Design of Comparative Experiments

R. A. Bailey

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May 2008

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BUT experiments to find out if A is better than B,
and, if so, by how much.

Philosophy

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BUT you cannot build a general theory until the reader has some pegs to hang it on.

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Show the reader that we are going to cover real experiments.
Get the reader thinking about experimental units, observational units, treatments

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Chapter 14 Backward Look
Putting it all together—
reflections that need most of the foregoing

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5. Linear model

Calf-feeding experiment

Calves were housed in pens, with ten calves per pen. Each pen was allocated to a certain type of feed. Batches of this type of feed were put into the pen; calves were free to eat as much of this as they liked. Calves were weighed individually.

Feed D Pen 1 10 calves	Feed C Pen 2 10 calves	Feed D Pen 3 10 calves	Feed B Pen 4 10 calves
Feed B Pen 5 10 calves	Feed A Pen 6 10 calves	Feed A Pen 7 10 calves	Feed C Pen 8 10 calves

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treatment = type of feed experimental unit = pen

observational unit = calf

Running example

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

Running example

0	160	240
160	80	80
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240	240	0

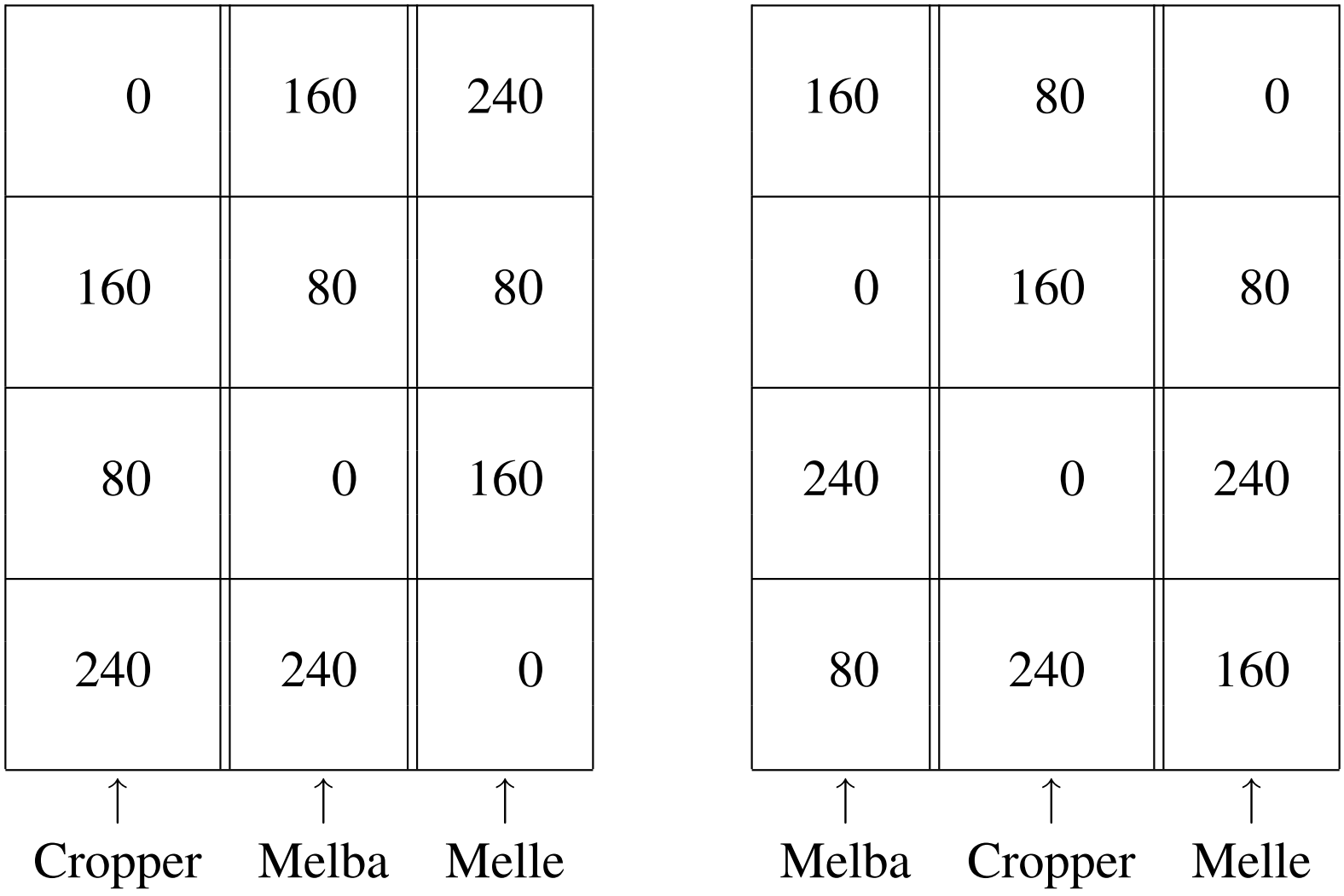
Cropper Melba Melle

160	80	0
0	160	80
240	0	240
80	240	160

Melba Cropper Melle

experimental unit $=$ observational unit $=$ plot

Running example



experimental unit $=$ observational unit $=$ plot

treatment $=$ combination of cultivar and amount of fertilizer

Treatments in the running example

Treatments are all combinations of:

factor	levels
Cultivar (C)	Cropper, Melle, Melba
Fertilizer (F)	0, 80, 160, 240 kg/ha

How many treatments are there?

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How many treatments are there?

Cultivar	Fertilizer			
	0	80	160	240
Cropper	√	√	√	√
Melle	√	√	√	√
Melba	√	√	√	√

Treatments in another example

Treatments are all combinations of:

factor	levels
Timing (T)	early, late
Fertilizer (F)	0, 80, 160, 240 kg/ha

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Timing (T)	early, late
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How many treatments are there?

Timing	Fertilizer			
	0	80	160	240
None	√			
Early		√	√	√
Late		√	√	√

Absolute basics.

First, some notation.

$$\omega = \text{plot} = \text{observational unit}$$

$$T(\omega) = \text{treatment on plot } \omega$$

$$Y_\omega = \text{response on plot } \omega$$

$$\mathrm{E}(Y_\omega) = \tau_{T(\omega)}$$

So if ω is the third plot with treatment 2 then $\mathrm{E}(Y_\omega) = \tau_2$.

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So if ω is the third plot with treatment 2 then $\mathrm{E}(Y_\omega) = \tau_2$.

Calling this response Y_{23}

- ignores the plots;
- encourages non-blindness;
- encourages operation by treatment instead of by inherent factors.

Chapter 2 Unstructured Experiments

- Completely randomized designs
- Why and how to randomize

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How do we randomize? Write down a systematic plan. Then choose a random permutation (from a computer, or shuffle a pack of cards) and apply it to the systematic plan.

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- Linear model, Estimation, Matrix notation
- Sums of squares, Variance
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Source	SS	df	MS	VR
mean	107161.3513	1	107161.3513	13147.39
diets	117.8964	2	58.9482	7.23
residual	236.3723	29	8.1508	–
Total	107515.62	32		

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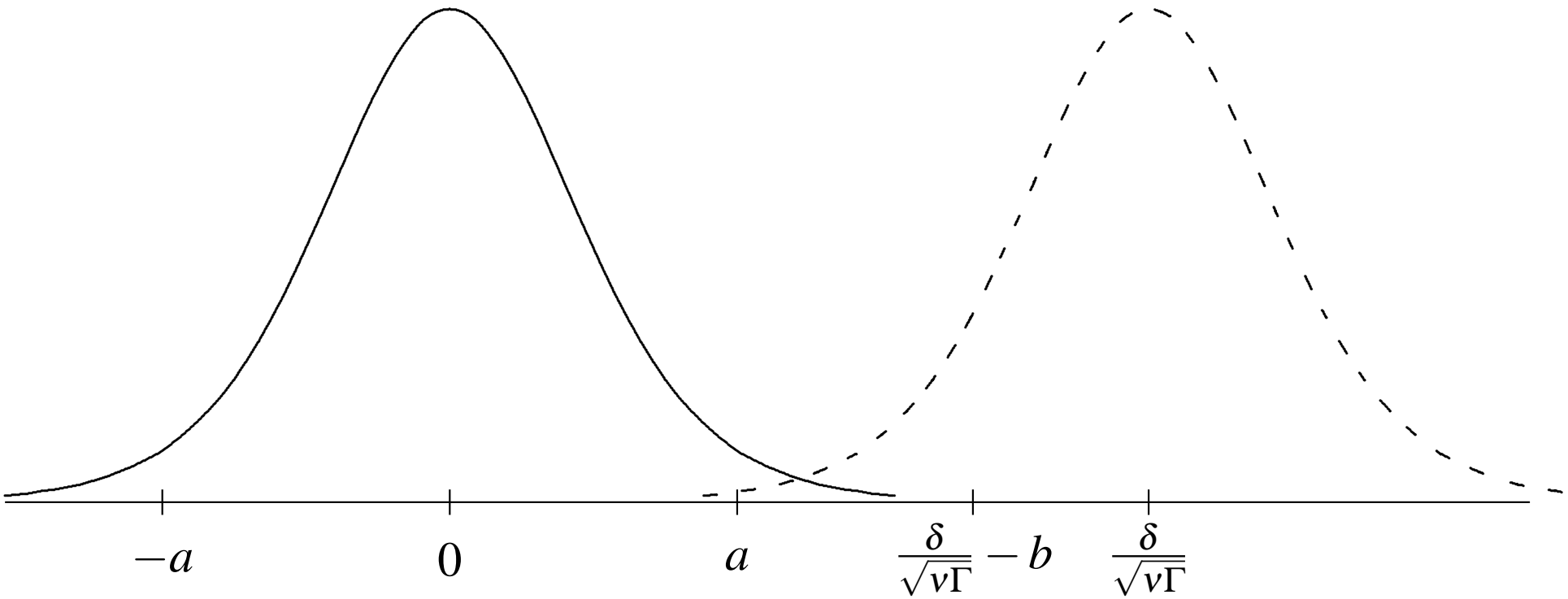
Fitting the grand mean as a submodel of the treatment space is a first taste of what we shall do many times with structured treatments: fit submodels and see what is left over.

Replication for power (two treatments)



Solid curve defines the interval $[-a, a]$ used for the hypothesis test;
dashed curve gives the probability density function of the
test statistic $\Delta/\sqrt{\nu\Gamma}$ if the real difference is δ;
$\Delta =$ estimate of δ; $\Gamma =$ estimate of variance per response;
$\nu =$ sum of reciprocals of replications.

- Replication of control treatments

Chapter 3 Simple Treatment Structures

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- Other treatment groupings
  Repeated splitting of groupings, obtaining nested submodels without the complication of understanding interaction.

Drugs at different stages of development

A pharmaceutical company wants to compare 6 treatments for a certain disease. There are are 3 different doses of formulation A, that has been under development for some time, and 3 different doses (not comparable with the previous 3) of a new formulation B, that has not been so extensively studied.

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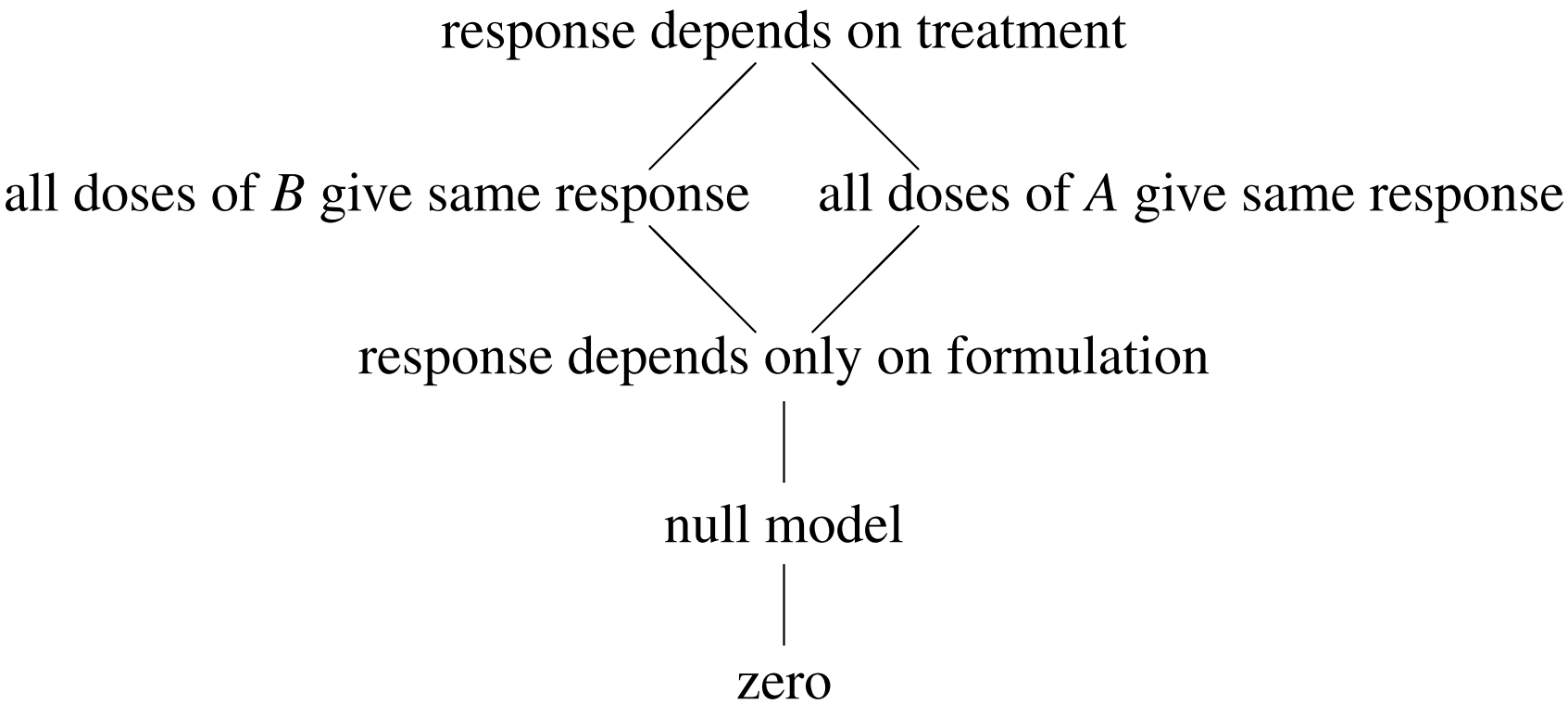

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- Loss of power with blocking

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factor	levels
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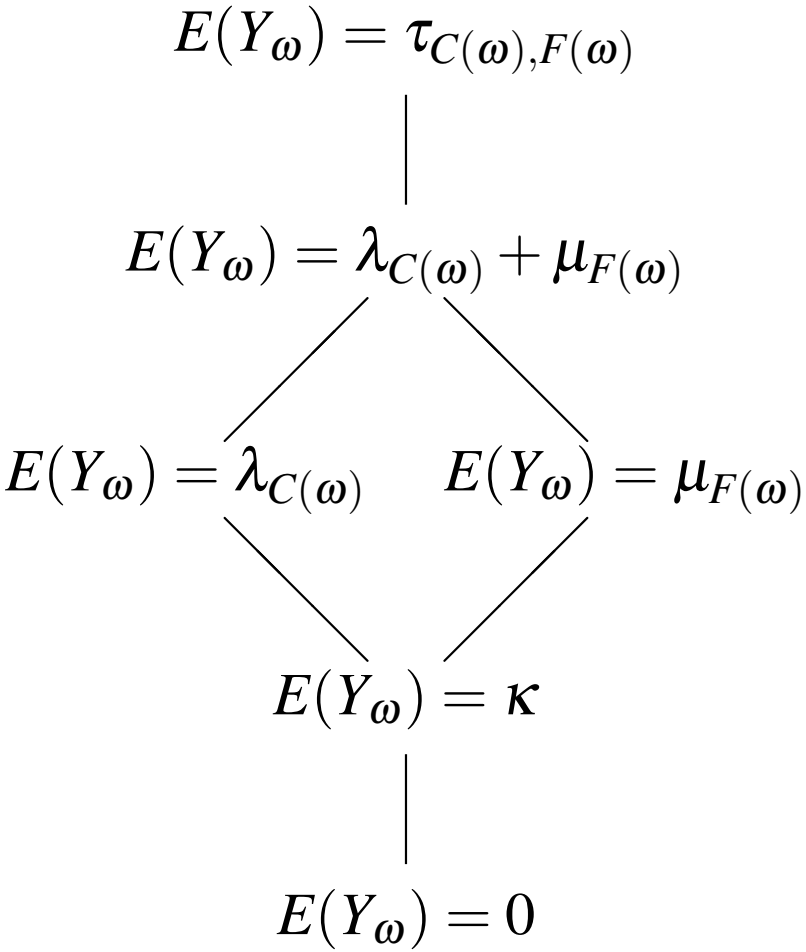

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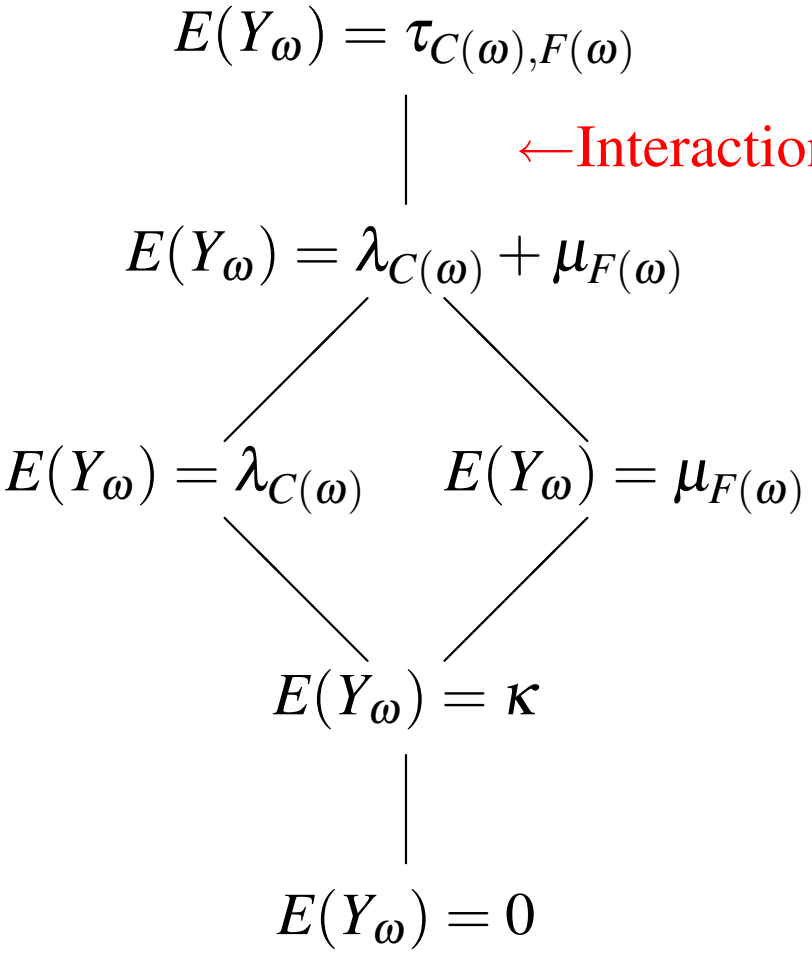

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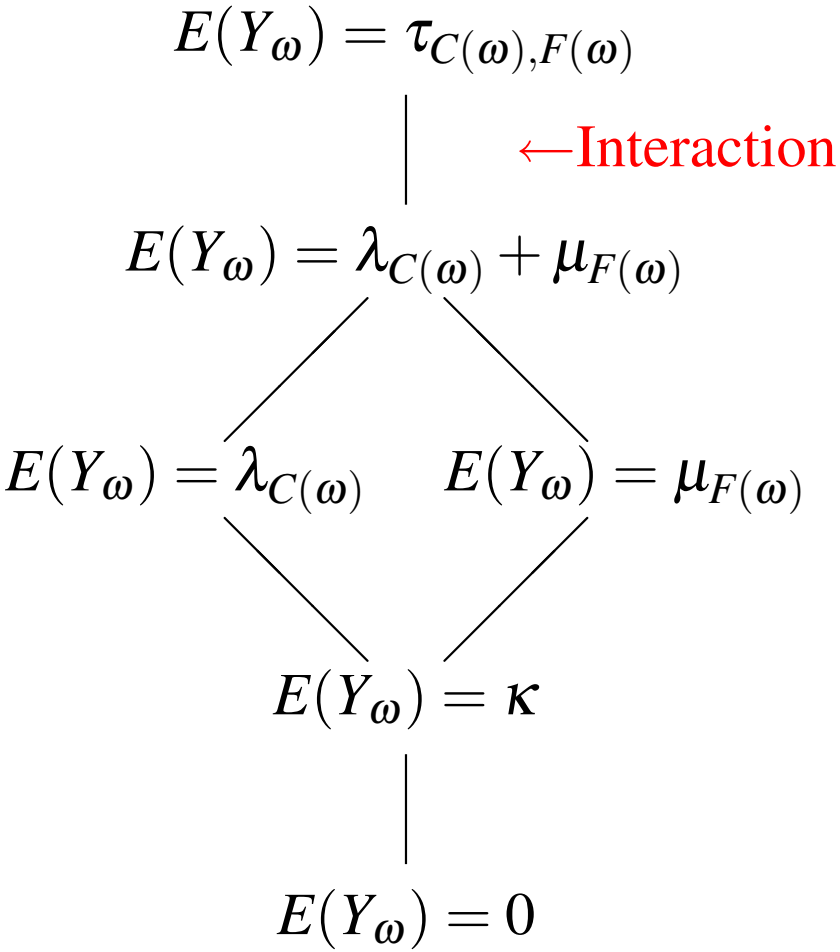

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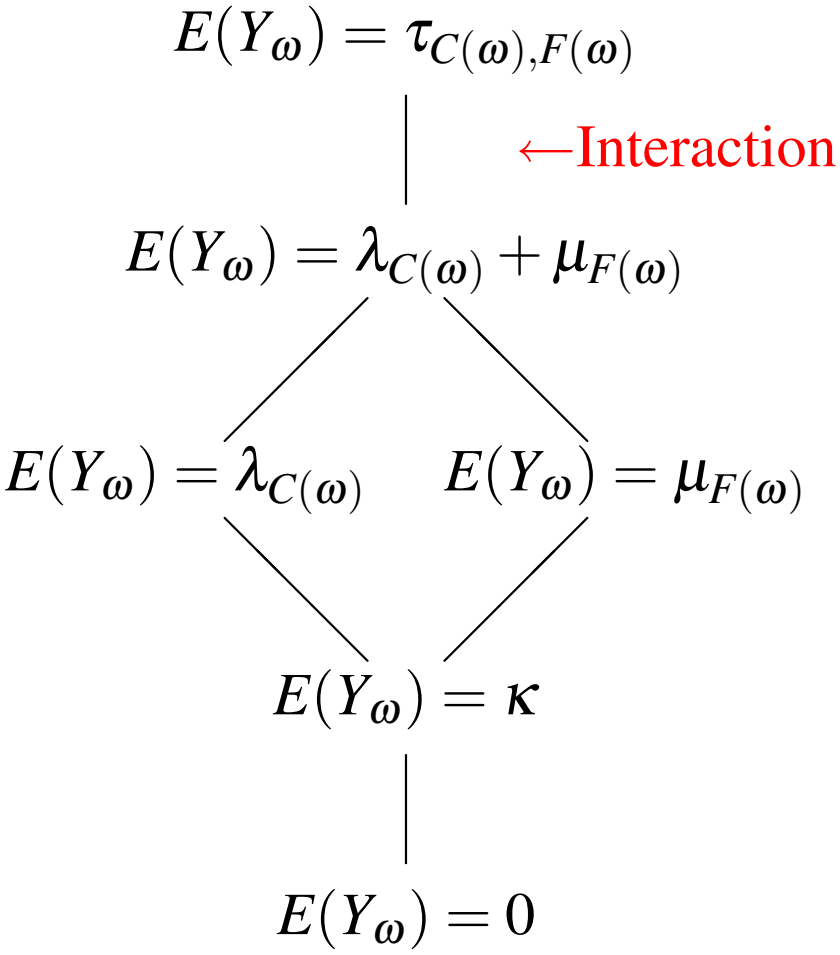
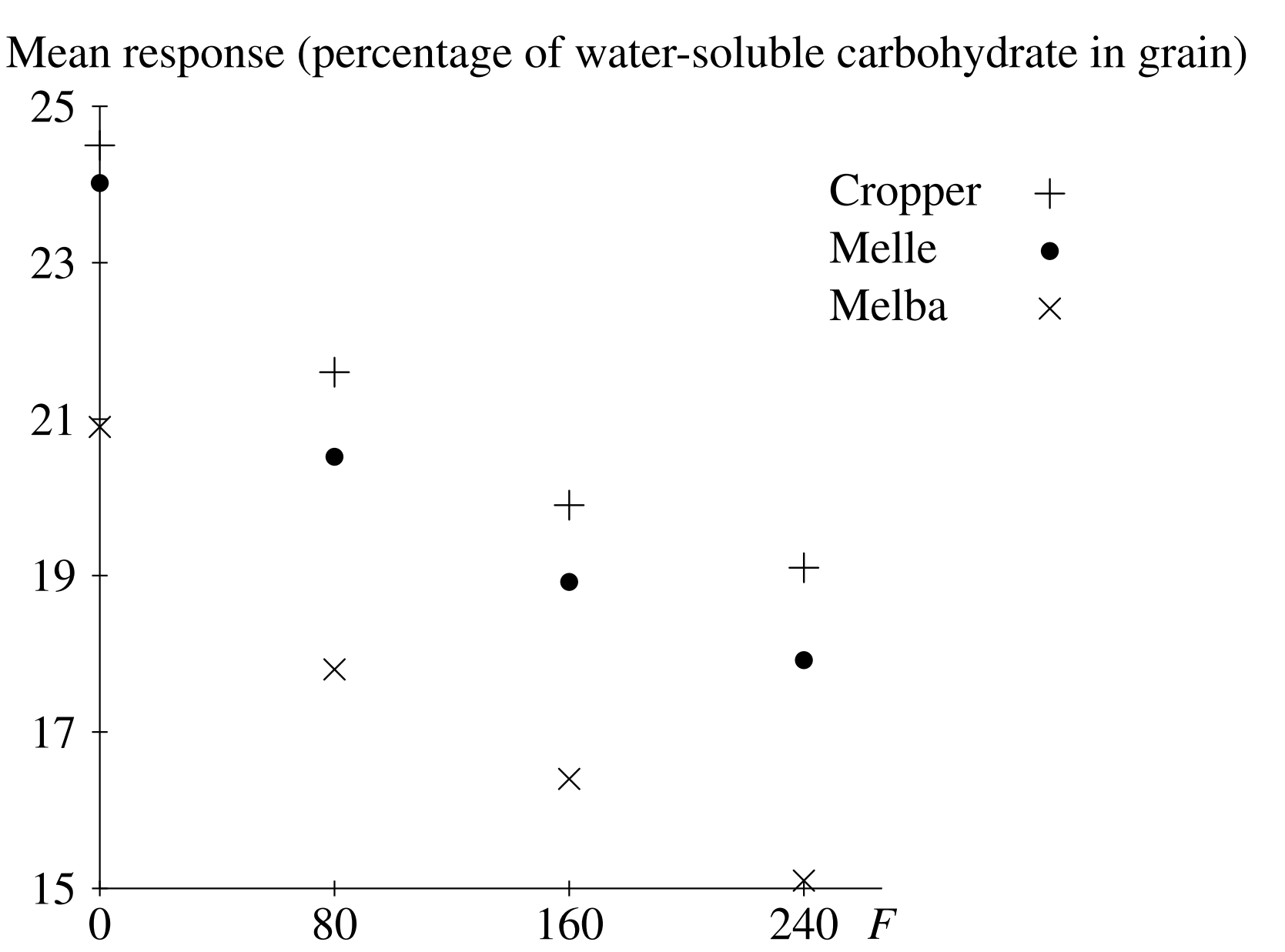
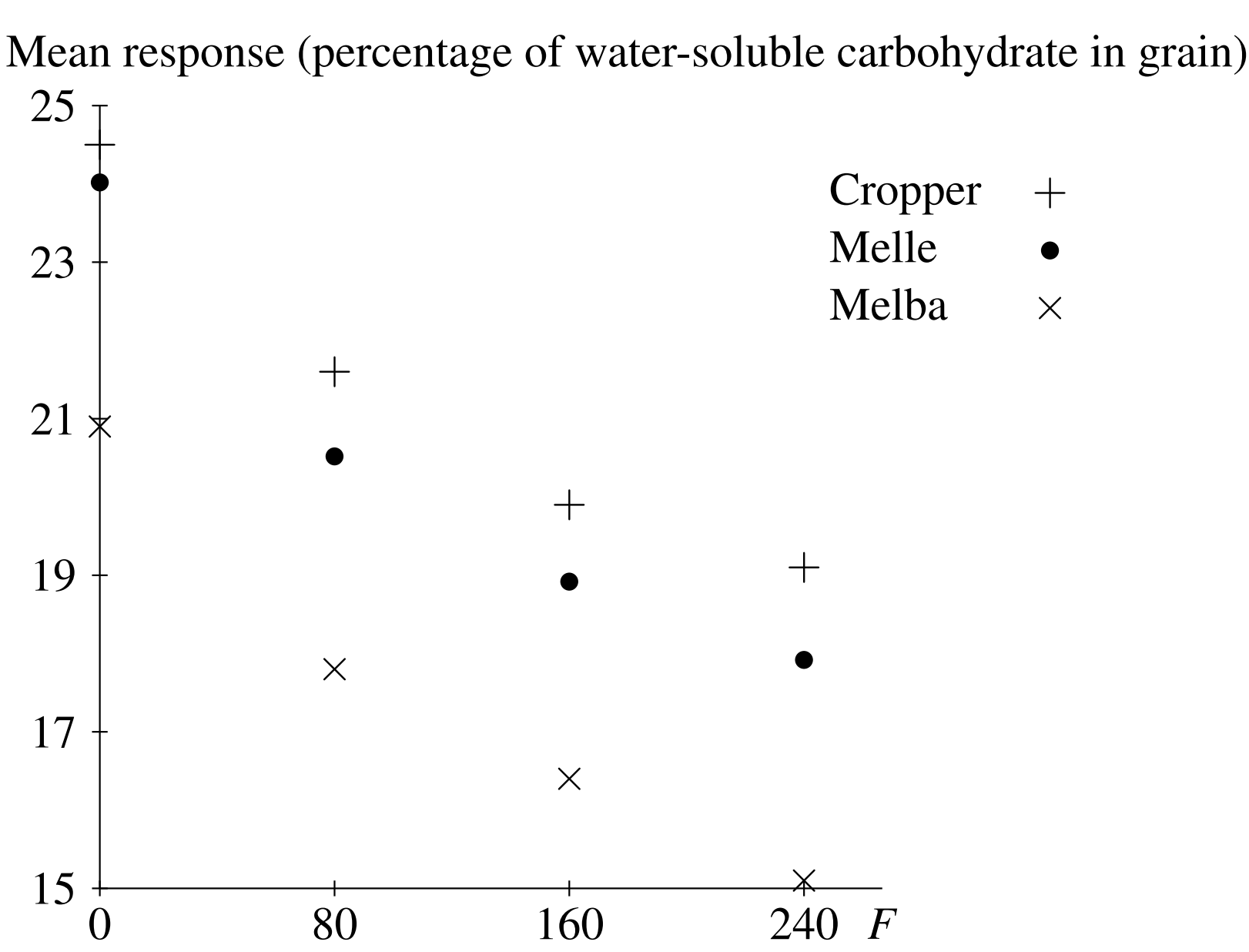
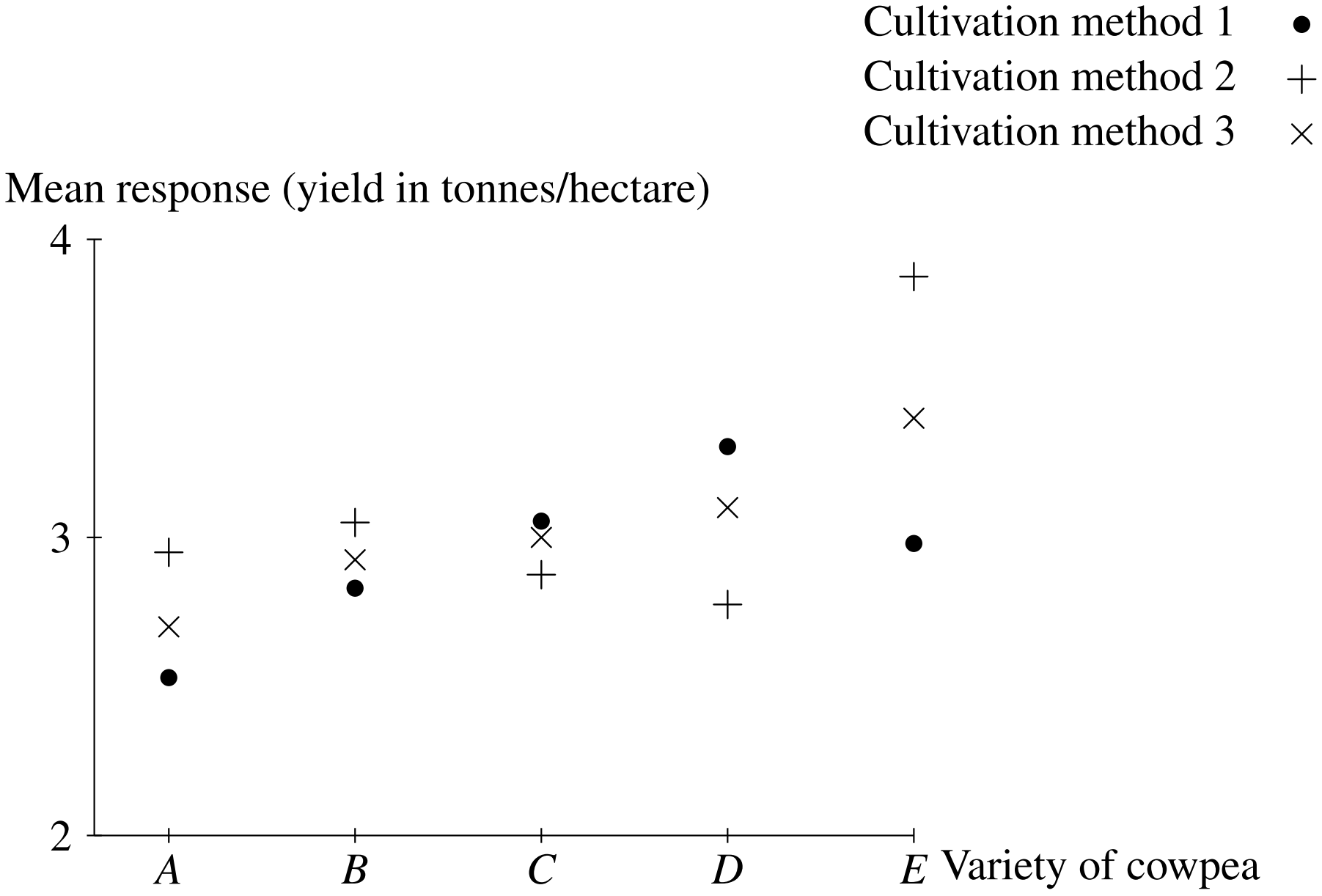


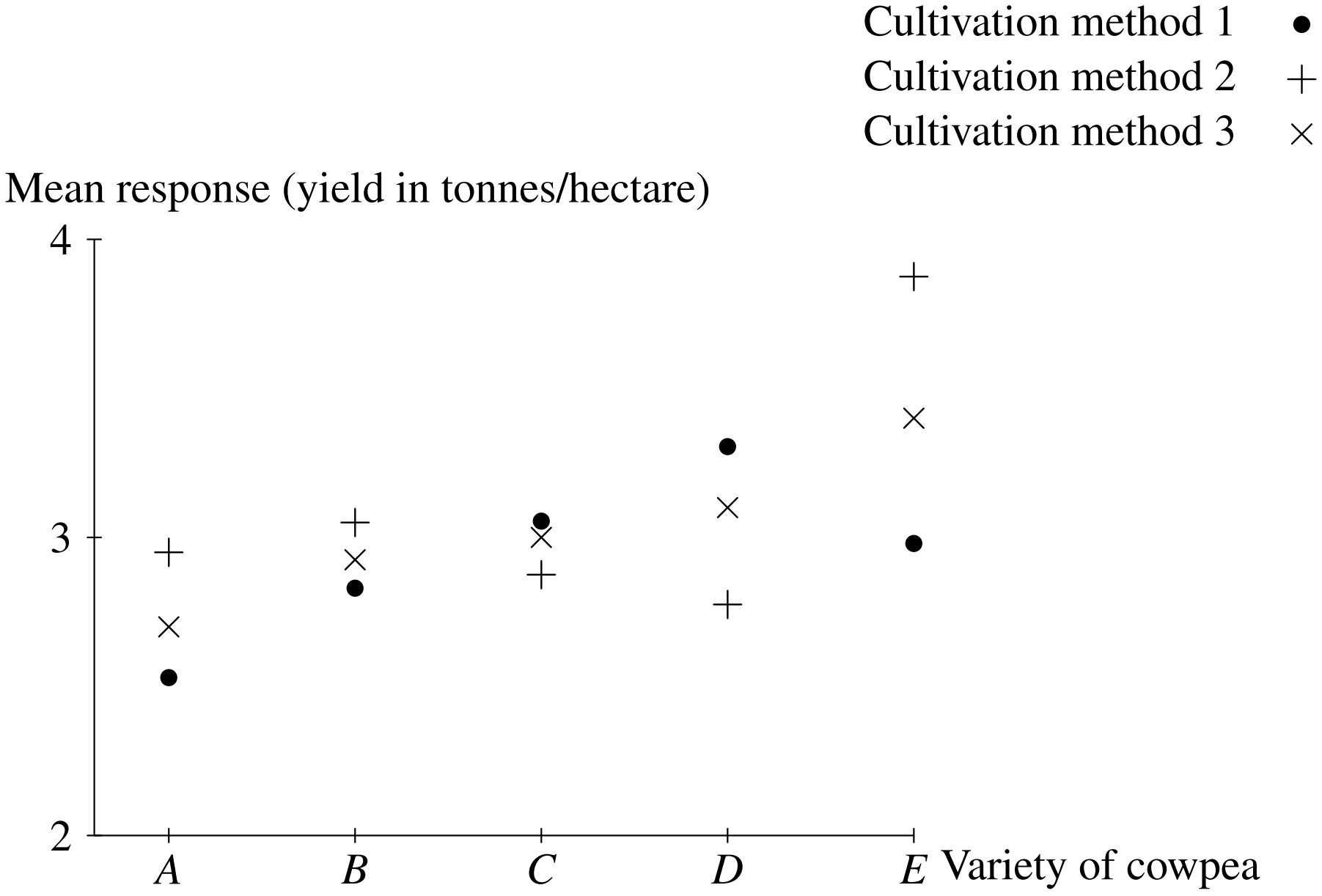
Most books give a single model which has these six models as special cases but which also specializes to some inappropriate models, which your software may let you fit.


Mean response (percentage of water-soluble carbohydrate in grain)
25
23
21
19
17
15
0
80
160
240
F
Cropper
Melle
Melba



The difference between cultivars is (essentially) the same at each quantity of fertilizer—no interaction.


Cultivation method 1
Cultivation method 2
Cultivation method 3
Mean response (yield in tonnes/hectare)
4
3
2
A
B
C
D
E
Variety of cowpea



There is interaction between Variety and Cultivation Method.

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3. There is no need to parametrize the other models.
4. Orthogonality $\Rightarrow$ different routes down the model diagram give consistent results.

Chapter 5 Factorial Treatment Structure

- ...
- Three (or more) treatment factors
- Factorial experiments (benefits)
- Construction and randomization of factorial designs
- Factorial treatments plus control

Double blocking.
Wine-tasting example: treatments are 4 wines

	Judge							
Tasting	1	2	3	4	5	6	7	8
1								
2								
3								
4								

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	Judge							
Tasting	1	2	3	4	5	6	7	8
1	*A*	*B*	*C*	*D*				
2	*D*	*A*	*B*	*C*				
3	*C*	*D*	*A*	*B*				
4	*B*	*C*	*D*	*A*				

a Latin square

Double blocking.
Wine-tasting example: treatments are 4 wines

	Judge							
Tasting	1	2	3	4	5	6	7	8
1	*A*	*B*	*C*	*D*	*C*	*D*	*A*	*B*
2	*D*	*A*	*B*	*C*	*D*	*C*	*B*	*A*
3	*C*	*D*	*A*	*B*	*A*	*B*	*C*	*D*
4	*B*	*C*	*D*	*A*	*B*	*A*	*D*	*C*
	a Latin square				and another			

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Wine-tasting example: treatments are 4 wines

	Judge							
Tasting	1	2	3	4	5	6	7	8
1	*A*	*B*	*C*	*D*	*C*	*D*	*A*	*B*
2	*D*	*A*	*B*	*C*	*D*	*C*	*B*	*A*
3	*C*	*D*	*A*	*B*	*A*	*B*	*C*	*D*
4	*B*	*C*	*D*	*A*	*B*	*A*	*D*	*C*
	a Latin square				and another			

Randomize the (order of) the 4 rows
Randomize the (order of) the 8 columns

Chapter 7 Experiments on People and Animals

- Applications of previous ideas

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  - Ethical issues
    - Best for this patient or best for the trial?
  - Analysis by intention to treat
    - One mouthwash is more effective at preventing gum disease than another, but also more unpleasant, so some subjects may give up taking it.

Chapter 8 Small Units inside Large Units

Feed D Pen 1 10 calves	Feed C Pen 2 10 calves	Feed D Pen 3 10 calves	Feed B Pen 4 10 calves
Feed B Pen 5 10 calves	Feed A Pen 6 10 calves	Feed A Pen 7 10 calves	Feed C Pen 8 10 calves

Chapter 8 Small Units inside Large Units

Feed D Pen 1 10 calves	Feed C Pen 2 10 calves	Feed D Pen 3 10 calves	Feed B Pen 4 10 calves
Feed B Pen 5 10 calves	Feed A Pen 6 10 calves	Feed A Pen 7 10 calves	Feed C Pen 8 10 calves

Stratum	Source	Degrees of freedom
mean	mean	1
pens	feed	3
	residual	4
	total	7
calves	calves	72
Total		80

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Stratum	Source	Degrees of freedom
mean	mean	1
pens	feed	3
	residual	4 no matter how many calves per pen
	total	7
calves	calves	72
Total		80

Modification

The 4 feeds consist of all combinations of

- 2 types of hay, which must be put in whole pens
- 2 types of anti-scour treatment, which are given to calves individually.

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- 2 types of hay, which must be put in whole pens
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Hay 2 Pen 1 5 calves A1 5 calves A2	Hay 2 Pen 2 5 calves A1 5 calves A2	Hay 2 Pen 3 5 calves A1 5 calves A2	Hay 1 Pen 4 5 calves A1 5 calves A2
Hay 1 Pen 5 5 calves A1 5 calves A2	Hay 1 Pen 6 5 calves A1 5 calves A2	Hay 1 Pen 7 5 calves A1 5 calves A2	Hay 2 Pen 8 5 calves A1 5 calves A2

Treatment factors in different strata

Stratum	Source	Degrees of freedom
mean	mean	1
pens	hay	1
	residual	6
	total	7
calves	anti-scour	1
	hay $\wedge$ anti-scour	1
	residual	70
	total	72
Total		80

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Stratum	Source	Degrees of freedom
mean	mean	1
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	residual	6
	total	7
calves	anti-scour	1
	hay $\wedge$ anti-scour	1
	residual	70
	total	72
Total		80

Residual df for hay increase from 4 to 6, so power increases.

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pens	hay	1
	residual	6
	total	7
calves	anti-scour	1
	hay $\wedge$ anti-scour	1
	residual	70
	total	72
Total		80

Residual df for hay increase from 4 to 6, so power increases.
Anti-scour and the interaction have smaller variance (between calves within pens rather than between pens) and substantially more residual df, so power increases.

(Classic) split-plot designs

Like the last one, but arrange the pens in complete blocks.

Chapter 9 More about Latin Squares

Using Latin squares for

- row-column designs
- two treatment factors with n levels each, in n blocks of size n, if it can be assumed that there is no interaction
- three treatment factors with n levels each, in n^2 experimental units, if it can be assumed that there is no interaction

Chapter 10 The Calculus of Factors

A factor F is a function for which we are more interested in knowing whether $F(\alpha) = F(\beta)$ than in knowing the value $F(\alpha)$.

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The universal factor U has just one class.

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The universal factor U has just one class.

The equality factor E has one class per observational unit.

Running example

0	160	240
160	80	80
80	0	160
240	240	0

Cropper Melba Melle

160	80	0
0	160	80
240	0	240
80	240	160

Melba Cropper Melle

Running example

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

$E = \text{plot} \prec \text{strip} \prec \text{field} \prec U$

Running example

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

$E = \text{plot} \prec \text{strip} \prec \text{field} \prec U$

$\text{strip} \prec \text{cultivar}$

Infimum of two factors

Given two factors F and G,
the factor $F \wedge G$ is defined by

$$(F \wedge G)[[\omega]] = F[[\omega]] \cap G[[\omega]].$$

Running example

0	160	240
160	80	80
80	0	160
240	240	0

Cropper Melba Melle

160	80	0
0	160	80
240	0	240
80	240	160

Melba Cropper Melle

$$\text{cultivar} \wedge \text{fertilizer} = \text{treatment}$$

Running example

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

$\text{cultivar} \wedge \text{fertilizer} = \text{treatment}$

$\text{field} \wedge \text{cultivar} = \text{strip}$

Supremum of two factors

Given two factors F and G,
the factor $F \vee G$ is the finest factor whose classes are unions of F-classes and unions of G-classes.

Supremum of two factors

Given two factors F and G,
the factor $F \vee G$ is the finest factor whose classes are
unions of F-classes and unions of G-classes.

If you try to fit F and G in a linear model, you will get into trouble
unless you fit $F \vee G$ first.

Running example

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

$\text{field} \vee \text{fertilizer} = U$

Running example

0	160	240
160	80	80
80	0	160
240	240	0

↑ Cropper ↑ Melba ↑ Melle

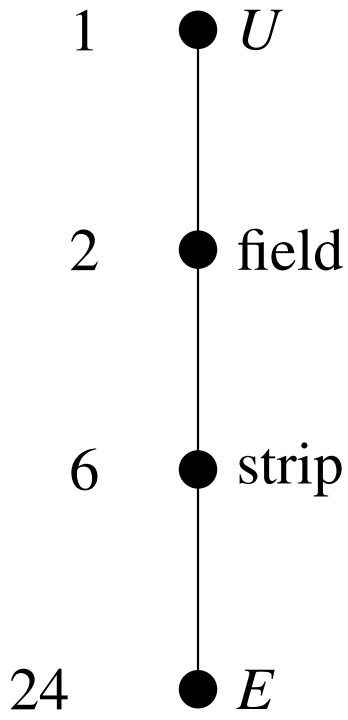
160	80	0
0	160	80
240	0	240
80	240	160

↑ Melba ↑ Cropper ↑ Melle

$\text{field} \vee \text{fertilizer} = U$

$\text{strip} \vee \text{treatment} = \text{cultivar}$

Hasse diagram for factors on the observational units



How many of each are there?

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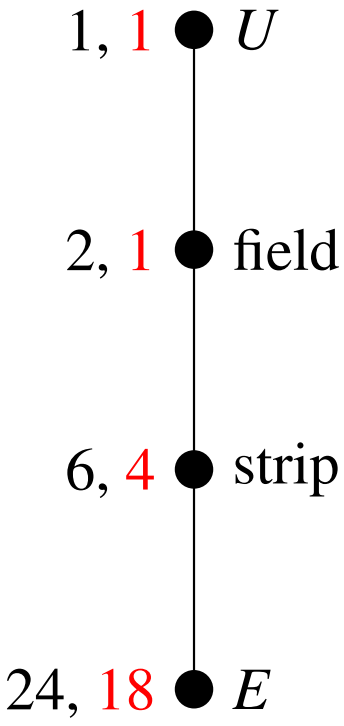
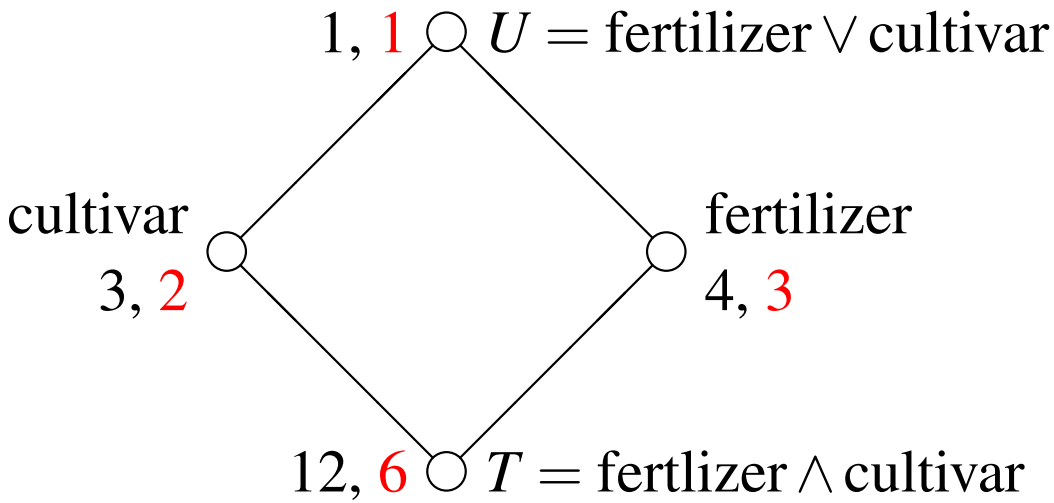


How many of each are there?

Degrees of freedom calculated by subtraction

Hasse diagram for factors on the treatments



Factorial treatments plus control

dose	type				
	Z	S	K	M	N
none	$\surd$				
single		$\surd$	$\surd$	$\surd$	$\surd$
double		$\surd$	$\surd$	$\surd$	$\surd$

Factorial treatments plus control

dose	type				
	Z	S	K	M	N
none	$\surd$				
single		$\surd$	$\surd$	$\surd$	$\surd$
double		$\surd$	$\surd$	$\surd$	$\surd$

$$\text{dose} \vee \text{type} = \text{fumigant}$$

Factorial treatments plus control

dose	type				
	Z	S	K	M	N
none	$\surd$				
single		$\surd$	$\surd$	$\surd$	$\surd$
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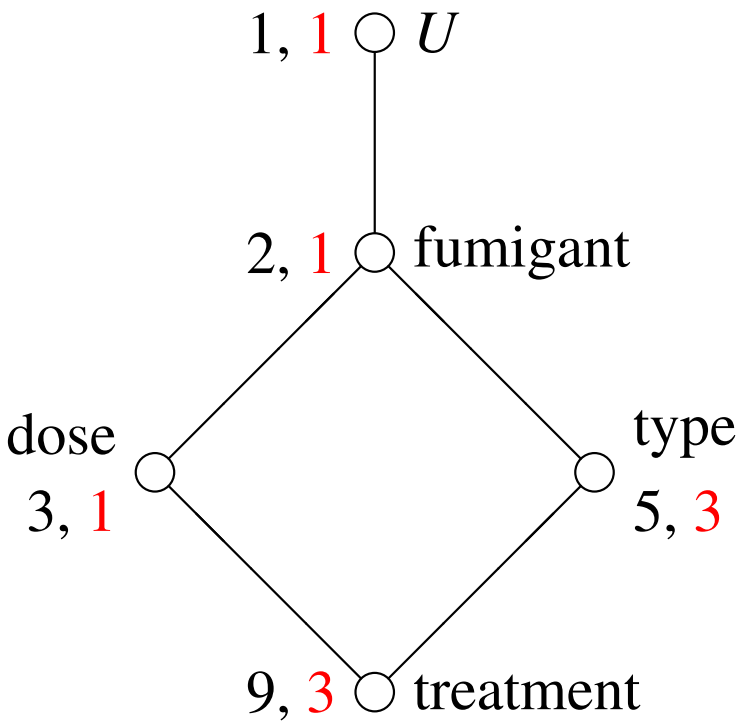
$$\text{dose} \vee \text{type} = \text{fumigant}$$



Hence a complete theory for orthogonal designs,
including the location of treatment subspaces in the correct strata.

This covers everything so far,
and there are many further examples.

Chapter 11 Incomplete-Block Designs

Blocks are incomplete if

- the block size is less than the number of treatments
- no treatment occurs more than once in any block.

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Balanced incomplete-block designs and square lattice designs.

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- the block size is less than the number of treatments
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Balanced incomplete-block designs and square lattice designs.

Inserting a control treatment in every block.

If the number of blocks is equal to the number of treatments, algorithm to arrange the blocks as the columns of a row-column design in such a way that each treatment occurs once per row.

Combining the above.

Characters	Treatments								
A	0	0	0	1	1	1	2	2	2
B	0	1	2	0	1	2	0	1	2
$A+B$	0	1	2	1	2	0	2	0	1
$A+2B$	0	2	1	1	0	2	2	1	0
$2A+B$	0	1	2	2	0	1	1	2	0
$2A+2B$	0	2	1	2	1	0	1	0	2
$2A$	0	0	0	2	2	2	1	1	1
$2B$	0	2	1	0	2	1	0	2	1
I	0	0	0	0	0	0	0	0	0

Characters	Treatments								
A	0	0	0	1	1	1	2	2	2
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$A+B$	0	1	2	1	2	0	2	0	1
$A+2B$	0	2	1	1	0	2	2	1	0
$2A+B$	0	1	2	2	0	1	1	2	0
$2A+2B$	0	2	1	2	1	0	1	0	2
$2A$	0	0	0	2	2	2	1	1	1
$2B$	0	2	1	0	2	1	0	2	1
I	0	0	0	0	0	0	0	0	0

$A \equiv 2A$	main effect of A
$B \equiv 2B$	main effect of B
$A+B \equiv 2A+2B$	2 degrees of freedom for the A-by-B interaction
$A+2B \equiv 2A+B$	2 degrees of freedom for the A-by-B interaction, orthogonal to the previous 2

Characters	Treatments								
A	0	0	0	1	1	1	2	2	2
B	0	1	2	0	1	2	0	1	2
$A+B$	0	1	2	1	2	0	2	0	1
$A+2B$	0	2	1	1	0	2	2	1	0
$2A+B$	0	1	2	2	0	1	1	2	0
$2A+2B$	0	2	1	2	1	0	1	0	2
$2A$	0	0	0	2	2	2	1	1	1
$2B$	0	2	1	0	2	1	0	2	1
I	0	0	0	0	0	0	0	0	0

$A \equiv 2A$	main effect of A
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$A+B \equiv 2A+2B$	2 degrees of freedom for the A-by-B interaction
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For 3 blocks of size 3, can alias blocks with any character.

Chapter 13 Fractional Factorial Designs

A factorial design is a fractional replicate if not all possible combinations of the treatment factors occur.

A fractional replicate can be useful if there are a large number of treatment factors to investigate and we can assume that some interactions are zero.

Chapter 9 constructed some fractional replicate designs from Latin squares.

Here we use characters to give us more types of fractional replicate.

Includes quantile plots for analysis.

Chapter 14 Backward Look

1. Randomization
2. Factors such as time, sex, age and breed—
   Are they treatment factors or plot factors?
3. Writing a protocol
4. ...

Examples, Questions and Exercises

Not all the examples are agricultural.

Almost all of the examples in this book are real.
On the other hand, almost none of them is the whole truth.

Each chapter ends with questions for discussion:
there is no single correct answer.

There are more general exercises at the end.

Sources of all these are given, as far as possible.

A question from Chapter 1

Several studies have suggested that drinking red wine gives some protection against heart disease, but it is not known whether the effect is caused by the alcohol or by some other ingredient of red wine. To investigate this, medical scientists enrolled 40 volunteers into a trial lasting 28 days. For the first 14 days, half the volunteers drank two glasses of red wine per day, while the other half had two standard drinks of gin. For the remaining 14 days the drinks were reversed: those who had been drinking red wine changed to gin, while those who had been drinking gin changed to red wine. On days 14 and 28, the scientists took a blood sample from each volunteer and measured the amount of inflammatory substance in the blood.

Identify the experimental units and observational units. How many are there of each? What is the plot structure?

What are the treatments? What is the treatment structure?

A question from Chapter 5

A group of people researching ways to reduce the risk of blood clotting are planning their next experiments. One says:

> *We know that aspirin thins the blood. Let's experiment with the quantity of aspirin. We could enrol about 150 healthy men into the trial, give 50 of them one aspirin tablet per day for a year, another 50 one and a half aspirin tablets a day, and the final 50 will get two aspirin tablets per day.*
> *When we have decided which quantity is best, we can run another trial to find out if there is any difference between taking the aspirin after breakfast or after dinner.*

How do you reply?

A question from Chapter 11

A horticulture research institute wants to compare nine methods of treating a certain variety of houseplant while it is being grown in a greenhouse in preparation for the Christmas market. One possibility is to ask twelve small growers to test three treatments each in separate chambers in their greenhouses. A second possibility is to ask three large commercial growers to test nine methods each, also in separate greenhouse chambers.

1. Construct a suitable design for the first possibility.
2. Randomize this design.
3. If the plots stratum variance is the same in both cases, which design is more efficient?
4. Compare the designs in terms of likely cost, difficulty and representativeness of the results.

The End

Thank you