Neighbour balance in a strip-block design for an experiment on irrigation

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This is called a strip-block design or criss-cross design.

One block of the design

	2 2	2 2	2 4	2 4	2 1	2 1	2 3	2 3	
	2 2	2 2	2 4	2 4	2 1	2 1	2 3	2 3	
	4 2	4 2	4 4	4 4	4 1	4 1	4 3	4 3	
	4 2	4 2	4 4	4 4	4 1	4 1	4 3	4 3	
	3 2	3 2	3 4	3 4	3 1	3 1	3 3	3 3	
	3 2	3 2	3 4	3 4	3 1	3 1	3 3	3 3	
	1 2	1 2	1 4	1 4	1 1	1 1	1 3	1 3	
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In order not to lose degrees of freedom,
it may be better not to model these neighbour effects explicitly but to use a neighbour-balanced design.

Constructing a neighbour-balanced design

- Start with a special column whose differences are all different modulo 4: $0 - 3 = 1; \quad 3 - 1 = 2; \quad 1 - 2 = 3.$

0
3
1
2

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- Develop this column modulo 4.

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3	0	1	2	3	0	1	2
1	2	3	0	1	2	3	0
2	3	0	1	2	3	0	1

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- Now every level has every other level on the immediate North the same number of times (ditto South).

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3	0	1	2	3	0	1	2
1	2	3	0	1	2	3	0
2	3	0	1	2	3	0	1

Design key modulo 4 to construct the design

factors	N, L (water), R (rows), C (columns), B_1	B_2
levels modulo 4	0, 1, 2, 3	0, 2

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Put $L \equiv R + B_1$ and $N \equiv C + B_1 + B_2$. (This is the design key.)

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One block of the design

This is the block in which $B_1 = 1$ and $B_2 = 2$.

$$\begin{aligned} L &= R + B_1 = R + 1 && \text{(shown first)} \\ N &= C + B_1 + B_2 = C + 3 && \text{(shown second)} \end{aligned}$$

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		C			
		0	3	1	2
R	0	1 3	1 2	1 0	1 1
	3	0 3	0 2	0 0	0 1
	1	2 3	2 2	2 0	2 1
	2	3 3	3 2	3 0	3 1

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Each level of **Position** has 8 treatments.
Because $L \equiv R + B_1$ and $N = C + B_1 + B_2$, these 8 are either those satisfying $2L + 2N = 0$ or those satisfying $2L + 2N = 2$.

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We should like this single degree of freedom confounded with **Position** to be orthogonal to the linear-by-linear component of the N-by-L interaction. If N and L both have equally spaced real levels, and the two extreme levels correspond to either notional levels 0, 2 or notional levels 1, 3, then the confounded contrast is the quadratic-by-quadratic component of the N-by-L interaction.

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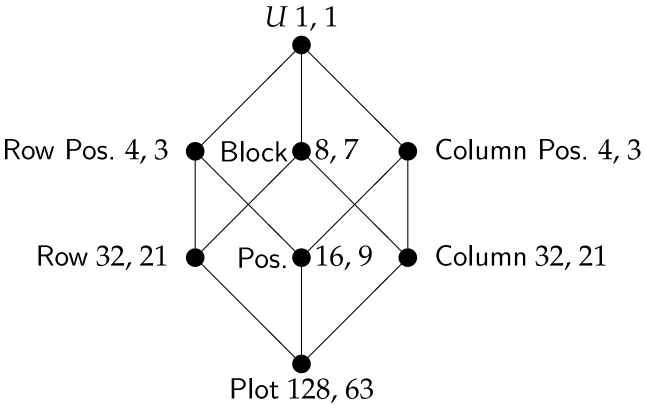
- For validity, strictly speaking we should introduce two 2-level factors $2R$ and $2C$ which distinguish 'outer' from 'inner'. However, this gives 18 strata in the ANOVA, and we do not believe that inner/outer corresponds to any real effect, especially as each block is surrounded by 'guard' pots. So we do not do this.

Hasse diagram for the plot structure



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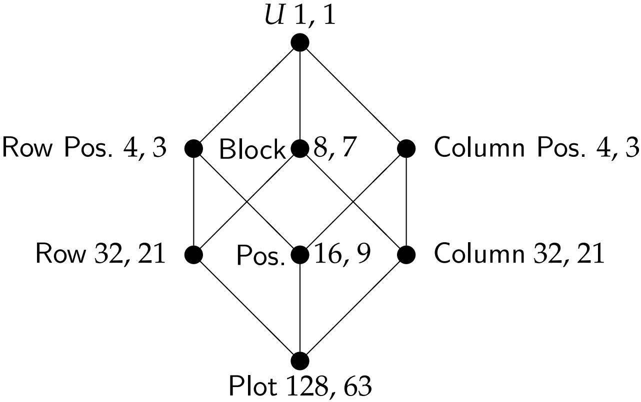
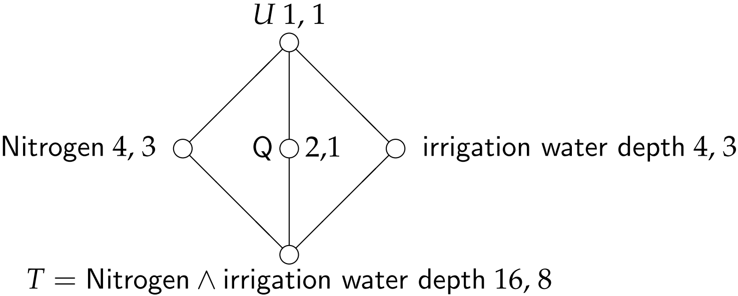


The response on each plot is the mean of the responses on the 4 pots in that plot.

Hasse diagram for the treatment structure



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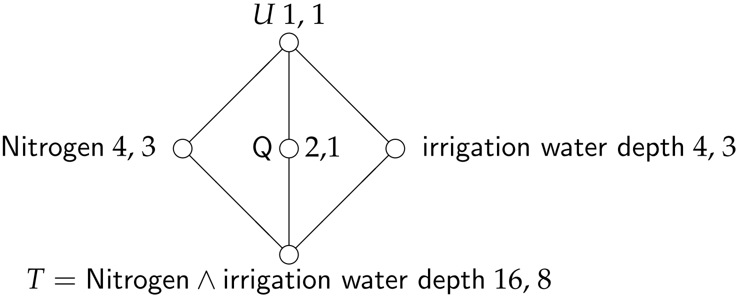

Q $= 2N + 2L$:
the contrast between its 2 levels is quadratic(N) $\times$ quadratic(L).

ANOVA table

stratum	df		
U	1		
Block	7		
Rowpos	3		
Colpos	3		
Position	9		
Row	21		
Column	21		
Plot	63		
Total	128		

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stratum	df	source	df
U	1		
Block	7		
Rowpos	3		
Colpos	3		
Position	9	Q	1
Row	21	L	3
Column	21	N	3
Plot	63	$\mathsf{N} \wedge \mathsf{L} - \mathsf{Q}$	8
Total	128		

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stratum	df	source	df
U	1		
Block	7		
Rowpos	3		
Colpos	3		
Position	9	Q	1
		residual	8
Row	21	L	3
		residual	18
Column	21	N	3
		residual	18
Plot	63	$\mathsf{N} \wedge \mathsf{L} - \mathsf{Q}$	8
		residual	55
Total	128		

ANOVA table

stratum	df	source	df
U	1	mean	1
Block	7	Block	7
Rowpos	3	Rowpos	3
Colpos	3	Colpos	3
Position	9	Q	1
		residual	8
Row	21	L	3
		residual	18
Column	21	N	3
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