QUEEN MARY, UNIVERSITY OF LONDON

MAS 108

Solutions 7

Probability I

Autumn 2005

Here is a possible solutions to Question 5 on Assignment 7.

- (a) If half the full-time workforce is male and one quarter is female, what is the remaining quarter? The people who wrote this cannot add up.
- (b) Choose a worker at random. Let M = "male", F = "female" and T = "full-time". Then the Theorem of Total Probability gives

$$P(T) = P(M) \times P(T \mid M) + P(F) \times P(T \mid F)$$

= $\frac{55}{100} \times \frac{91}{100} + \frac{45}{100} \times \frac{55}{100} = \frac{55 \times 136}{10000}.$

By Bayes' Theorem,

$$P(M \mid T) = \frac{P(T \mid M) \times P(M)}{P(T)} = \frac{\frac{55}{100} \times \frac{91}{100}}{\frac{55 \times 136}{10000}} = \frac{91}{136}$$

Then

$$P(F \mid T) = 1 - P(M \mid T) = \frac{45}{136}$$

In words, the proportion of the full-time workforce that is male is 91/136 and the proportion that is female is 45/136.

(c) They seem to have calculated

$$P(T \cap M) = P(M) \times P(T \mid M) = \frac{55}{100} \times \frac{91}{100} \approx 0.5005$$

and

$$P(T \cap F) = P(F) \times P(T \mid F) = \frac{45}{100} \times \frac{55}{100} \approx 0.2475$$

In words, half the workforce is full-time male and one quarter of the workforce is full-time female. This makes it clear that the missing quarter of the workforce consists of the part-time workers.

It seems that the authors confused P(M | T) with $P(M \cap T)$.