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MAS 108 Probability I

Solutions 4 Autumn 2005

Here is one possible solution to Question 5 on Assignment 4.

(a) Sally's parents must both be NC or CN. Let $S =$ "Sally has at least one C gene" and $T =$ "Sally does not have cystic fibrosis". Before Sally is born, $P(T) = 1 - P(\text{Sally is } CC) = 3/4$ and $P(S \cap T) = P(\text{Sally is } NC \text{ or } CN) = 1/2$. We *know* that T is true, so we are really being asked for the conditional probablity

$$P(S \mid T) = \frac{P(S \cap T)}{P(T)} = \frac{2}{3}.$$

(b) Let $M =$ "Michael has at least one C gene" and $N =$ "Michael does not have cystic fibrosis". In the general population, the probability that each gene is C is $1/50$ and the probability that it is N is $49/50$. So if we knew nothing about Michael we would say that $P(M) = 1 - \left(\frac{1}{50}\right)^2 = \frac{49\times 51}{2500}$ and $P(M \cap N) = 2 \times \frac{49}{50} \times \frac{1}{50} = \frac{2\times 49}{2500}$. Once again we are really being asked for the condtional probability

$$P(M \mid N) = \frac{P(M \cap N)}{P(N)} = \frac{2}{51}.$$

(c) Let $B =$ "child has cystic fibrosis". Then $B \subseteq S \cap M$. So

$$P(B \mid T \cap N) = P(B \cap S \cap M \mid T \cap N) = \frac{P(B \cap S \cap M \cap T \cap N)}{P(T \cap N)}$$

$$= \frac{P(B \mid S \cap M \cap T \cap N) P(S \cap M \cap T \cap N)}{P(T \cap N)} = \frac{1}{4} \frac{P(S \cap T) P(M \cap N)}{P(T) P(N)},$$

because Sally and Michael inherit their genes independently,

$$= \frac{1}{4} P(S \mid T) P(M \mid N) = \frac{1}{4} \times \frac{2}{3} \times \frac{2}{51} = \frac{1}{153}.$$