Queen Mary
University of London

MAS 108 Probability I

Test **11 November 2005, 1610–1655**

Write your name and student number in the spaces below.

Answer all questions. Write all your answers in the boxes provided.

Electronic calculators must not be used.

Name: _____

Student Number: _____

1 (10 marks) In a certain town, 60% of the population read the *Sun*, 30% read the *Independent*, and 5% read both the *Sun* and the *Independent*. A person is chosen at random in that town. What is the conditional probability that the chosen person reads the *Sun* given that the person reads the *Independent*?

Let $S =$ "read Sun" and $I =$ "read Independent".

$$P(S \mid I) = \frac{P(S \cap I)}{P(I)} = \frac{5\%}{30\%} = \frac{1}{6}$$

2 (15 marks) A student keeps taking an examination until he passes. At each attempt the probability that he passes is $2/3$, independent of all other attempts.

(a) Write down the sample space.

$\mathcal{S} = \{P, FP, FFP, FFFP, \ldots\}$, where $P =$ pass and $F =$ fail.

(b) Write down the probability that he passes at the third attempt.

$$P(FFP) = \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}.$$

(c) Calculate the probability that he passes within the first four attempts.

$$P(\text{passes within first 4 attempts}) = 1 - P(\text{fails 4 times}) = 1 - \left(\frac{1}{3}\right)^4 = \frac{80}{81}.$$

3 (20 marks) A couple has four children. Each child has probability p of being a boy, independent of all other children, where $0 < p < 1$.

(a) Find the probability that they have exactly 2 boys.

$$^4\mathrm{C}_2 p^2 q^2 = 6p^2q^2$$

(b) Find the probability that they have exactly 3 boys.

$$^4\mathrm{C}_3 p^3 q = 4p^3q$$

(c) For which values of p are they more likely to have 2 boys than 3 boys?

$6p^2q^2 > 4p^3q \Rightarrow 3q > 2p$ (because $p^2q > 0$)
$\Rightarrow 3(1-p) > 2p \Rightarrow 3 - 3p > 2p \Rightarrow 3 > 5p \Rightarrow p < \frac{3}{5}$

4 (15 marks) A fish is caught at random on the Great Barrier Reef. The probability that the fish is striped is $7/20$; the probability that the fish is luminous is $1/5$; and the probability that the fish is both striped and luminuous is $1/20$.

Find the probability that the fish is neither striped nor luminous.

Let $S =$ "striped" and $L =$ "luminous".

$$P(S \cup L) = P(S) + P(L) - P(S \cap L) = \frac{7}{20} + \frac{1}{5} - \frac{1}{20} = \frac{10}{20} = \frac{1}{2}.$$

$$P(\text{neither striped nor luminous}) = P((S' \cap L')) = P((S \cup L)') = 1 - P(S \cup L) = \frac{1}{2}.$$

5 (20 marks) Let A and B be events with $0 < P(A) < 1$ and $0 < P(B) < 1$.

(a) State what it means for A and B to be *independent* of each other.

$P(A \cap B) = P(A) \times P(B)$

(b) Prove that if A and B are independent of each other then A and B' are independent of each other.

$A \cap B' = A \setminus A \cap B$, so $A \cap B'$ and $A \cap B$ are disjoint and their union is A, so

$$P(A \cap B') + P(A \cap B) = P(A),$$

by Axiom 3. Therefore

$$P(A \cap B') = P(A) - P(A \cap B) = P(A) - P(A)P(B),$$

because A and B are indepedent, so

$$P(A \cap B') = P(A)(1 - P(B)) = P(A)P(B'),$$

by Proposition 1. Since $P(A \cap B') = P(A) \times P(B')$, the events A and B' are independent.

6 (10 marks) A discrete random variable X has the following probability mass function.

x	1	5
$p(x)$	$\frac{3}{4}$	$\frac{1}{4}$

(a) Find $E(X)$.

$$E(X) = 1 \times \frac{3}{4} + 5 \times \frac{1}{4} = \frac{3+5}{4} = 2.$$

(b) Find $\mathrm{Var}(X)$.

$$E(X^2) = 1 \times \frac{3}{4} + 25 \times \frac{1}{4} = \frac{3+25}{4} = 7,$$

so

$$\mathrm{Var}(X) = E(X^2) - (E(X))^2 = 7 - 2^2 = 3.$$

7 (10 marks) There are nine sheep and seven goats in a field. The farmer randomly chooses three of these animals to show to a visitor. Find the probability that exactly two of the chosen animals are sheep.

$$P(\text{exactly 2 sheep are chosen}) = \frac{{}^{9}\mathrm{C}_2 \times {}^{7}\mathrm{C}_1}{{}^{16}\mathrm{C}_3} = \frac{9 \times 8}{2} \times 7 \times \frac{3 \times 2}{16 \times 15 \times 14} = \frac{9}{20}.$$