QUEEN MARY, UNIVERSITY OF LONDON

MAS 108 Probability I

Solutions 10 Autumn 2005

Here are possible solutions to Questions 1 and 2 on Assignment 10. If your method of solution is different it may still be correct. Come and ask me if you are not sure.

1 (a)

$$P(Y = 1 \text{ and } Z = 1) = P(1 \text{ yellow, } 1 \text{ blue, } 1 \text{ red}) = \frac{{}^{5}C_1 \times {}^{5}C_1 \times {}^{5}C_1}{{}^{15}C_3} = \frac{25}{91}.$$

The other entries are calculated similarly. Six of them are

$$\frac{{}^{5}C_0 \times {}^{5}C_1 \times {}^{5}C_2}{{}^{15}C_3} = \frac{10}{91}$$

and the remaining three are

$$\frac{{}^{5}C_0 \times {}^{5}C_0 \times {}^{5}C_3}{{}^{15}C_3} = \frac{2}{91}.$$

These give the joint probability mass function for Y and Z in this table.

		values of Z			
		0	1	2	3
	0	$\frac{2}{91}$	$\frac{10}{91}$	$\frac{10}{91}$	$\frac{2}{91}$
values	1	$\frac{10}{91}$	$\frac{25}{91}$	$\frac{10}{91}$	0
of Y	2	$\frac{10}{91}$	$\frac{10}{91}$	0	0
	3	$\frac{2}{91}$	0	0	0

(b) The row totals give the probability mass function of Y as follows.

y	0	1	2	3
$P(Y = y)$	$\frac{24}{91}$	$\frac{45}{91}$	$\frac{20}{91}$	$\frac{2}{91}$

Then

$$E(Y) = \frac{1 \times 45 + 2 \times 20 + 3 \times 2}{91} = 1$$

and

$$E(Y^2) = \frac{1 \times 45 + 4 \times 20 + 9 \times 2}{91} = \frac{11}{7},$$

so

$$\text{Var}(Y) = \frac{11}{7} - 1^2 = \frac{4}{7}.$$

(c) (i)

$$E(YZ) = \frac{1 \times 1 \times 25 + 1 \times 2 \times 10 + 2 \times 1 \times 10}{91} = \frac{5}{7}$$

so

$$\text{Cov}(Y, Z) = E(YZ) - E(Y)E(Z) = \frac{5}{7} - 1^2 = -\frac{2}{7}.$$

Are you surprised that this is negative?

(ii)

$$\text{corr}(Y, Z) = \frac{\text{Cov}(Y, Z)}{\sqrt{\text{Var}(Y)\,\text{Var}(Z)}} = \frac{-\frac{2}{7}}{\frac{4}{7}} = -\frac{1}{2}.$$

(d) The probability mass function for Z is the same as the pmf for Y, so $E(Z) = E(Y)$ and $\text{Var}(Z) = \text{Var}(Y)$.

(i) $E(Y - Z) = E(Y) - E(Z) = 0$.

(ii) $\text{Var}(Y - Z) = \text{Var}(Y) - 2\,\text{Cov}(Y, Z) + \text{Var}(Y) = \frac{4}{7} + 2 \times \frac{2}{7} + \frac{4}{7} = \frac{12}{7}$.

(e) Y and Z are not independent of each other, because $\text{Cov}(Y, Z) \neq 0$.

2 (a) $X \sim \text{Bin}(208, \frac{1}{4})$ so $E(X) = \frac{208}{4} = 52$.

(b) $\text{Var}(X) = 208 \times \frac{1}{4} \times \frac{3}{4} = 39$, so X is approximately distributed like Y, where $Y \sim N(52, 39)$. Using the continuity correction,

$$\begin{aligned}
P(50 \leq X \leq 60) &= P\left(49\frac{1}{2} \leq X \leq 60\frac{1}{2}\right) \\
&\approx P\left(49\frac{1}{2} \leq Y \leq 60\frac{1}{2}\right) \\
&= P\left(\frac{49\frac{1}{2} - 52}{\sqrt{39}} \leq \frac{Y - 52}{\sqrt{39}} \leq \frac{60\frac{1}{2} - 52}{\sqrt{39}}\right) \\
&= P(-0.40 \leq Z \leq 1.36) \qquad \text{where } Z \sim N(0,1) \\
&= \Phi(1.36) - \Phi(-0.40) \\
&= \Phi(1.36) - (1 - \Phi(0.40)) \\
&= 0.9131 - (1 - 0.6554) \qquad \text{from Table 4 in } \textit{Cambridge Tables} \\
&= 0.5685.
\end{aligned}$$