



MAS 108

Probability I

Test

11 November 2005, 1610–1655

Write your name and student number in the spaces below.

Answer all questions. Write all your answers in the boxes provided.

Electronic calculators must not be used.

Name: _____

Student Number: _____

1 (10 marks) In a certain town, 60% of the population read the *Sun*, 30% read the *Independent*, and 5% read both the *Sun* and the *Independent*. A person is chosen at random in that town. What is the conditional probability that the chosen person reads the *Sun* given that the person reads the *Independent*?

2 (15 marks) A student keeps taking an examination until he passes. At each attempt the probability that he passes is $\frac{2}{3}$, independent of all other attempts.

(a) Write down the sample space.

(b) Write down the probability that he passes at the third attempt.

(c) Calculate the probability that he passes within the first four attempts.

3 (20 marks) A couple has four children. Each child has probability p of being a boy, independent of all other children, where $0 < p < 1$.

(a) Find the probability that they have exactly 2 boys.

(b) Find the probability that they have exactly 3 boys.

(c) For which values of p are they more likely to have 2 boys than 3 boys?

4 (15 marks) A fish is caught at random on the Great Barrier Reef. The probability that the fish is striped is $7/20$; the probability that the fish is luminous is $1/5$; and the probability that the fish is both striped and luminous is $1/20$.

Find the probability that the fish is neither striped nor luminous.

5 (20 marks) Let A and B be events with $0 < P(A) < 1$ and $0 < P(B) < 1$.

(a) State what it means for A and B to be *independent* of each other.

(b) Prove that if A and B are independent of each other then A and B' are independent of each other.

6 (10 marks) A discrete random variable X has the following probability mass function.

x	1	5
$p(x)$	$\frac{3}{4}$	$\frac{1}{4}$

(a) Find $E(X)$.

(b) Find $\text{Var}(X)$.

7 (10 marks) There are nine sheep and seven goats in a field. The farmer randomly chooses three of these animals to show to a visitor. Find the probability that exactly two of the chosen animals are sheep.