	7df	contrasts between blocks
all other combinations of $R,\ B_1,$ and B_2	21 df	contrasts between rows within blocks and row-positions

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Each level of **Position** has a constant level of R and a constant level of C.
Because $L \equiv R + B_1$ and $N = C + B_1 + B_2$,
each level of **Position** has 8 treatments.
Because $2L + 2N \equiv 2R + 2C + 4B_1 + 2B_2 = 2R + 2C$,
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We should like this single degree of freedom confounded with **Position** to be orthogonal to the linear-by-linear component of the N-by-L interaction.

Real levels of L and N

L	0	1	2	3	$\leftarrow$ 3 df
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The confounded contrast is $2L + 2N$.
If N and L both have equally-spaced real levels, and the two extreme levels of each correspond to either notional levels 0, 2 or notional levels 1, 3, then the confounded contrast is the quadratic-by-quadratic component of the N-by-L interaction.

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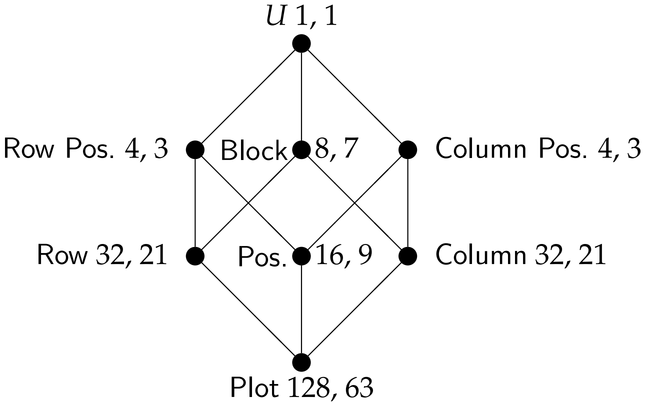
- For validity, strictly speaking we should introduce two 2-level factors $2R$ and $2C$ which distinguish 'outer' from 'inner'. However, this gives 18 strata in the ANOVA, and we do not believe that inner/outer corresponds to any real effect, especially as each block is surrounded by 'guard' pots. So we do not do this.

Hasse diagram for the plot structure



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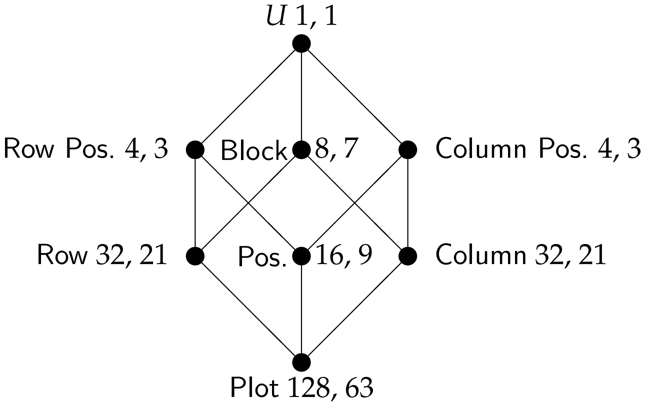
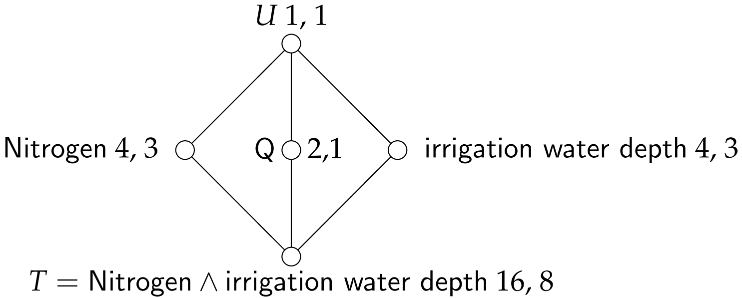


The response on each plot is the mean of the responses on the 4 pots in that plot.

Hasse diagram for the treatment structure



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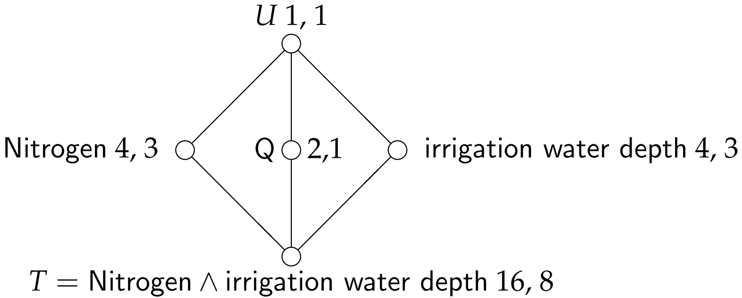

Q $= 2N + 2L$:
the contrast between its 2 levels is quadratic(N) $\times$ quadratic(L).

ANOVA table

stratum	df		
U	1		
Block	7		
Rowpos	3		
Colpos	3		
Position	9		
Row	21		
Column	21		
Plot	63		
Total	128		

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stratum	df	source	df
U	1		
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Position	9	Q	1
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Plot	63	$\mathsf{N} \wedge \mathsf{L} - \mathsf{Q}$	8
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		residual	8
Row	21	L	3
		residual	18
Column	21	N	3
		residual	18
Plot	63	$N \wedge L - Q$	8
		residual	55
Total	128		

ANOVA table

stratum	df	source	df
U	1	mean	1
Block	7	Block	7
Rowpos	3	Rowpos	3
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